Synthesis of Application-Specific Multiprocessor Architectures

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Abstract

This paper describes a formal technique for automated synthesis of multiprocessor systems for given applications. The application task is specified in terms of a graph, and the architecture synthesized includes a set of processing elements and the interconnection architecture between them. The technique generates a task execution schedule along with the architecture. The technique involves creation of a Mixed Integer-Linear Programming (MILP) model and solution of the model. A primary component of the model is the set of relations that must be satisfied to ensure proper ordering of various events in the task execution as well as to ensure completeness and correctness of the system. Some example architectures have been synthesized, and these results are reported.

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1 Introduction

Application-specific systems have become so complex that system-level design decisions cannot be made without the aid of computer tools. As system complexity and size have increased, designers have relied increasingly on analysis techniques like simulation and queueing models for assistance during the design process. However, in most cases, system design decisions have been left to the human designer, who often uses a "generate and test" approach to confirm the validity of his or her decisions. There is a growing need to develop tools for system-level design. This paper addresses a technique for design of multiprocessor systems for given applications.

Our focus is on the design of the system architecture, which is the first step in the design of an application-specific multiprocessor system. We assume the application domain is specified in terms of a task data flow graph. The task data flow graph specifies a set of subtasks (nodes in the graph) that need to be performed and the data precedence between them (arcs in the graph). Given the task data flow graph, the goal is to synthesize a multiprocessor architecture which meets various cost and performance requirements and constraints. Synthesizing an architecture involves making decisions about the number and types of processing elements selected, the overall interconnection between the processing elements, and the scheduling of subtasks on the processing elements.

This paper describes a new approach which is aimed at producing a custom multiprocessor architecture, as well as mapping the subtasks onto the architecture and providing a schedule for the task execution. *This research focuses on the automatic design of the multiprocessor architecture itself, not merely the mapping of tasks onto a given architecture.* A distinguishing feature of the research is the fact that we are designing a truly heterogenous system, in terms of the functionality and the cost-speed characteristics of the processing elements, which allows a more precise tailoring of the synthesized architecture to a specific application. Also, our approach can be used to explore different interconnection styles; e.g., bus, point-to-point, ring, or a mixture of these. We assume there is no global clock and communications between subtasks are asynchronous. With the exception of some early work by Talukdar and Mehrotra [9], we believe this is the first publication to describe a solution to this problem.

2 Previous Related Research

Our research builds on past research done by others in the areas of multiprocessor task allocation and data path synthesis. Most of the previous related work on multiprocessors is directed at the problem of task allocation for a given architecture.

The assignment of tasks to a fixed multiprocessor system was inspired by Stone's work on the two-processor problem [10]. More recently, Chu et. al. [2] have described an integer 0-1 programming approach to the problem of task allocation in distributed data processing. The problem they have considered involves the allocation of a set of m subtasks to a set of p (fixed) processors already interconnected in some fashion. Their model does not consider the data precedence relations among the subtasks. Houstis [6] describes task allocation for real-time applications with concurrent selection of the optimal number of processing elements. She does consider data precedence, but assumes identical processing elements and does not consider timing constraints. Indurkhya et.al.[8] use random models for distributed programs, and confirm intuitive results for the 2-processor and n-processor cases. Haddad[3] described a load allocation problem solved with continuous partition sizes to minimize total execution time. An early study mapping algorithms such as the Fast Fourier Transform onto the CM* multiprocessor in order to meet real time constraints was undertaken by Brantley [1]. Talukdar and Mehrotra [9] describe a problem which is a simplified and similar version of our problem. They also model the problem using mathematical programming, though they use heuristics to solve it.

Mathematical programming has also been applied to the data path synthesis problem. Hafer and Parker [4] used a mixed-integer linear programming approach to automatically synthesize register-transfer level datapaths, given a data flow/control flow graph description of the hardware. The approach involves developing various timing relationships to be satisfied, but does not include interconnection styles or delays, and does not consider the detailed timing of multiple outputs. Hwang et. al. [7] have described an integer linear programming model for the scheduling problem in data path synthesis under resource constraints, and they present a heuristic technique called *Zone Scheduling* for solving large size problems.

3 The Problem Definition

We are addressing the problem of multiprocessor architecture synthesis for a given application task. Our approach involves creation of a formal model of the multiprocessor synthesis problem and the solution of this model, using mathematical programming.

The task consists of a set of subtasks. Each subtask requires certain input data and produces certain output data. Inputs to a subtask may come from other subtasks and outputs from a subtask may go to other subtasks. The set of subtasks and the input-output relationships among them can be expressed by a task data flow graph as shown in Figure 1. The subtask nodes are labeled S_1, S_2, etc. (S_a in general). The input end of a data arc is labeled $i_{a,b}$ if it provides b^{th} input to subtask S_a, and the output end is labeled $o_{a,c}$ if it transmits the c^{th} output from the subtask S_a. Although we represent our task by a data flow graph, there is a subtle distinction between our meaning and the traditional meaning attached to a data flow graph. With the traditional meaning, a subtask would require all the inputs before starting its execution and none of the outputs would be available until after its exection is over. However, in our model subtasks do not require all the inputs before starting their execution and they may produce some outputs even before their completion. To express this possibility, each input $i_{a,b}$ has a parameter $f_R(i_{a,b})$ associated with it which specifies that up to $f_R(i_{a,b})$ fraction of the subtask S_a can proceed without requiring the input $i_{a,b}$. Similarly, each output $o_{a,c}$ has a parameter $f_A(o_{a,c})$ associated with it which specifies that the output $o_{a,c}$ becomes available when $f_A(o_{a,c})$ fraction of the subtask S_a is completed.

