Queueing Analysis of an ATM Switch with Multichannel Transmission Groups

by

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Technical Report No. CRI 89-25

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ABSTRACT

The discrete-time $D^{[A]}/D/c/B$ queueing system is studied. We consider both a bulk arrival process with constant bulk inter-arrival time (D) and general bulk-size distribution (A) and a periodic arrival process ($D_1 + \cdots + D_N$). The service/transmission times are deterministic (D) and the system provides for a maximum of c servers with a buffer size B. The motivation for studying this queueing system is its application in performance modeling and analysis of an *asynchronous transfer mode* (ATM) switch with multiple-link transmission groups.

Contents

1 INTRODUCTION

The prevalence of fiber optics and new switching technologies will lead us to a new generation of communication system generally called Broadband Integrated Service Digital Network (B-ISDN) able to support diverse user demands such as data, voice, graphics, video, images, and their possible combinations in a cost-effective manner. ATM will be the target transfer mode (i.e., multiplexing and switching techniques) solution[1] for implementing B-ISDN [20]. ATM is a specific packet-oriented transfer mode using asynchronous (statistical) time division multiplexing and switching mechanism, providing integrated network access as well as an unique basic network transport service. Information is transported in small fixed-length "packets" called *cells* [2] prefixed with a header containing a label that uniquely identifies the virtual channel[3] (VC) over a User-Network Interface (UNI) or Network-Node Interface (NNI). ATM is a connection-oriented technique allowing the network to support both connection-oriented as well as connectionless services [6], [20].

In this paper we investigate a queueing system which is appropriate to evaluate the performance of an ATM UNI (concentrator/multiplexer and/or switch) or NNI (switch). Stream traffic (e.g. voice, FAX, file transfer) can be modeled as the superposition of a set of periodic arrival processes, while more general traffic can be modeled as a bulk arrival process with constant bulk inter-arrival time and general bulk-size distribution. It is assumed that the time axis is equally divided into time slots (discrete-time) and, without loss of generality, all arrivals and departures[4] are synchronized with slot boundaries. This assumption is indeed very reasonable due

[1]Synchronous Optical Network (SONET) provides the target transmission infra-structure.

[2]Most recently, the size of ATM cell has been standardized by CCITT to have a 48-octet information field plus a 5-octet header.

[3]also called virtual connection, virtual circuit, or virtual link by others.

[4]Assume that an arriving cell is transmitted immediately without any delay if there is a free channel.

to the synchronous operation of standardized underlying SONET frame [21] and the fixed cell size of ATM. The transmission time of a cell is one time slot. While most of the previous work has studied the single-channel system [3], [4], [5], [8], [12], [14], [22], this paper analyzes a more general case - multi-channel system[5]. Both infinite and finite buffer systems (for the case of periodic arrival, these correspond to no-loss and loss systems respectively) are considered. The resulting queueing system with bulk arrivals, deterministic inter-bulk arrival time (D), general bulk-size distribution (A), multi-server (c), deterministic service (D), and buffers of size B is then generally called the discrete-time $D^{[A]}/D/c/B$ queueing system. Note that the buffer size B is defined in this paper as the maximum number of cells *excluding* those in transmission that the system can hold.

Much work has appeared in literature on the single-channel (c=1) system. Karol, Hluchyj, and Morgan [14] compared the performance of input versus output queueing on a time-slotted space-division fast packet switch. In their analytical model for queues on outputs, they solved the system with infinite buffer and assuming binomial bulk size. The finite buffer case was studied later by Karol and Hluchyj in [8]. In addition, they considered the problem of using a packet switch for circuit-switched traffic and addressed the tradeoff between the size of buffer required in a switch and the probability that an existing circuit connection is disrupted. Their analysis was based on a discrete-time queueing system with periodic arrival processes. The system was solved by considering all possible arrival patterns (i.e., exhaustive search) [12], [13]. Eckberg, in his early work [4], derived an efficient algorithm for computing the exact delay distribution for the *continuous-time* single server queue with periodic arrivals, deterministic service time and infinite buffer size. More recently, Eckberg and Hou investigated the effects of output buffer sharing on buffer requirements in an ATM packet switch using a discrete-time $D^{[A]}/D/1$ queueing model with binomial distributed bulk size [5]. In [22], Tran-gia and Ahmadi solved a discrete-time

[5]with a single shared queue.

$G^{[X]}/D/1/S$, according to their terminology, queueing system, motivated by its applications in packet-switching systems. Note that for the case of a single-channel with finite buffer their analysis is somehow more general by incorporating general bulk inter-arrival times. Morris proposed an algorithmic method to analyze a discrete-time $D/D/1$ queue with infinite as well as finite buffer applied to packet-switching applications [17]. Most recently, efficient recursive computational algorithms were developed by Cidon and Sidi for analyzing the performance of a single-channel ATM system with periodic arrivals.

The concept of the multichannel transmission group (MCTG) originally comes from System Network Architecture (SNA) of IBM [9], [11]. A transmission group is a set of parallel (SDLC) links joining two adjacent nodes with similar characteristics[6]. These links can be treated as a single *logical* link that has higher bandwidth and also provides better availability/reliability. Information moving through the network is distributed among parallel channels within any MCTG, permitting more efficient use of network resources. The X.25 and X.75 link layer protocols also support the so called multilink procedure (MLP) applying the same concept. This procedure provides for the use of multiple links between network interfaces - signaling terminal exchanges (STEs). While enjoying performance and reliability advantages, the MCTG architecture in conventional packet-switched networks suffers from the sequencing problem caused by having variable length packets, or different transmission rates associated with different channels in a MCTG[7]. To restore the (end-to-end) ordering of packet transfer for any given call, a *resequencing* mechanism which might be implemented on a hop-by-hop and/or an end-to-end basis is required [1], [26]. However, this problem is not present in ATM networks since the cells are fixed length and it is reasonable to assume that all channels in a given MCTG have the same transmission rate (sequenc-

[6]In ATM networks, two adjacent communication nodes (UNI/NNI) are commonly connected by several parallel communication channels.

