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Abstract

In ATM networks supporting B-ISDN, the traffic is highly correlated and neglecting its correlation leads to dramatic underestimation of the end-to-end delay and end-to-end loss probability. Being able to accurately estimate end-to-end performance is of paramout importance for traffic control. The purpose of this paper is to introduce a procedure for modelling the output process of a finite buffer discrete-time queue loaded with a Markovian Arrival Process (D-BMAP/D/1/K). The procedure matches the output process long-term index of dispersion for counts and correlation of the number of arrivals in consecutive slots with the same statistics of a two-state Markov Modulated Bernoulli Process. We show through numerical examples that this procedure is very accurate. We also introduce a framework for the analysis of queueing networks with Markov Modulated type of flow.

I) INTRODUCTION

The future Broadband Integrated Services Digital Network will carry video, voice and data applications with different Quality of Service requirements. The cell arrival stream from integrated traffic is highly correlated and neglecting its correlations leads to a dramatic underestimation of the delay and loss probability [1]-[3]. Therefore, many researchers have evaluated the impact of correlated traffic on the performance of an isolated multiplexer [4]-[9]. However, little attention has been given to the estimation of end-to-end performance. An ATM multiplexer is viewed as a discrete-time finite buffer queue with Markovian Modulated input. In this paper, we introduce a procedure for modelling the output process of an ATM switching node and specify a framework for queueing networks with Markov modulated flow which can be used for the evaluation of end-to-end values.

In delay sensitive applications, if a cell arrives at the destination after a certain treshold, it becomes useless for signal reconstruction. While the propagation and transmission delay account for the fixed part of the end-to-end delay, the queueing delay is mainly responsible for the variable component of end-to-end delay (jitter). A precise estimation of end-to-end loss metrics such as loss probability and length of loss gap [10] depends on the accurate estimation of these metrics at each individual node. Finding an appropriate queueing network representation for B-ISDN networks is of paramount importance for the computation of end-to-end values.

We assume that the input traffic to the queueing network is modelled as a Discrete Time Batch Markovian Arrival Process (D-BMAP) [11]. The D-BMAP process has been successfully used for modelling the integration of voice, video and data sources [15]-[16]. Our procedure consists of matching the statistics of the output process of a D-BMAP/D/1/k queue with the statistics of a two-state Markov Modulated Bernoulli Process (MMBP). In the MMBP process there are only single arrivals in each slot and the arrival probabilities depend on the state of an underlying Markov chain. Insofar as the MMBP is a sub-case of the more general D-BMAP we are able to maintain an uniform representation of the flows in a queueing network. We show that we can model the output process accurately by considering the long term index of dispersion for counts and the correlation of the number of arrivals between consecutive slots (at lags equal to 1 and 2).

This paper is organized as follows. In section II, we briefly describe the traffic model. Section III details the framework for queueing networks with Markov modulated flow. In section IV we introduce the matching procedure for the output process. Section V shows how the matching procedure was validated, and finally some conclusions are in section VI.

II) THE TRAFFIC MODEL

In this section we briefly describe the stochastic processes which are used to represent the traffic flow in our queueing network model. In the Discrete Time Batch Markovian Arrival Process [11], a batch may arrive at every dicrete time. The batch size probability mass function depends on the state of an underlying discrete time Markov chain. A D-BMAP is completely specified by the matrices D_n whose elements $(d_{ij})_n$ give the probability that a transition from state i to state j occurs and a batch of size n arrives. Clearly, we have that:

$$\sum_{n=0}^{\infty} \sum_{j=1}^{m} (d_{ij})_n = 1$$

where m is the number of states of the underlying Markov chain.