The multiprocessor architecture is specified in terms of the processors selected and the interconnection architecture between them. A simple example multiprocessor system is shown in Figure 2. The specific model described in this paper assumes point-to-point interconnection; i.e., if a processor p_{d1} needs to send data to another processor p_{d2}, then there must be a direct communication link from p_{d1} to p_{d2}. However, it must be emphasized that the approach is capable of handling other styles of interconnection.

For each subtask S_a, a set P_a represents the set of processors capable of executing it. However, only one processor actually performs the subtask in the synthesized architecture, and the execution time for the subtask depends on the processor on which it is performed. A parameter, denoted as $D_{pS}(p_d, S_a)$, specifies the execution time for the subtask S_a if the processor p_d is selected to perform it.

A data arc from node S_{a1} to node S_{a2} implies that some data is transferred from the subtask S_{a1} to the subtask S_{a2}. The volume of data transferred varies from arc to arc, and a parameter $V_{a1,a2}$ specifying the volume is associated with each arc. The data transfer maybe a *remote transfer* (if the two subtasks are mapped to different processors in the synthesized system), where the data is transferred from a proccesor to another; or it maybe a *local transfer* within the same processor (if the two subtasks are mapped to the same processor). Delay associated with a data transfer depends on whether it is a remote transfer or a local transfer. Local transfer delay could be negligible compared to the remote transfer delay. In any case, the local transfer delay is represented by the parameter D_{CL} which specifies the time taken in transferring a unit volume of data locally. The remote transfer delay is represented by the parameter D_{CR} which specifies the time taken in transferring a unit volume of data remotely.

A set P represents the set of all the processors available for selection as part of the synthesized architecture, where

$$P = \bigcup_{a} P_a$$

Associated with each processor $p_d \in P$ is a parameter C_d which specifies the cost of the processor. Another cost parameter is C_L which specifies the cost of creating a communication link between two processors.

Certain constraints related to the cost of the system as well as timing of arbitrary events may also be specified. In summary, the following are the problem inputs and outputs:

- Problem inputs:
  - A task data flow graph specifying the overall application task, along with the associated parameters
  - A set of processing elements with varying functionality, cost and performance
  - Communication link characteristics: cost and performance
  - Constraints on total system cost
  - Constraints on timing of arbitrary events
- Problem outputs:
  - A multiprocessor architecture, including
    - the chosen set of processing elements
    - the interconnection style for the elements
  - A schedule for the subtasks
  - Detailed timing information for computation and transfer of data

4 The Problem Approach: Formal Model

Our approach is a natural outgrowth of the work described by Chu [2], Talukdar [9], and Hafer [4]. We are using mathematical programming to produce a formal model for the problem. Such a mathematical model allows us a deep understanding of the problem and allows us to verify our software more easily, even if future run-time problems with larger examples force us to resort to heuristics. Such an approach allows us to modify, extend and enhance the model to include more design possibilities and variations without significant reconstruction of existing code. Also, the approach offers a great degree of flexibility in handling arbitrary constraints.

4.1 The Model

A complete mathematical programming formulation of the problem requires specification of an objective function that has to be optimized and a set of constraints that have to be satisfied. The objective function can be whatever the designer wishes; e.g., the total system cost, or the overall system performance. The set of constraints consists of the constraints that must be satisfied for the overall task to be performed correctly as well as the arbitrary timing and cost constraints imposed by the designer.

4.1.1 The Constraints

The constraints that must be satisfied for the overall task to be performed correctly consist primarily of the relations that ensure proper ordering of the subtasks and the data transfers taking into account the timing involved in carrying them out and the relations that express the conditions for complete and correct system configuration. In order to express the various constraints, one needs to define certain variables related to the system. The necessary variables fall into two basic categories:

- *Timing variables:* These are real variables which represent timings of various critical events in the operation of the system. There are three classes of timing variables defined:

- *Data availability timing variables:*
    - * *Input data availability, $T_{IA}(i_{a,b})$:* Time when the data required by input $i_{a,b}$ of subtask S_a is available for use.
    - * *Output data availability, $T_{OA}(o_{a,c})$:* Time when the output data value $o_{a,c}$ computed by subtask S_a has become available.
- *Subtask execution timing variables:*
    - * *Subtask execution start, $T_{SS}(S_a)$:* Time when the execution of subtask S_a actually begins.
    - * *Subtask execution end, $T_{SE}(S_a)$:* Time when the execution of subtask S_a is completed.
- *Data transfer timing variables:*
    - * *Data transfer start, $T_{CS}(i_{a,b})$:* Time when the communication/transfer of the data required by input $i_{a,b}$ of subtask S_a actually begins.
    - * *Data transfer end, $T_{CE}(i_{a,b})$:* Time when the communication/transfer of the data required by input $i_{a,b}$ of subtask S_a ends.