[7]Transmission errors on one of the channels may also cause out-of-sequence packet delivery.

ing problems due to errors are inherent and are properly resolved on an end-to-end basis in ATM networks due to the very low error rates found in optical fibers) [16].

Recently, Turner proposed the same concept (called link group according to his terminology) to allow dynamic load distribution across a set of communication links by cyclically selecting them in a fast packet switching network [23]. Buffers are dedicated at each input port of the switch fabric. Therefore, a link group is functionally equivalent to a set of single-server queues in parallel. A detailed simulation study is provided in [2]. Hui used the trunk group concept and developed a three-phase algorithm for optimally allocating network resources in ATM networks in [10]. The analysis in his paper concentrated on evaluating the blocking probability within a trunk group at the burst level using the central limit theorem and large deviation theory. In [18], Pattavina introduced a similar concept of channel group to address the problem of bandwidth allocation in a broadband packet switch and showed a feasible hardware implementation in a Batch-banyan switch with input queueing. The channel group can be functionally modeled as a multi-server queue with shared buffers. Some performance results obtained using simulation and simplified analysis were also provided.

In this paper, we analyze the performance of applying the MCTG architecture in an ATM switch with *partially* shared output buffers using the proposed $D^{[A]}/D/c/B$ queueing model. Note that buffers are dedicated to each individual MCTG. A typical system model is shown in figure 1.

The remainder of this paper is organized in the following fashion. In section 2, we present the analysis of the general discrete-time $D^{[A]}/D/c/B$ queueing system. Next, in section 3, we extend the recursive algorithms developed by Cidon and Sidi in [3] to a multi-channel system with N superposed periodic arrival processes ($D_1 + \cdots + D_N/D/c/B$). Numerical results and application to an ATM switch with MCTGs are provided in section 4. Finally, we conclude the paper in section 5.

2 GENERAL ARRIVALS

We first introduce the notation used in this paper. For a discrete-time random variable (r.v.) X, we define the following:

- probability mass function (pmf) : $x(k) \triangleq P[X = k]$
- mean : $\bar{X} \triangleq E[X]$
- variance : $\sigma_X^2 \triangleq Var(X)$
- coefficient of variation : $c_X \triangleq \sigma_X / \bar{X}$
- probability generating function (p.g.f.) : $X(z) \triangleq \sum_{k=-\infty}^{\infty} x(k) z^k$

2.1 Infinite Buffer

We shall assume that the arrival process consists of the superposition of N independent arrival sources. Cells from a particular source, say n $(1 \leq n \leq N)$, arrive at the system (a tagged MCTG under consideration in an ATM switch) according to a random process[8]

$$\mathbf{A^{(n)}} = \{A_i^{(n)} : 1 \leq i < \infty\}$$

where $A_i^{(n)}$ is the r.v. of the number of arrivals to the system at the beginning of time slot i from source n. For any n $(1 \leq n \leq N)$, the random variables $\{A_i^{(n)} : 1 \leq i < \infty\}$ are assumed to be independent and identically distributed (i.i.d.) with known pmf $a_i^{(n)}(k) \triangleq P[A_i^{(n)} = k] = a^{(n)}(k)$ and have finite mean, $\lambda^{(n)} = E[A_i^{(n)}]$, and variance, $\sigma_{A_i}^{(n)^2} = Var(A_i^{(n)})$. The overall arrival process to the system can thus be modeled as a bulk (or batch) arrival process with deterministic bulk inter-arrival time (i.a.t.)

[8]Here we adopt the convention of using subscripts to indicate evolution in time and superscripts to indicate the particular source.

equaling to one slot and general bulk-size distribution $\{A_i : 1 \leq i < \infty\}$. The total number of cells arriving to the system at the beginning of slot i is given by

$$A_i = A_i^{(1)} + A_i^{(2)} + \cdots + A_i^{(N)}$$

Again the random variables $\{A_i : 1 \leq i < \infty\}$ are also i.i.d. with pmf $a_i(k) \triangleq P[A_i = k] = a(k)$ resulting from

$$a(k) = a^{(1)}(k) * a^{(2)}(k) * \cdots * a^{(N)}(k)$$

where the symbol $*$ denotes the discrete convolution operator and have finite mean, $\lambda = E[A_i]$ and variance, $\sigma^2_{A_i} = Var(A_i)$.

Let Q_i be the r.v. of the number of cells waiting in the queue (i.e., the backlog) at the end of slot i. For the moment let us assume infinite buffer. The evolution of the system can thus be described by the following equation:

$$Q_i = [Q_{i-1} + A_i - c]^+ \tag{1}$$

where we employ the notation $X^+ = \max(X, 0)$. This queueing system can be modeled as a discrete-time Markov chain (DTMC). For a stable system (i.e., $\lambda \leq c$), the state probability $q_i(k)$ of Q_i converges to an equilibrium (steady state) pmf $q(k)$.

We now introduce the generating functions:

$$A(z) = \sum_{k=-\infty}^{\infty} a(k) z^k$$

$$Q(z) = \sum_{k=-\infty}^{\infty} q(k) z^k$$

where $A(z)$ and $Q(z)$ are analytic in the unit disk $|z| < 1$.

We first multiply equation (1) by z^k and sum over the index k assuming the system at steady state (i.e. $i \to \infty$). After some manipulation, this leads to

$$Q(z) = \frac{\sum_{m=0}^{c-1} r_m (z^c - z^m)}{z^c - A(z)} \tag{2}$$

Now we are in a position to get rid of the c unknown terms, $\{r_m\}_{m=0}^{c-1}$, in the numerator of equation (2). This can be done by invoking the requirement that $Q(z)$ is analytic in $|z| < 1$ and the fact that $Q(1) = 1$. Using Rouche's theorem, it can be shown that the equation

$$z^c - A(z) = 0 \tag{3}$$

has exactly $(c-1)$ complex zeros inside $|z| \leq 1$ while the c^{th} is clearly $z = 1$; we denote these zeros by $\theta_1, \ldots, \theta_{c-1}$ and $\theta_c = 1$[9]. The analyticity of $Q(z)$ requires that all the zeros of the denominator within and on the unit circle must also make the numerator vanish. That is, $\theta_1, \ldots, \theta_{c-1}$ and $\theta_c = 1$ are the coincident roots of the denominator and numerator in (2). Therefore, applying the fact that $Q(1) = 1$ and carrying out the necessary manipulation, we rewrite equation (2) as