The matrices D_n are related to the transition probability matrix D of the underlying Markov chain by:

$$D = \sum_{n=0}^{\infty} D_n$$

The Discrete Time Batch Markovian Arrival Process is the discrete time version of the Batch Markovian Arrival Process [12]-[14]. The D-BMAP is a non-renewal process. Such a process is very attractive for modelling integrated traffic due to the flexibility that we have to set the parameters $((d_{ij})_n)$ to generate a specific correlation pattern [15]-[18]. Wang and Silvester [15] developed an accurate procedure for matching the statistics of integrated (video, voice and data) traffic with the statistics of a two state D-BMAP. Blondia [16] showed how to map the short and long-term correlations of a video source into a D-BMAP. In [11], Blondia provides a comprehensive description of the D-BMAP process and specifically of its counting process. Hashida et al. [17] derived the statistics of both the counting and interarrival processes for the specific case where the underlying Markov Chain has only two states. In [11], it was demonstrated that the mean arrival rate, the variance of number of arrivals and the covariance at lag k are given by:

$$\lambda = \pi\left(\sum_{k=1}^{\infty} kD_k\right)\bar{e}$$

$$var = \pi\left(\sum_{k=1}^{\infty} k^2 D_k\right)\bar{e} - \lambda^2$$

$$cov(x_1, x_k) = \pi\left(\sum_{n=1}^{\infty} nD_n\right)D^{k-2}\left(\sum_{n=1}^{\infty} nD_n\right)\bar{e} - \lambda^2$$

where $\bar{e}$ is the unit column vector and π is the steady state probability of the underlying Markov chain, i.e;

$$\pi D = \pi \qquad \pi\bar{e} = 1$$

The Markov Modulated Bernoulli Process (MMBP) is a subcase of D-BMAP of interest in this paper. In a Markov Modulated Bernoulli Process, at most a single cell may arrive at each time epoch. We focus our attention on the two-state MMBP which is totally specified by $(p_1, p_2, \alpha_1, \alpha_2)$ where p_i (i=1,2) is the probability of having an arrival when the underlying Markov chain is at state i, and α_i (i=1,2) is the probability of remaining in state i at each time epoch. The matrices D_n of a two-state MMBP are given by:

$$D_0 = \begin{bmatrix} (1-p_1)\alpha_1 & (1-p_1)(1-\alpha_1) \\ (1-p_2)(1-\alpha_2) & (1-p_2)\alpha_2 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} p_1\alpha_1 & p_1(1-\alpha_1) \\ p_2(1-\alpha_2) & p_2\alpha_2 \end{bmatrix}$$

The Markov Modulated Bernoulli Process has also been widely applied to the modelling of integrated traffic. Le Boudec used the MMBP as input traffic to evaluate the cell loss rate in a multiplexer with buffer priority [19]. Guillermin et al. [20] studied the burstiness concept in B-ISDN networks via an MMBP model. They solved the MMBP/D/1/K and proved a relationship between the stationary queue length distribution and the arrival time distribution in a MMBP/D/1/K queue.

II) QUEUEING NETWORKS WITH MARKOV MODULATED FLOWS

The global flow of a communication network is normally described by queueing network models. In the so-called product form networks, the flow is assumed to be Poisson and the probability mass function of the distribution of customers among the network nodes can be computed as the product of the probability mass function of the number of customers at each node [21]. Although the product form assumptions allowed the development of computationally efficient algorithm, they are too restrictive to be used in many practical situations [22]. Alternatively, a more realistic approach is to represent the flow in communication networks as a renewal process specified by the mean and variance of the number of arrivals [23]-[25]. The parametric decomposition approximation is used in this approach [23]. The parametric decomposition evaluates each queue in the network as if they were stochastically independent. The queues are analyzed in isolation only after the input flow parameters (mean and variance) are computed via renewal approximations. The parametric decomposition can be seen as a generalization of the product form concept in which the dependencies among the queues are captured by the estimation of the flow parameters. Whitt used the parametric decomposition together with renewal approximations of point processes to develop the Queueing Network Analyzer [24].