- *Binary variables:* These are 0-1 variables which represent the implementation decisions regarding the system configuration. There are two types of binary variables defined:
    - *Subtask-to-processor-mapping variable, $\sigma_{d,a}$:* The variables of this type specify the mapping between the subtasks and the processors. $\sigma_{d,a} = 1$ indicates processor p_d will implement subtask S_a.
    - *Data-transfer-type variable, $\gamma_{a1,a2}$:* The variables of this type specify the data transfer type for the various data arcs. $\gamma_{a1,a2} = 1(0)$ indicates that data transfer from subtask S_{a1} to subtask S_{a2} is a remote (local) transfer.

The necessary constraints have been classified into ten categories:

- *Processor-selection constraint:* For each subtask S_a, a set of processors P_a is available to implement it. In order for the implementation to be correct, one and only one processor should be selected to implement the subtask. Thus, for each subtask S_a, the following must be satisfied:

$$\sum_{d|p_d \in P_a} \sigma_{d,a} = 1 \tag{4.1.1}$$

- *Data-transfer-type constraint:* $\gamma_{a1,a2}$ is a variable which indicates whether the data transfer from the subtask S_{a1} to the subtask S_{a2} is a local transfer or a remote transfer. Now, if the subtasks S_{a1} and S_{a2} are mapped to the same processor (say p_d, where $p_d \in P_{a1}$ and $p_d \in P_{a2}$), then we know that it is a local transfer, and thus $\gamma_{a1,a2} = 0$. However, if they are mapped to different processors, then the data transfer is remote, and thus $\gamma_{a1,a2} = 1$. Thus, the defining equation for $\gamma_{a1,a2}$ is:

$$\gamma_{a1,a2} = 1 - \sum_{d \mid p_d \in P_{a1} \wedge p_d \in P_{a2}} \sigma_{d,a1}\sigma_{d,a2} \tag{4.1.2}$$

  We will have such an equation for each pair of subtasks communicating with each other.

- *Input-availability constraint:* $T_{IA}(i_{a,b})$ is the time the data required at input $i_{a,b}$ will be available, which will be the time $T_{CE}(i_{a,b})$ when the data transfer has ended. So, for each input $i_{a,b}$, we have:

$$T_{IA}(i_{a,b}) = T_{CE}(i_{a,b}) \tag{4.1.3}$$

- *Output-availability constraint:* Once execution of the subtask S_a begins, a certain time elapses before an output data value $o_{a,c}$ produced by the subtask becomes available. The time elapsed would be the time taken in executing $f_A(o_{a,c})$ fraction of the subtask; and so the time $T_{OA}(o_{a,c})$ must satisfy the following relation:

$$T_{OA}(o_{a,c}) = T_{SS}(S_a) + f_A(o_{a,c})(T_{SE}(S_a) - T_{SS}(S_a)) \tag{4.1.4}$$

  We will have such a relation for each output.

- *Subtask-execution-start constraint:* $T_{SS}(S_a)$ is the time the subtask S_a begins execution. There must be a certain relationship between the time a given subtask begins its execution and the times at which its various inputs become available. Since $f_A(i_{a,b})$ fraction of the subtask S_a can proceed without requiring the input $i_{a,b}$, the following relation must be satisfied for all the inputs $i_{a,b}$ to the subtask:

$$T_{IA}(i_{a,b}) \leq T_{SS}(S_a) + f_A(i_{a,b})(T_{SE}(S_a) - T_{SS}(S_a)) \tag{4.1.5}$$

- *Subtask-execution-end constraint:* Once execution of a subtask begins, a time equal to the execution time of the subtask must elapse before the subtask is completed. Execution time of the subtask depends on the processor being used for it. A priori we do not know which processor a given subtask S_a is going to be mapped to. Any processor from the set P_a could be selected to execute the subtask S_a. The uncertainty can be expressed by the following relation. The summation acts as a selection since only one $\sigma_{d,a} = 1$ for each a:

$$T_{SE}(S_a) = T_{SS}(S_a) + \sum_{d|p_d \in P_a} \sigma_{d,a} D_{pS}(p_d, S_a) \tag{4.1.6}$$

For each subtask S_a, we need such a relation.

- *Data-transfer-start constraint:* The time at which transfer of data begins must be after the output data is produced. For each input data (except for external inputs) $i_{a2,b2}$ (to the subtask S_{a2}) being supplied by another subtask's output, if the output supplying the data is $o_{a1,c1}$, the following relation must be satisfied by $T_{CS}(i_{a2,b2})$:

$$T_{CS}(i_{a2,b2}) \geq T_{OA}(o_{a1,c1}) \tag{4.1.7}$$

- *Data-transfer-end constraint:* The time at which transfer of data ends depends on whether the transfer is remote or local. A priori we do not know which case will occur. However, the two possibilities can be combined into one single relation using the variable $\gamma_{a1,a2}$. Thus, for each input data $i_{a2,b2}$ being supplied by another subtask S_{a1}, we have:

$$T_{CE}(i_{a2,b2}) = T_{CS}(i_{a2,b2}) + \gamma_{a1,a2} D_{CR} V_{a1,a2} + (1 - \gamma_{a1,a2}) D_{CL} V_{a1,a2} \tag{4.1.8}$$