$$Q(z) = \frac{(c-\lambda)(z-1)}{z^c - A(z)} \prod_{m=1}^{c-1} \frac{z-\theta_m}{1-\theta_m} \tag{4}$$

where

$$\lambda = \left.\frac{dA(z)}{dz}\right|_{z=1}$$

The queue size distribution, $\{q(k)\}_{k=0}^{\infty}$, may now be theoretically obtained by

$$q(k) = P[Q = k] = \left.\frac{d^k Q(z)}{dz^k}\right|_{z=0}$$

although this frankly gets to be rather difficult as c gets large. With some tedious computations, we can, however, find the mean and variance of the queue size:

$$\bar{Q} = \frac{A''(1) - c(c-1)}{2(c-\lambda)} + \sum_{m=1}^{c-1} \frac{1}{1-\theta_m} \tag{5}$$

$$\begin{aligned} \sigma_Q^2 = & \left\{\frac{2}{3} + \frac{2[A''(1) - c(c-1)]}{c-\lambda}\right\} \sum_{m=1}^{c-1} \frac{1}{1-\theta_m} \\ & + \frac{A'''(1) - c(c-1)(c-2)}{3(c-\lambda)} + \frac{[A''(1) - c(c-1)]^2}{2(c-\lambda)^2} + \bar{Q} - (\bar{Q})^2 \end{aligned} \tag{6}$$

where

$$A''(1) = \left.\frac{d^2 A(z)}{dz^2}\right|_{z=1} = E[A_i^2]$$

[9]This is similar to the analysis for $M/D/c$ [7].

$$A'''(1) = \left.\frac{d^3A(z)}{dz^3}\right|_{z=1} = E[A_i^3]$$

The average waiting time of any cells in the queue is obtained by applying the well known Little's result[10]

$$\bar{W} = \frac{\bar{Q}}{\lambda} \tag{7}$$

Example: single-server with binomial bulk size

Now, considering a single-server (c=1) system, $Q(z)$ becomes

$$Q(z) = \frac{(1-\lambda)(z-1)}{z - A(z)}$$

If A_i has the binomial pmf

$$a(k) = P[A_i = k] = \begin{pmatrix} N \\ k \end{pmatrix} p^k (1-p+pz)^{N-k} \qquad 0 \le k \le N$$

and its corresponding p.g.f.

$$A(z) = (1-p+pz)^N$$

where p $(=\lambda)$ is the probability that a cell will arrive at the system from any source in any given time slot. After some algebra, we find the mean and variance of queue size for $c = 1$ case from (5) and (6) as follows:

$$\bar{Q} = \frac{N(N-1)p^2}{2(1-p)} \tag{8}$$

$$\sigma_Q^2 = \frac{N(N-1)(N-2)p^3}{3(1-p)} + \frac{N(N-1)p^2}{2(1-p)} + \frac{[N(N-1)]^2p^4}{4(1-p)^2} \tag{9}$$

These are identical to the results in [5] and [14].

[10]Because all moments of the queue size and arrival bulk-size may be obtained from (4) and $A(z)$, which, in turn, enable us to find all moments of the waiting time via the extension of Little's result [7].

2.2 Finite Buffer

In this subsection, we consider the same queueing system except the buffer space is now assumed to be finite with size B ($B < N - c$). The system evolves according to the following equation:

$$Q_i = \min\{[Q_{i-1} + A_i - c]^+, B\} \tag{10}$$

The system state Q_i constitutes a stationary finite-state DTMC with state transition probabilities $p_{kl} \triangleq P[Q_{i+1} = l | Q_i = k]$ given by

$$p_{kl} = \begin{cases} \sum_{n=0}^{c-l} a(n) & k = 0, 0 \le l \le c \\ a(c + k - l) & 1 \le k \le B - 1, 0 \le l \le c + k \\ \sum_{n=B+c-l}^{N} a(n) & k = B, 0 \le l \le B \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Both transient and steady (equilibrium) state probabilities can be obtained [15]. In this paper, we focus on the steady state probabilities which we find by numerical solution techniques such as LU decomposition together with forward substitution and backsubstitution, or Gaussian elimination with backsubstitution [19]. The average queue length, $\bar{Q}$, can then be determined by

$$\bar{Q} = \sum_{k=1}^{B} k \cdot q(k)$$

where $q(k)$ is the steady state probability found above.

To obtain the probability of cell loss, ϕ, we observe a tagged cell contained in an arriving bulk of size n. The probability of the tagged cell being in bulk of size n is $n \cdot a(n)/\lambda$. The conditional probability of cell loss given that the queue size is k upon arrival, ϕ_k, is thus

$$\phi_k = \sum_{n=B+c+1-k}^{N} P[\text{the tagged cell is lost} \mid \text{arriving batch-size} = n]$$

$$\cdot P[\text{the tagged cell in an arriving bulk of size } n]$$

$$= \sum_{n=B+c+1-k}^{N} \frac{(n+k-B-c)}{n} \cdot \frac{n \cdot a(n)}{\lambda} \tag{12}$$

Then, by the theorem of total probability, we have

$$\phi = \frac{1}{\lambda} \sum_{k=0}^{B} q(k) \sum_{n=B+c+1-k}^{N} (n+k-B-c) \cdot a(n) \tag{13}$$

So the (effective) waiting time, $\bar{W}^*$, averaged over cells admitted into the queue is given by

$$\bar{W}^* = \frac{\bar{Q}}{\lambda^*}$$

where

$$\lambda^* = \lambda(1-\phi)$$

λ^* is the effective arrival rate of the system defined as the rate of cells actually entering the system. For the case of single-server (c=1), the results presented here correspond to the result in [8].

3 PERIODIC ARRIVALS

In this section, we consider the same queueing system but with periodic arrival process which can then be modeled as a discrete-time $D_1 + \cdots + D_N/D/c/B$ queueing system. An efficient computational algorithm for analyzing the performance of the queueing system is presented by extending the methods developed in [3] to multiple channels.