In the future B-ISDN network, the traffic will be highly correlated; a renewal representation of the network flow can no longer be accepted. Recently, some studies have modelled B-ISDN networks as queueing networks with non-renewal flows. Kroner et al. [26] considered a queueing network in which each connection at the transport level was represented as an on-off source. The long and the short term fluctuations in the links were computed respectively by a fluid flow approximation and by an M/D/1-S model. They showed how to compute the end-to-end delay and the distribution of the transfer delay at each queue in a tandem network. Reising analyzed two queues in tandem [27]. The input of the first queue is a two-state Markov Modulated Bernoulli Process and the interfering traffic of both queues is a Bernoulli process. He derived an approximation model for the output process of the first queue as a two-state MMBP. Grienenfield [28] studied two queues in tandem by using a pertubation method to compute approximations for the mean and the variance of the end-to-end delay too. He considered that both the input and the interfering traffic were wide sense stationary processes, and he used a procedure for matching intermediate results in a Weibuill distribution to compute the jitter delay which is associated with the input traffic (pertubation).

In our investigation, we consider open queueing networks with multiple class of clients. In each node there is a single server with finite buffer space and constant service time. Service is provided in a First-Come-First-Served fashion. Both internal and external traffic are represented as Discrete Time Batch Markovian Arrival process. The discrete time assumption derives from the ATM standard. In order to solve this queueing network with non-renewal flow, we employ the parametric decomposition approximation. The fact that the correlation structure of a cell stream tends to be preserved along the network reinforces the idea of solving the queues in isolation [26]. The network elementary operations are defined as:

The output process of A D-BMAP/D/1/K

The output process of a D-BMAP/D/1/K queue is also correlated, and neglecting its correlations leads to inaccurate results. In [29] the output process of an ATM switch was modelled as two different memoryless processes: i) a Bernoulli process and ii) an Interrupted Poisson Process with the mean duration of the active state equal to the mean busy period. In both cases, the model failed to produce accurate results. At each time slot, at most one cell may depart from a queue. The Markov Modulated Bernoulli Process, a correlated process with single arrivals, is a good candidate for modelling the output process of a D-BMAP/D/1/K queue. We, therefore, developed a procedure for matching the statistics of the output process with the statistics of a two-state MMBP (figure 1). Moreover, by modelling the output process as a MMBP, we were able to represent all the flows in the network as D-BMAP process (see figure 1).

Joining

The superposition of two D-BMAP processes with m_1, m_2 states and n_1, n_2 maximum batch size is also a D-BMAP with $m_1 \times m_2$ states and $n_1 + n_2$ maximum batch size. The matrix D_k which elements $(d_{ij})_k$ which give the probability of going from state i to state j and having a batch arrival of size k is computed as:

$$D_k = \sum_{q=0}^{n_1} D_q^{(1)} \otimes D_{k-q}^{(2)}$$

For instance, the superposition of an MMBP and a D-BMAP with maximum batch size of 2 is given by:

$$D_0 = D_0^{(1)} \otimes D_0^{(2)}$$

$$D_1 = D_0^{(1)} \otimes D_1^{(2)} + D_1^{(1)} \otimes D_0^{(2)}$$

$$D_2 = D_0^{(1)} \otimes D_2^{(2)} + D_1^{(1)} \otimes D_1^{(2)} + D_2^{(1)} \otimes D_0^{(2)}$$

$$D_3 = D_0^{(1)} \otimes D_3^{(2)} + D_1^{(1)} \otimes D_2^{(2)} + D_2^{(1)} \otimes D_1^{(2)} + D_3^{(1)} \otimes D_0^{(2)}$$

where $A \otimes B$ denotes the Krockener product of matrix A by matrix B.

Splitting

We assume that routing is state independent. It means that the probability of a cell departing from one node and going to another node is fixed. When characterizing the flow between two nodes, we represent the output process of the first queue as an MMBP process, and then model the flow that goes to the second queue as an MMBP with parameters:

$$(p_{ij} \times p_1, p_{ij} \times p_2, \alpha_1, \alpha_2)$$

where p_{ij} is the probability that a cell leaves node i and goes to node j.