The next two categories of constraints ensure that the hardware resources (processors, communication links) are shared correctly. These constraints ensure that the same hardware resource is not scheduled to perform more than one function during any given time interval. In order to express these constraints concisely, we need to define a special function called an overlap function L (as defined in [4]). The function is defined on two closed intervals of time, $[t1, t2]$ and $[t3, t4]$ (where $t1 < t2$ and $t3 < t4$), as:

$$L([t1, t2], [t3, t4]) = \begin{cases} 1, & \text{if the intervals overlap} \\ 0, & \text{otherwise} \end{cases}$$

- *Processor-usage-exclusion constraint:* If two subtasks S_{a1} and S_{a2} are being executed by the same processor p_d, then the two subtasks must not be scheduled to be executed at the same time. The situation that two subtasks S_{a1} and S_{a2} are being implemented by the same processor p_d implies $\sigma_{d,a1} = \sigma_{d,a2} = 1$. For each processor p_d and each pair of subtasks S_{a1} and S_{a2} such that the sets of processors P_{a1} and P_{a2} available to implement the subtasks contain the processor p_d, the following relation ensures that the overlap in the usage of the processor by the two subtasks is prevented:

$$\sigma_{d,a1}\sigma_{d,a2} L([T_{SS}(S_{a1}), T_{SE}(S_{a1})], [T_{SS}(S_{a2}), T_{SE}(S_{a2})]) = 0 \tag{4.1.9}$$

- *Communication-link-usage-exclusion constraint:* If the data required by two inputs $i_{a1,b1}$ and $i_{a2,b2}$ are being transmitted over the same communication link, then the two data transfers must not be scheduled at the same time. Let us say the input data $i_{a1,b1}$ is supplied by the subtask S_{a3} and the input data $i_{a2,b2}$ is supplied by the subtask S_{a4}. The two inputs $i_{a1,b1}$ and $i_{a2,b2}$ will be transmitted over the same communication link if the two subtasks S_{a1} and S_{a2} are mapped to the same processor, say p_{d2}, and also the subtasks S_{a3} and S_{a4} are mapped to the same processor, say p_{d1} (in that case, both the inputs will be transmitted over the communication link from processor p_{d1} to processor p_{d2}). For each processor pair (p_{d1}, p_{d2}) and each pair of inputs $i_{a1,b1}$ and $i_{a2,b2}$ (to subtasks S_{a1} and S_{a2} respectively, and from subtasks S_{a3} and S_{a4} respectively) such that the sets of processors P_{a1} and P_{a2} available to implement the subtasks S_{a1} and S_{a2} contain the processor p_{d2} and the sets of processors P_{a3} and P_{a4} available to implement the subtasks S_{a3} and S_{a4} contain the processor p_{d1}, the following relation ensures that the overlap in the usage of the communication link from processor p_{d1} to processor p_{d2} by the two data transfers is prevented:

$$\sigma_{d2,a1}\sigma_{d2,a2}\sigma_{d1,a3}\sigma_{d1,a4}L([T_{CS}(i_{a1,b1}), T_{CE}(i_{a1,b1})], [T_{CS}(i_{a2,b2}), T_{CE}(i_{a2,b2})]) = 0 \tag{4.1.10}$$

The set of constraints described here should be treated as an example set. *The exact form of constraints used can be tailored to meet the characteristics of the design problem at hand.* Our approach offers a great degree of flexibility in this regard.

4.1.2 Objective Functions

Two of the most important goals that the designer may wish to optimize are the overall system performance and the total system cost.

Overall System Performance: The performance is usually measured by how fast the system can perform the task. So, it can be represented by the time at which the task is completed (or all the subtasks are completed). If T_F is a real variable representing the time at which the task is completed, then the objective is to *minimize* T_F.

To ensure that T_F represents the time at which all the subtasks are completed, we need to introduce the following constraint in the model (for each subtask S_a):

$$T_F \geq T_{SE}(S_a) \tag{4.1.11}$$

Total System Cost: The total cost of the system can be expressed as the sum of the costs of the processors selected and the costs of the links created. In order to do so, we need to define two types of binary variables:

- *Processor-selection variable, β_d:* The variables of this type specify which processors have been selected in the synthesized architecture. $\beta_d = 1$ indicates the processor p_d is being included in the system.
- *Communication-link-creation variable, $\chi_{d1,d2}$:* The variables of this type specify what communication links are present in the synthesized architecture. $\chi_{d1,d2} = 1$ indicates there exists a communication link from the processor p_{d1} to the processor p_{d2} in the designed system.