3.1 No Loss System

Following the approach in [3], a *frame* is defined to be any F consecutive time slots. The arrival process consists of N $(D_1, \cdots, D_N)$ independent periodic arrival sources and each source generates exactly one arrival per frame. For a stable system, the obvious condition $N \leq c \cdot F$ must hold. In this subsection, the system is assumed to be capable of storing $N - c$ cells (excluding those cells in transmission) to guarantee no loss. Let N_i be the number of arrivals at the beginning of slot i and Q_i be the number of cells waiting in the queue at the end of slot i. For a given set of arrival sources, the N cells arrive in the same pattern and can be described by [3], [12],

$$N_i = N_{jF+i} \qquad 1 \leq i \leq N, \;\; j = 1, 2, \cdots$$

As in the last section, the system equation is:

$$\begin{aligned} Q_0 &= 0 \\ Q_i &= [Q_{i-1} + A_i - c]^+ \end{aligned} \tag{14}$$

The resulting queue size can be shown to be periodic by extending lemma 2 and corollary 1 of [3]. Therefore, we only need to calculate q_i's for $F \leq i \leq 2F - 1$ using the system evolution equation given in (14). In addition, it can be easily proved that the system reaches the steady-state periodic behavior for any given set of arrival sources after at most N time slots (one frame). Applying the approach presented in [3], we construct an alternate system by enforcing $Q_{j-F} = 0$ for an arbitrary slot j ($F \leq j \leq 2F - 1$). Based on theorem 1 of [3], we can solve the system using the alternate (disconnected) system instead of the original cyclic system.

In this paper, we generalize the algorithms given in [3] to the multi-channel case. It is assumed that the N arriving cells are uniformly distributed over a frame of length F. Let $a_{F,N}(k)$ be the probability that k out of the N cells arrive at the beginning of any slot in a frame of size F and $q_{F,N}(k)$ be the probability that a typical slot in the system has a queue size of k. We thus have

$$a_{F,N}(k) = \binom{N}{k} \left(\frac{1}{F}\right)^k \left(1 - \frac{1}{F}\right)^{N-k} \qquad 0 \leq k \leq N \tag{15}$$

The set of recursive computational equations for calculating the queue size pmf of a no-loss $D_1 + \cdots + D_N/D/c/B$ queueing system is then given by

$$q_{F,0}(k) = 0 \qquad F \geq 1,\ k \geq 1 \tag{16}$$

$$q_{F,N}(k) = 0 \qquad F \geq 2,\ k \geq 1,\ 1 \leq N \leq k \tag{17}$$

$$q_{1,N}(k) = \begin{cases} 1 & k = N - c,\ N > c \\ 1 & k = 0,\ 0 \leq N \leq c \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$q_{F,N}(k) = \begin{cases} 1 & k = 0,\ N \leq c \\ 0 & k \neq 0,\ N \leq c \end{cases} \tag{19}$$

$$q_{F,N}(0) = \sum_{n=0}^{c} a_{F,N}(n) \left[\sum_{k=0}^{c-n} q_{F-1,N-n}(k) \right] \qquad F \geq 2,\ N \geq c \tag{20}$$

$$q_{F,N}(k) = \sum_{n=0}^{k+c} a_{F,N}(n) q_{F-1,N-n}(k - n + c) \qquad F \geq 2,\ 1 \leq k \leq N - c \tag{21}$$

Note that $q_{F,N}(k) = 0$ whenever $N > c \cdot F$. Now the average queue size, $\bar{Q}_{F,N}$, is simply

$$\begin{aligned} \bar{Q}_{F,N} &= \sum_{k=1}^{N-c} k q_{F,N}(k) \\ &= \sum_{n=0}^{N} a_{F,N}(n) \bar{Q}_{F-1,N-n} + \frac{F}{N} - c + c \left[\sum_{n=0}^{c-1} a_{F,N}(n) \sum_{k=0}^{c-n-1} q_{F-1,N-n}(k) \right] \end{aligned} \tag{22}$$

We can apply Little's result to obtain the average waiting time in the queue, $\bar{W}_{F,N}$, as

$$\bar{W}_{F,N} = \frac{F}{N} \bar{Q}_{F,N}$$

To find the waiting time pmf, we consider a tagged cell arriving at the system that has frame size F and $N-1$ arrivals together with N_1 other cells (i.e., bulk size$=N_1+1$) out of $(N-1)$ arriving cells in slot N. The position of the tagged cells in this arriving bulk is assumed to be uniformly distributed over the bulk of size $N_1 + 1$. The tagged cell has to wait for the transmission of those cells ahead of him within the same bulk, N_2, plus all cells backlogged from the previous slot, Q_{N-1}. If $Q_{N-1} + N_1 \leq c - 1$, there is no waiting for the tagged cell. Otherwise, the tagged cell will wait for k slots provided that $kc \leq Q_{N-1} + N_2 \leq (k+1)c - 1$ where $1 \leq k \leq \lfloor \frac{N-1}{c} \rfloor$. The pmf of the waiting time, $W_{F,N}(k)$, of a typical cell can thus be found as follows:

$$W_{F,N}(k) = \sum_{N_1=0}^{N-1} \frac{a_{F,N}(N_1)}{N_1+1} \sum_{N_2=0}^{N_1} \sum_{l=kc-N_2}^{(k+1)c-1-N_2} q_{F-1,N-N_1-1}(l) \tag{23}$$

3.2 Loss System

We now proceed to consider the case of a loss system in which the maximum number of cells (excluding those cells in transmission), B (buffer size), that may be stored is less than $N - c$. When an arriving cell from a particular source is lost, all cells from this source will be lost [24], [25]. In this paper, we are interested in determining the loss probability, $\phi_{F,N}$, of a typical cell for a specific set of arriving sources.