Using the variables defined above, the objective is to:

$$MINIMIZE \sum_{d|p_d \in P} \beta_d C_d + C_L(\sum_{d1,d2|p_{d1} \in P \wedge p_{d2} \in P} \chi_{d1,d2})$$

where C_d is the cost of a processor p_d and C_L is the cost of building a communication link between two processors, as defined in Section 3. The variables of type β_d are related to the variables of type $\sigma_{d,a}$. A processor p_d will be included in the system if and only if at least one of the subtasks S_a ($p_d \in P_a$) is mapped to it, which implies that the variable β_d is the logical OR of all the $\sigma_{d,a}$ variables. This can be expressed by introducing the following constraint in the model (for all a such that $p_d \in P_a$):

$$\beta_d \geq \sigma_{d,a} \tag{4.1.12}$$

The variables of type $\chi_{d1,d2}$ are also related to the variables of type $\sigma_{d,a}$. A communication link is created from processor p_{d1} to processor p_{d2} if and only if at least one of the subtasks S_{a1} ($p_{d1} \in P_{a1}$) mapped to the processor p_{d1} needs to send data to at least one of the subtasks S_{a2} ($p_{d2} \in P_{a2}$) mapped to the processor p_{d2}. So, the variable $\chi_{d1,d2}$ is the logical OR of all the product terms of the form $(\sigma_{d1,a1}\sigma_{d2,a2})$, where the subtask S_{a1} supplies some data to the subtask S_{a2}. This condition leads to the introduction of following constraint in the model (for all $a1, a2$ such that $p_{d1} \in P_{a1}$ and $p_{d2} \in P_{a2}$ and subtask S_{a1} sends data to subtask S_{a2}):

$$\chi_{d1,d2} \geq \sigma_{d1,a1}\sigma_{d2,a2} \tag{4.1.13}$$

The essence of the model has been presented. It is easy to see that *arbitrary constraints imposed by the designer can be expressed using the timing and binary variables defined in the model.*

4.2 Synthesis Using the Model

Several constraints comprising the formulation presented in Section 4.1 are non-linear relations. These relations were linearized and the model was converted into a MILP (Mixed Integer-Linear Programming) formulation. *Bozo* [5], a branch-and-bound program to solve an MILP problem has been developed by L. J. Hafer of Simon Fraser Univ. Bozo invokes a commercial linear programming package, XLP, developed by XMP Software, Inc. We are using this program to solve our MILP model. Architectures for some small example tasks have been synthesized by solving the MILP problem. These results are discussed in Section 5.

5 Experiments and Results

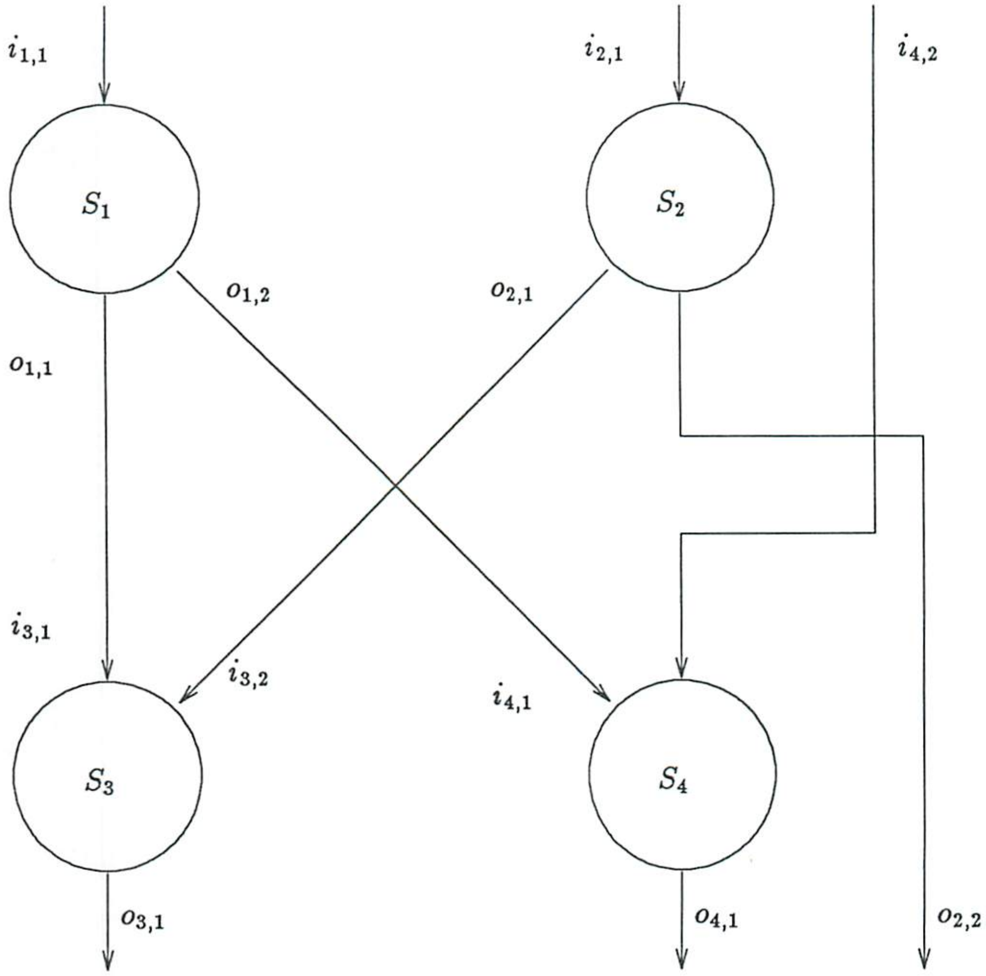
We have considered two example task graphs. The first example consists of four subtask nodes, while the second consists of nine. Some of the data related to these examples is taken from [9].