The analysis of the loss system follows the same procedure as the no-loss system presented in last subsection except equation (18) becomes

$$q_{1,N}(k) = \begin{cases} 1 & k = 0,\ 0 \leq N \leq c \\ 1 & k = N - c,\ c \leq N \leq c + B \\ 1 & k = B,\ N \geq c + B \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

and equation (21) is still valid for $k < B$. In addition, we have

$$\begin{aligned} q_{F,N}(B) &= 0 \qquad F \geq 2,\ N \leq c + B \\ q_{F,N}(B) &= \sum_{n=c}^{B+c-1} a_{F,N}(n) q_{F-1,N-n}(B + c - n) + \sum_{n=B+c}^{N} a_{F,N}(n) \qquad F \geq 2 \end{aligned} \tag{25}$$

Adopting the same argument as for computing the waiting time pmf in previous subsection, we are now in a position to evaluate the loss probability, $\phi_{F,N}$, as follows:

$$\phi_{F,N} = \sum_{l=0}^{B} \sum_{n=l+c}^{N-1} \frac{(n+1-l-c)a_{F,N-1}(n)}{n+1} q_{F-1,N-n-1}(B-l) \tag{26}$$

The effective arrival rate, λ^*, is thus

$$\begin{aligned}\lambda^* &= \lambda(1-\phi_{F,N}) \\ &= \frac{N}{F}(1-\phi_{F,N})\end{aligned}$$

So the average waiting time in the queue, $\bar{W}^*_{F,N}$, averaged over those cells admitted into the system is given by:

$$\bar{W}^*_{F,N} = \frac{\bar{Q}_{F,N}}{\lambda^*}$$

where

$$\bar{Q}_{F,N} = \sum_{k=1}^{B} k \cdot q_{F,N}(k)$$

4 NUMERICAL RESULTS

We demonstrate some numerical results for various performance measures of interest in this section. The system model of an internally non-blocking ATM switch with partially shared output buffers employing the MCTG architecture is shown in figure 1. We focus on a tagged output MCTG with c channels having N out of N_{total} superposed input processes. Both input and output traffics are assumed to be balanced[11]. For the arrival process, a binomial distributed bulk-size is assumed[12]. Each input source generates a cell in any time slot with probability p. Figure 2 shows the average waiting time as a function of the offered load per input source (p) under the infinite buffer assumption for various cases such that the number of arriving sources is twice the number of servers ($N = 2c$). The scaling advantage favoring a larger c is clearly shown. For the finite buffer case, we plot the loss probability as a function of B and N for the cases of 4 channels with $p = 0.25$ and 8 channels with $p = 0.5$

[11]It is assumed that the N input processes are i.i.d. and each input cell is destined for any output port with equal probability.

[12]More general bulk-size distribution and the effects of traffic burstiness can also be taken in account without much difficulty.

in figure 3. Note that for a given loss probability, the 8-channel system requires less buffers than the 4-channel one (for constant utilization Np/c). In figure 4, we show the cell loss probability against the offered load per source p for the 4-channel system with 10 sources and the 8-channel system with 20 sources, and two different values of B. Again, for a given loss probability and buffer size, we observe that the 8-channel system can support higher traffic load per input channel than its 4-channel counterpart[13].

For the periodic input traffic, figures 5 and 6 show the waiting time and the cell loss probability respectively as a function of the normalized buffer size B (i.e., the average size of buffers per channel) for a fixed frame size, $F = 10$, and various c under the same loading condition (80%). Note that in figure 5 the waiting times become fixed and there is no loss for $B > N - c$. We find similar performance advantages as in non-periodic input traffic case. The larger the number of channels, the greater the performance improvement.

5 CONCLUSIONS

In this paper, we have analyzed the performance of an ATM switch employing a MCTG architecture based on the discrete-time $D^{[A]}/D/c/B$ queueing model. For input traffic with general bulk-size distribution, we consider both infinite and finite buffer cases. For a periodic arrival process we extended the results of [3] to a multi-channel system for both loss and no-loss systems.

As indicated and quantified in [8] and [14], both output queueing and completely shared buffering achieve the optimal delay-throughput performance in an ATM switch with single-channel transmission groups because cells are delayed or lost only when

[13]In other words, the 8-channel system can support a greater number of sources than two 4-channel systems could.

two or more cells from different inputs simultaneously arrive at the same output. This delay and/or loss probability can be further reduced by using the proposed MCTG architecture in an ATM switch with partially shared output buffers, especially under the condition of (transient) imbalanced input and/or output traffic. The MCTG concept provides the additional advantages of super-rate switching which can naturally support services with peak bandwidth requirement exceeding the capacity of any single link, better link reliability/availability, less total buffer requirement and efficient bandwidth allocation.

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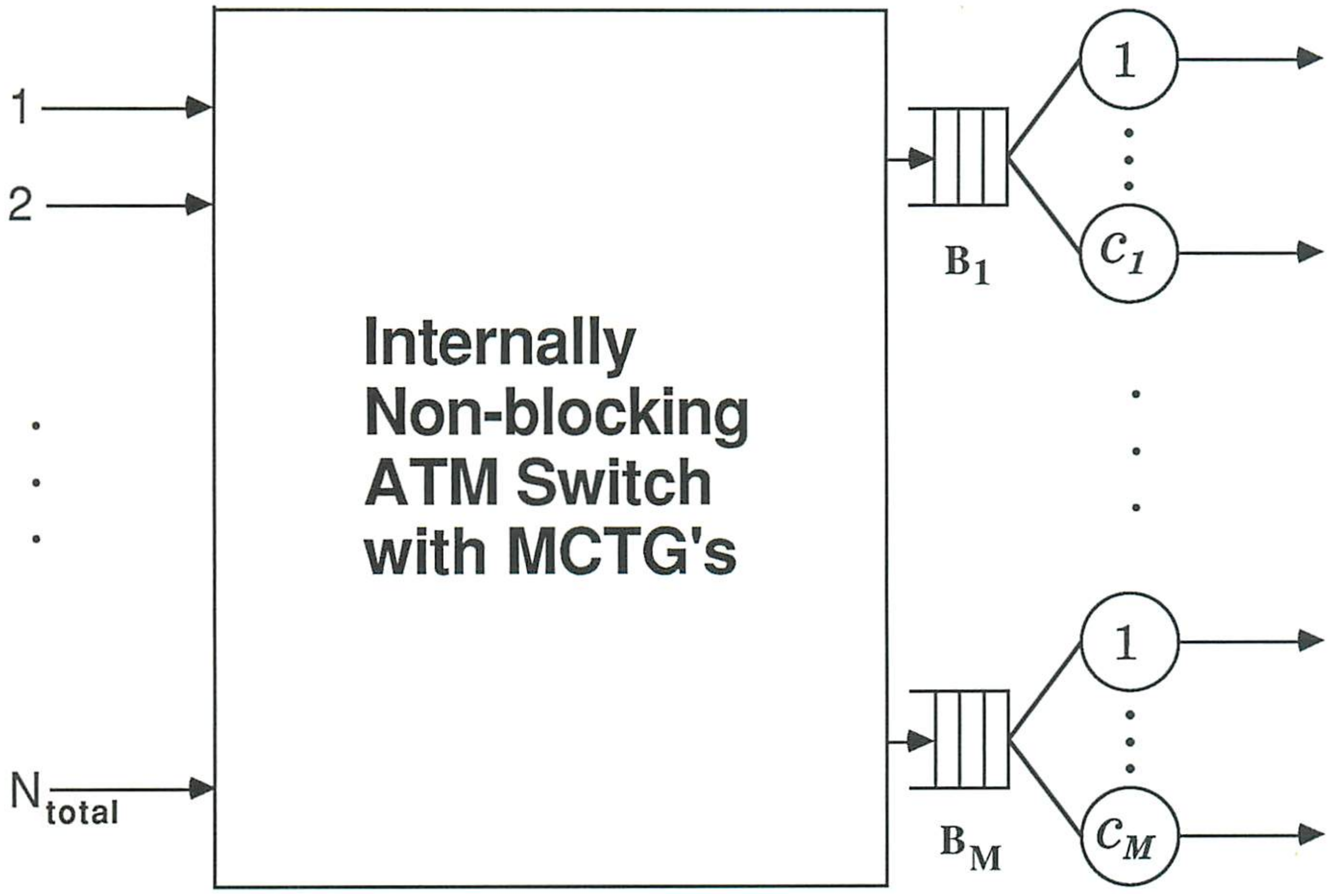

Fig. 1: The system model of an internally nonblocking ATM switch with multichannel transmission groups (MCTG).

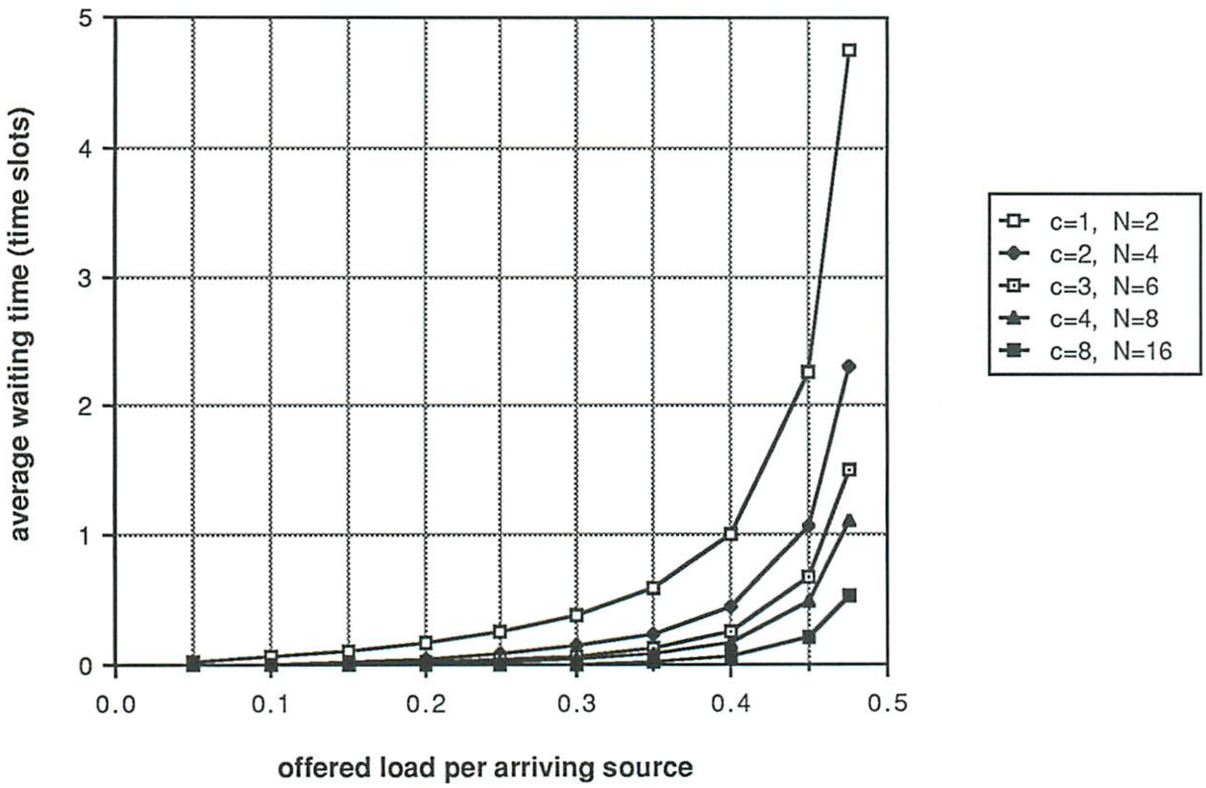

Fig. 2: The average waiting time for various cases of N=2c assuming infinite buffer.