5.1 Example 1: Four-Node Task Graph

This example data flow graph is shown in Figure 1. Associated f_R and f_A parameters are also given in the figure, constraining input/output timing for the subtasks.

We assume we have available three types of processors: p_1, p_2, p_3. The costs of these processors and the execution times of various subtasks on the processors are given in Table 1. An entry of '—' in the table implies that the particular processor is functionally not capable of performing the particular subtask. As is obvious from the table, different processors have different cost-speed-functionality characteristics.

In this example the volume of data that needs to be communicated is one unit at each



$$\begin{aligned}
f_R(i_{1,1}) &= 0.25 \\
f_R(i_{2,1}) &= 0.25 \\
f_R(i_{3,1}) &= 0.25 \\
f_R(i_{3,2}) &= 0.50 \\
f_R(i_{4,1}) &= 0.25 \\
f_R(i_{4,2}) &= 0.50
\end{aligned}$$

$$\begin{aligned}
f_A(o_{1,1}) &= 0.50 \\
f_A(o_{1,2}) &= 0.75 \\
f_A(o_{2,1}) &= 0.50 \\
f_A(o_{2,2}) &= 0.75 \\
f_A(o_{3,1}) &= 0.75 \\
f_A(o_{4,1}) &= 0.75
\end{aligned}$$

Figure 1: Four-Node Task Graph

		Execution Time			
Proc.	Cost	S_1	S_2	S_3	S_4
p_1	4	1	1	-	3
p_2	5	3	1	2	1
p_3	2	-	3	1	-

Table 1: Execution Time and Cost Table for Four-Node Graph

Design	Runtime	Cost	Performance
1	11	14	2.5
2	24	13	3
3	28	7	4
4	37	5	7

Table 2: Architectures for Four-Node Graph

arc in the graph. Local transfer delay is given to be negligible; i.e., $D_{CL} = 0$. We are also given the communication link characteristics. The cost of a link, C_L, is one unit; and the remote transfer delay for a unit volume of data over a link, D_{CR}, is also one unit.

The MILP model for the example consists of 93 variables, 21 timing and 72 binary, and 174 constraints. Bozo was used to generate 4 non-inferior architectures. These different architectures were generated by changing the constraint value for the total cost of the system, and optimizing the overall performance of the system. Bozo's runtime to generate each of these designs is of the order of a few seconds. These runtimes are on a system with CPU type Solbourne Series5e/900 (similar to Sun SPARCsystem 4/490) with 128 MB of memory. Cost, performance and runtime (in seconds) for the four designs are given in Table 2.

A brief discussion of these designs follows:

- *Design 1:* This design consists of 3 processors: p_{1a} - a processor of type p_1, p_{2a} - a processor of type p_2, and p_{3a} - a processor of type p_3. Processor p_{1a} performs