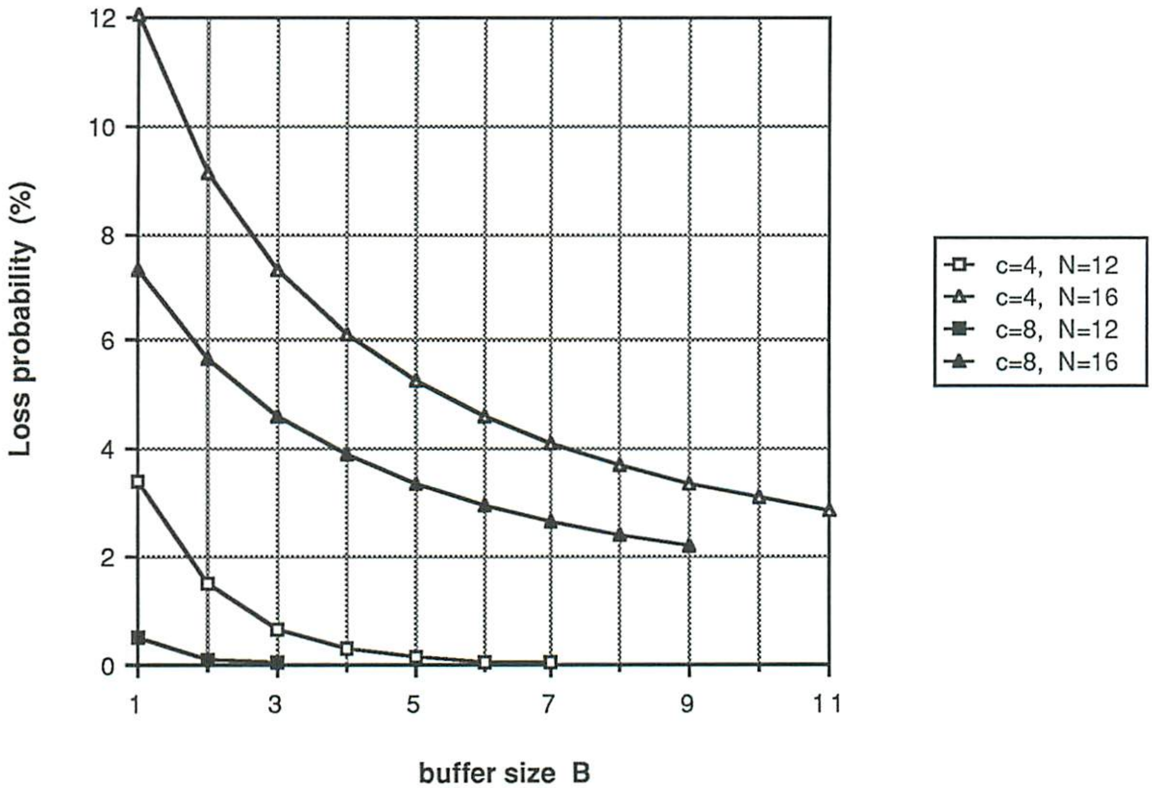

Fig. 3: The loss probability as a function of B and N, for c=4, p=0.25 and c=8, p=0.5.

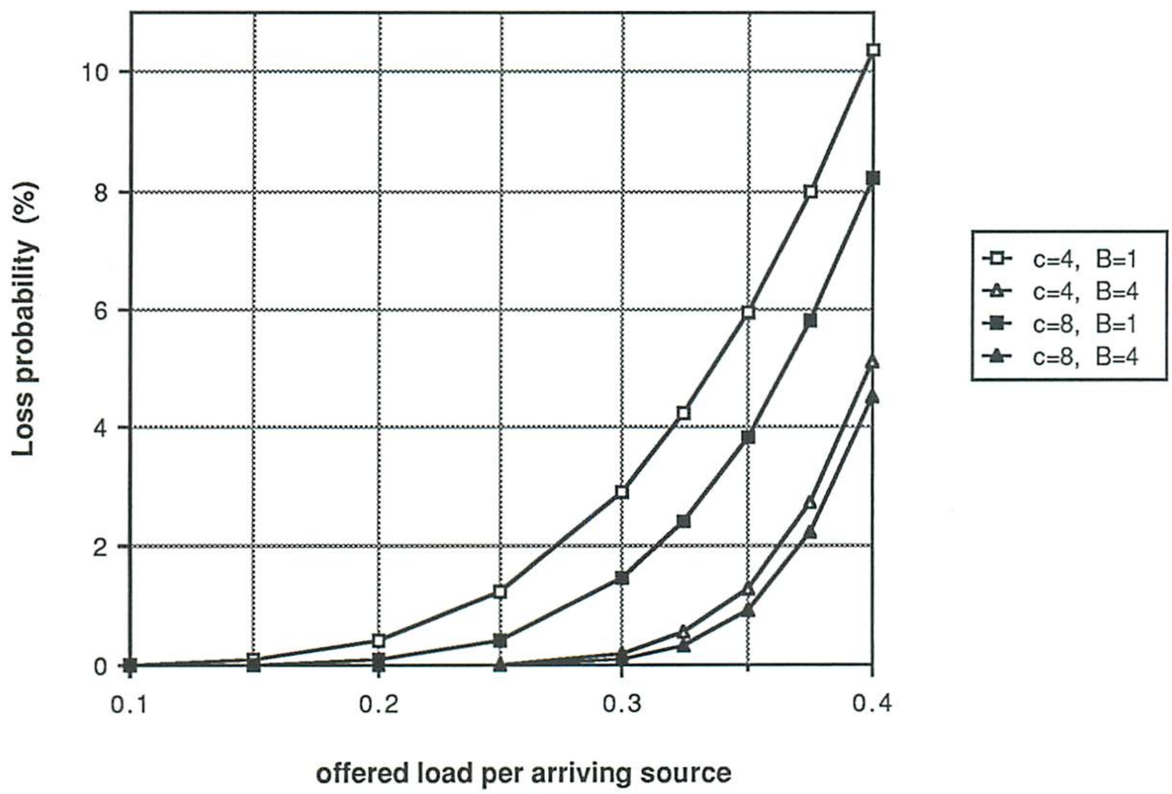

Fig. 4: The loss probability for c=4,N=10 and c=8,N=20, and various B.

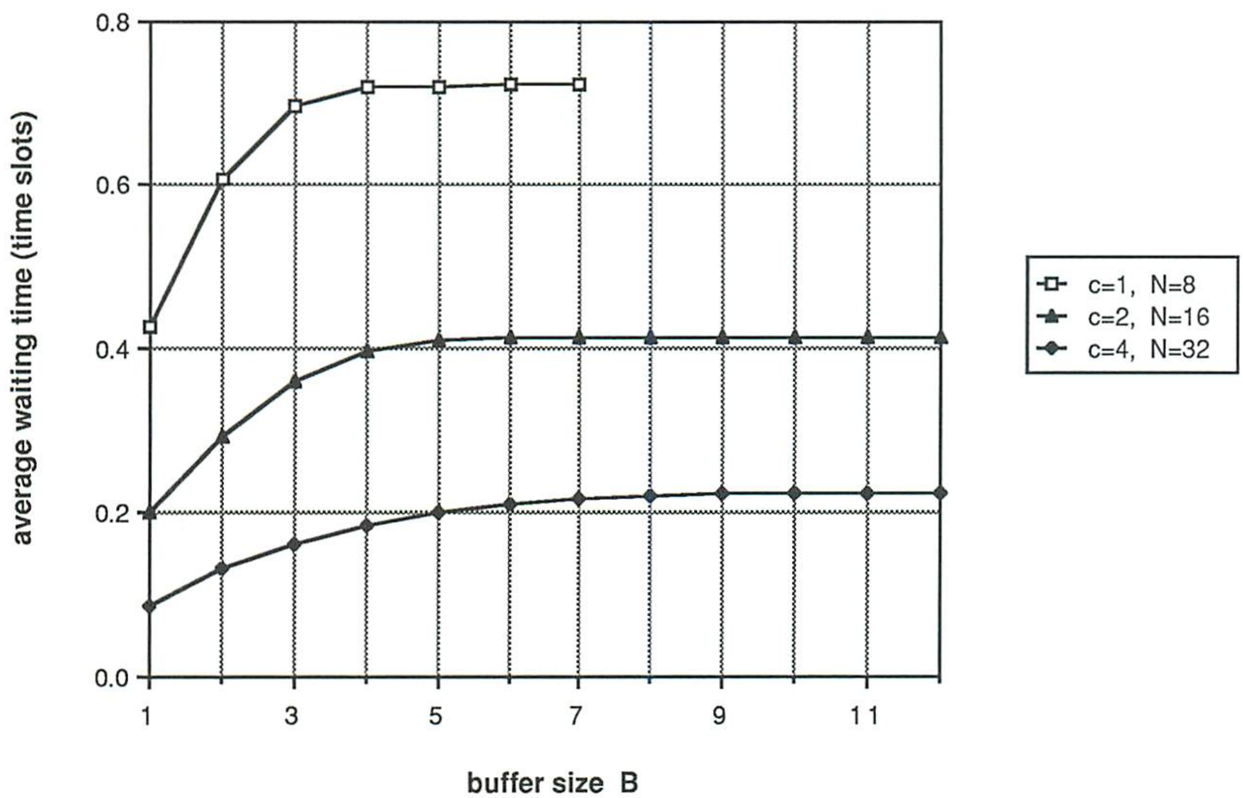

Fig. 5: The average waiting time as a function of B for F=10 and various c.

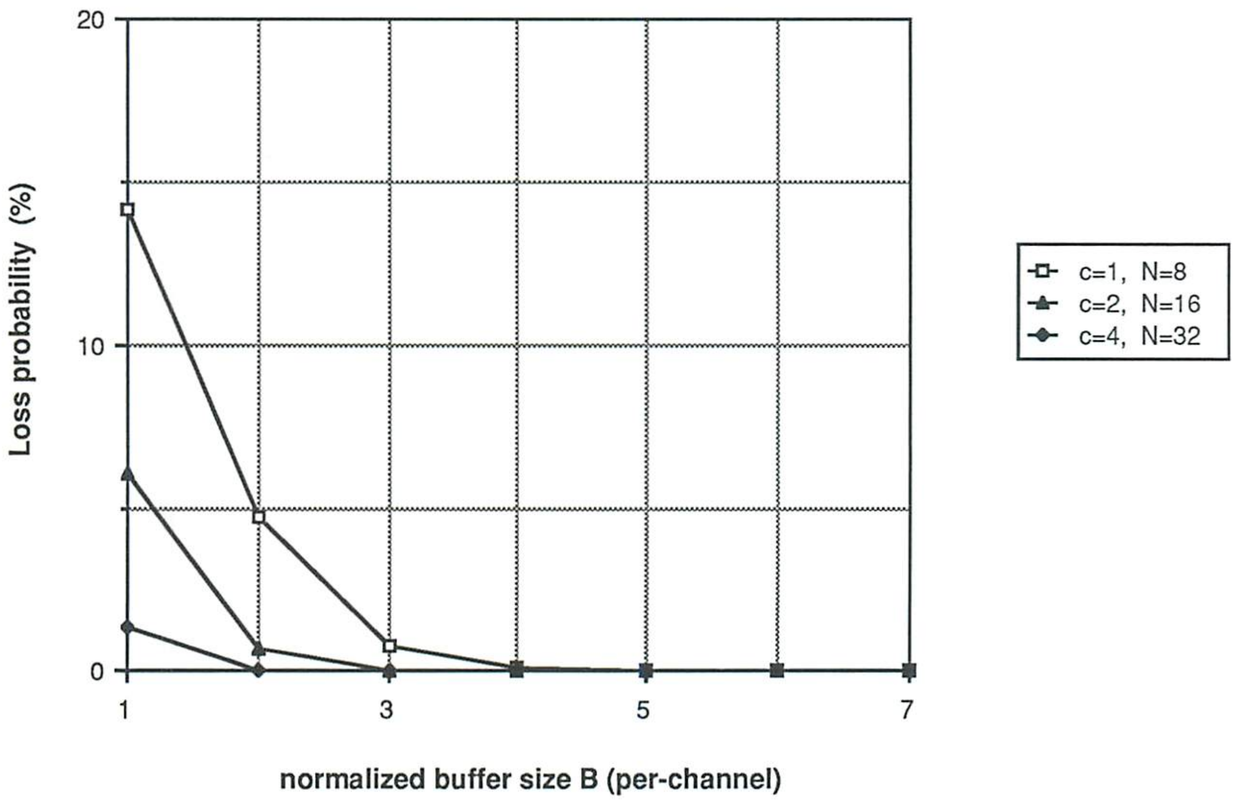

Fig. 6: The loss probability as a function of normalized B for F=10 and various c.