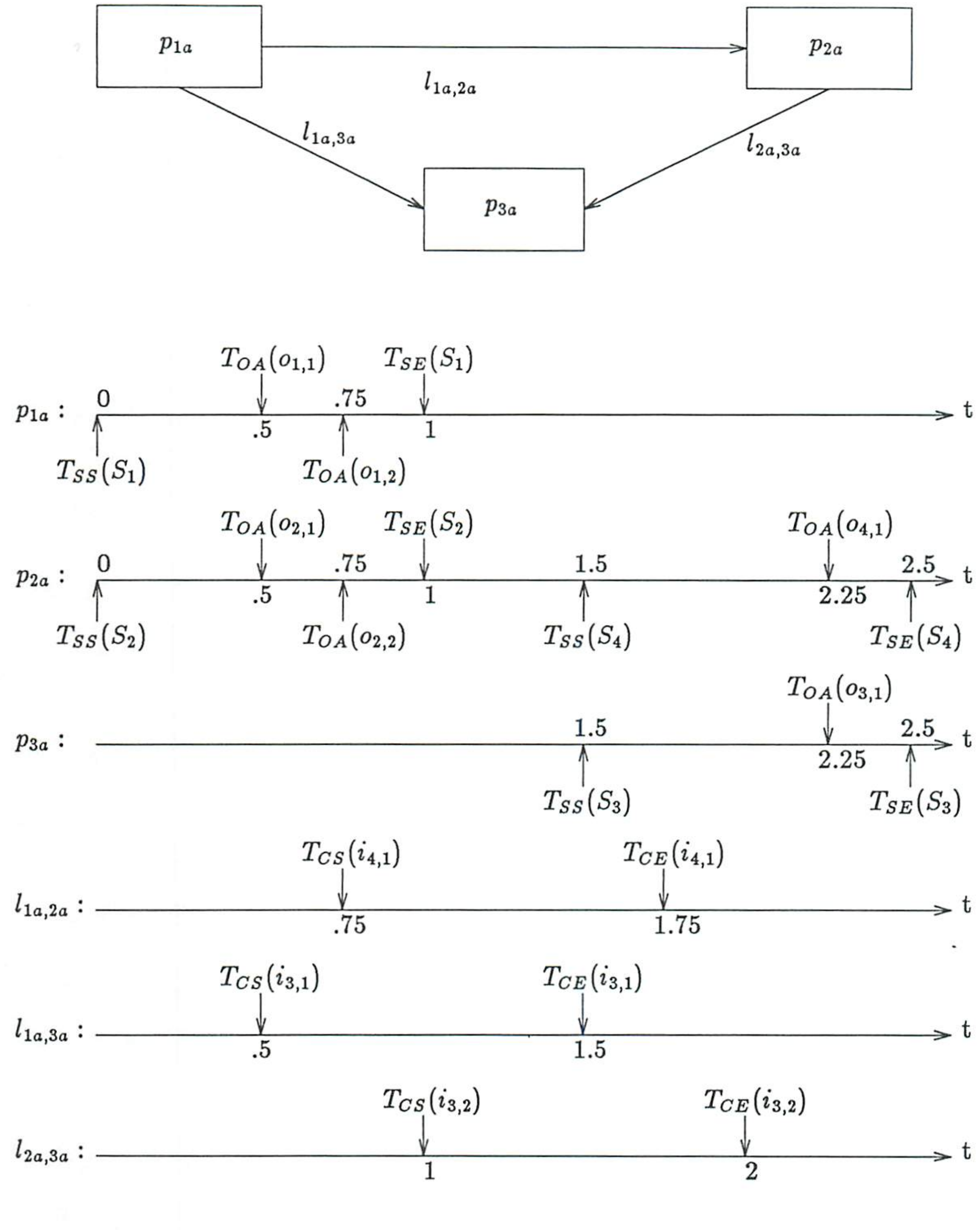


Figure 2: Synthesized Multiprocessor System I and Schedule for Four-Node Graph

subtask S_1, processor p_{2a} performs subtasks S_2 and S_4 in that order, and processor p_{3a} performs subtask S_3. There are three communication links: $l_{1a,2a}$, $l_{1a,3a}$, and $l_{2a,3a}$. Data $i_{4,1}$ gets transmitted on link $l_{1a,2a}$, data $i_{3,1}$ gets transmitted on link $l_{1a,3a}$, and data $i_{3,2}$ gets transmitted on link $l_{2a,3a}$. As an illustration, this architecture is shown in Figure 2. A detailed schedule for the various events is also shown in the figure.

- *Design 2:* This design is similar to design 1, and also consists of 3 processors: p_{1a}, p_{2a}, and p_{3a}. However, it has only two links: $l_{1a,2a}$, and $l_{1a,3a}$. Presence of fewer links forces a change in the mapping between the resources and the events. Processor p_{1a} performs subtasks S_1 and S_2 in that order, processor p_{2a} performs subtask S_4, and processor p_{3a} performs subtask S_3. Data $i_{4,1}$ gets transmitted on link $l_{1a,2a}$, data $i_{3,1}$ and data $i_{3,2}$ get transmitted on link $l_{1a,3a}$ in that order.
- *Design 3:* This design consists of 2 processors: p_{1a} - a processor of type p_1, and p_{3a} - a processor of type p_3. Processor p_{1a} performs subtasks S_1 and S_4 in that order, and processor p_{3a} performs subtasks S_2 and S_3 in that order. There is a communication link: $l_{1a,3a}$. Data $i_{3,1}$ gets transmitted on link $l_{1a,3a}$.
- *Design 4:* This design consists of just 1 processor: p_{2a} - a processor of type p_2. The processor performs the subtasks S_2, S_1, S_3, and S_4 in that order.

5.2 Example 2: Nine-Node Task Graph

The data flow graph is shown in Figure 3. For this example, we assumed that a subtask requires all the inputs before it can start and that none of the outputs from a subtask become available until its execution is over. Again, there are three types of processors, with the costs and the execution times given in Table 3. The volume of data is one unit for each arc. We are given: $D_{CL} = 0$, $D_{CR} = 1$. For this graph, we synthesized architectures for two different styles of interconnection.

5.2.1 Point-to-Point Interconnection

Here, as before, if two processors need to communicate, then there must be a direct link between them; and the cost of building a link $C_L = 1$.

The MILP model consists of 272 variables, 47 timing and 225 binary, and 1081 constraints. We generated 5 non-inferior architectures by changing the constraint value for

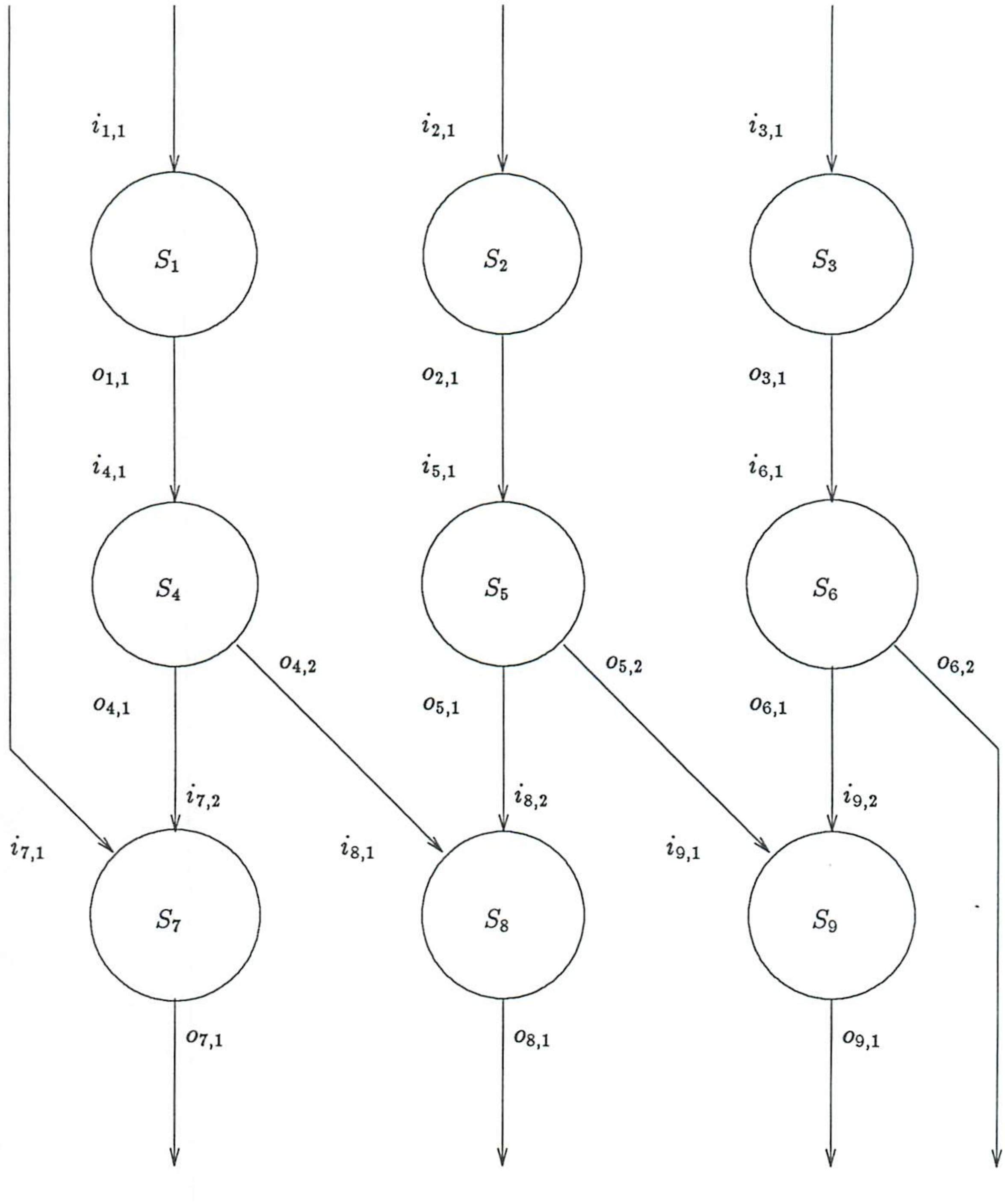


Figure 3: Nine-Node Task Graph

		Execution Time								
Proc.	Cost	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
p_1	4	2	2	1	1	1	1	3	–	1
p_2	5	3	1	1	3	1	2	1	2	1
p_3	2	1	1	2	–	3	1	4	1	3

Table 3: Execution Time and Cost Table for Nine-Node Graph

Design	Runtime	Cost	Performance
1	62.2	15	5
2	445.17	12	6
3	538.67	8	7
4	75.18	7	8
5	6416.87	5	15

Table 4: Architectures for Nine-Node Graph (Point-to-Point)

the total system cost, and optimizing the system performance. Bozo's runtime for each of these designs is of the order of a few hours, except for design 5. Cost, performance and runtime (in minutes) for the five designs are given in Table 4. Discussion of the designs is omitted.

5.2.2 Bus-Style Interconnection

In this interconnection style, the system consists of a set of processors and a bus connecting all the processors to each other. So, the cost of the system is dominated by the costs of the processors selected. Our approach is capable of modeling such a system.

The MILP bus-architecture model consists of 200 variables, 47 timing and 153 binary, and 416 constraints. Three non-inferior architectures were generated by changing the constraint value for the total system cost, and optimizing the system performance. Runtime for each of these designs is of the order of a few hours. Table 5 gives the statistics

Design	Runtime	Cost	Performance
1	107.3	10	6
2	89.53	6	7
3	61.52	5	15

Table 5: Architectures for Nine-Node Graph (Bus-Style)

for the three designs (runtime in minutes). Discussion of the designs is omitted.

6 Conclusions and Future Work

In this paper, we have presented a formal model for the multiprocessor synthesis problem. It has been shown that the model can be solved fairly quickly for small size problems. Even for larger problems, the runtime is not prohibitive but there is much room for improvement. The model offers a great degree of flexibility. The model described in the paper should be treated as an example. The overall approach has potential to be applied to model more generalized design situations (e.g.; more design parameters, more design styles, etc.). Further research is required in two directions:

- Enhancing the model to handle a more generalized multiprocessor design problem.
- Designing techniques to improve the runtime for solution of the model.

Work is in progress at USC in both the directions. Memory design and mixed-style interconnect design are some of the aspects related to the multiprocessor synthesis problem that are being addressed. Incorporation of some heuristics for performing the branch-and-bound search seems to be a promising idea for improving the runtime.

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