IV) THE MATCHING PROCEDURE

Finding an appropriate representation for the output process of a queue is of paramount importance for defining a queueing network framework. With the advent of traffic integration, we now face the challenge of obtaining a suitable representation for the output process of queues with correlated input. Only recently, have some studies derived the statistical properties of the output process of queues with Markov Modulated input [30]-[32]. Saito [30] studied the output process of an N/G/1 queue and particularly of the MMPP/D/1 queue. He analyzed the interdeparture time process and explicitly detailed the expressions for the mean, variance and covariance at lag 1. The burstiness of a process (C_p (z)) was defined as the Z-transform of the covariance of interdeparture time (in [31] C_p (z) was used to analyze integrated traffic). By comparing the C_p (z) curves for the input and for the output processes, Saito concluded that covariances are likely to be preserved. He also pointed out that the reduction of the coefficient of variation is larger in heavily loaded systems than it is in lightly loaded ones. Thus, the departure process of a lightly loaded system with bursty arrival process tends to be bursty too. Takine et al. [32] studied the output process of a D-BMAP/D/1/K queue. They derived not only the expression for the m^{th}-factorial moment of the interdeparture time process, but also the distribution of the idle and the busy periods. They pointed out that, when the variation of arrivals is high, both the mean and the idle periods become larger as the correlation increases. This observation is also valid for the coefficient of variation of busy periods and of the interdeparture time.

Before showing how to match the statistics of the output process with the statistics of a two-state MMBP, we need to characterize the output process itself. Having one exactly departure at each time epoch of a busy period suggests that we can represent the output process as a D-MAP in which the matrices D'_1 and D'_0 correspond respectively to busy and idle periods. In order to capture the behavior of busy/idle periods, we need to associate each state of the D-MAP with the phase of the arrival process and with the number of enqueued cells at the end of each time slot [11]. If we have a gated server (i.e., if a cell finds the server empty at its arrival slot, it can only be transmitted at the next slot) then, the output process is given by [11]:

$$D'_0 = \begin{bmatrix} D_0\ D_1\ \dots\ D_{k-1} & \sum_{n=k}^{\infty} D_n \\ 0\ \ 0\ \dots\ \ 0 & 0 \\ \dots\ \dots\ \dots\ \ \dots & \dots \\ 0\ \ 0\ \dots\ \ 0 & 0 \end{bmatrix}$$

$$D'_1 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ D_0 & D_1 & D_2 & \dots & D_{k-1} & \sum_{n=k}^{\infty} D_n \\ 0 & D_0 & D_1 & \dots & D_{k-2} & \sum_{n=k-1}^{\infty} D_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & D_0 & \sum_{n=1}^{\infty} D_n \end{bmatrix}$$

On the other hand, if we have a cut-through type of service (i.e. the cell can be transmitted in the same slot in which it arrives) the output process is specified by:

$$D'_0 = \begin{bmatrix} D_0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$D'_1 = \begin{bmatrix} D_1 & D_2 & D_3 & \dots & D_K & \sum_{n=K+1}^{\infty} D_n \\ D_0 & D_1 & D_2 & \dots & D_{K-1} & \sum_{n=K}^{\infty} D_n \\ 0 & D_0 & D_1 & \dots & D_{K-2} & \sum_{n=K-1}^{\infty} D_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & D_0 & \sum_{n=1}^{\infty} D_n \end{bmatrix}$$

The index of dispersion time curve completely defines the correlation structure of a counting process. Consequently, to accurately approximate the output process, it is important to provide a good match with the index of dispersion time curve. In our procedure, we chose to match the long-term index of dispersion and the covariance of the number of arrivals between consecutive slots and between two consecutive slots (at lags of 1 and 2). Our procedure is:

$$output_{mean} = MMBP_{mean}$$

$$output_{variance} = MMBP_{variance}$$

$$output_{covariance\ lag=1} = MMBP_{covariance\ lag=1}$$

$$output_{covariance\ lag=2} = MMBP_{covariance\ lag=2}$$

V) NUMERICAL EXAMPLES

To validate the matching procedure, we consider two different D-BMAP processes. The data shown in this section correspond to a server with gated service and buffer size 100. Time is normalized to one slot. The first process is a two state D-BMAP with the same transition probability in each state (α) [32]. The batch size is Poisson distributed with mean $(1+c)\rho$ (state 1) and $(1-c)\rho$ (state 2) where ρ is the overall traffic intensity and *c* is a parameter. it was demonstrated in [32] that the square coefficient of variation (C_v^2) and the correlation coefficient of the number of arrivals at lag *n* ($C_c(n)$) are respectively given by:

$$C_v^2 = \rho^{-1} + c^2$$

$$C_c(n) = \frac{c^2\rho}{1+c^2\rho} \times (2\alpha-1)^n$$

In the first set of experiments, we simulated two queues in tandem. The input to the first queue is a D-BMAP and the input to the second queue is the output process of the first queue plus interfering traffic. The interfering traffic belongs to the same type of process of the input process as the first queue. We also simulated an isolated queue in which the input is given by the same interfering traffic of the two queues in tandem plus a two-state MMBP which substitutes the output of the first queue (figure 2). Then, we compared the delay and the loss probability of an arriving cell to the second queue of the two queues in tandem with the delay and loss probability of an arriving cell to the isolated queue. In order to avoid both the non-queueing phenomenon in tandem queues with constant service time and to make the output process the major responsible for the delay and loss probability, we kept the interfering traffic intensity low.

Takine et al. [32] pointed out that when the coefficient of variation of the input stream is moderate (c = 0.5) to high (c = 0.9), the correlation of arrivals plays a key role in determining the characteristics of the output process (mean interdeparture time, mean duration of busy period and idle periods) In our experiment, we kept constant two of the three statistics $(\rho, C_v, C_c(1))$ and varied the third one by changing either ρ, c, or α. We observed that the matching procedure was sensitive to the fluctuations in the output process due to the combination of a high coefficient of variation and correlation.

In figures 3 to 6, we show the percentual error $\left(\frac{d_{out} - d_{match}}{d_{out}} \times 100\right)$ where d_{out} is the delay suffered by a cell at the second queue of the two queues in tandem, and d_{match} is the delay seen by a cell at the isolated queue when we substituted the output process by an MMBP. In figures 3 and 4, we varied C_v by changing c from 0.1 to 0.9. We verified that the matching procedure accurately captures the variations in C_v. However, we noticed that it performs much better at moderate loads ($\rho = 0.4$, figure 3) than at high loads ($\rho = 0.8$, figure 4). The percentual error was less than 1% at moderate load and was below 7% even at high loads. By varying α from 0.1 to 0.9, we evaluated the impact of both positively and negatively correlated streams in the precision of the matching procedure. No significant impact was observed (figures 5 and 6). The same trend seen in figures 3 and 4 is carried over to figures 5 and 6. The matching procedure gives better delay estimations for lightly loaded systems ($\rho = 0.2$, figure 5) than for highly loaded systems ($\rho = 0.8$, figure 6)

The aforementioned two queues in tandem were also analyzed to evaluate the estimation given by the matching procedure over the whole range of (mean) delays. We varied ρ and C_v of the input process to the first queue, changing them so that the (mean) delay in the second queue was in the range from 2 to 90 time units. We plotted the estimated mean delay and the mean delay at the second queue. The x-axis represents varied loaded so that the delay is the value shown. No error above 5% was observed.

In order to verify the accuracy of the matching procedure in a generally connected network, we analyzed a group of parallel queues feeding into a second tandem queue. We then compared the delay and the loss probability seen by a cell at the second queue with the delay and loss probability seen by a cell at a queue in isolation. In the latter queue, the output process of each parallel queue was substituted by a two-state MMBP (figure 8). In figure 9 and10 we show respectively the delay and the loss probability at the second queue and at the queue in isolation, considering four parallel queues. Each parallel queue had an initial load of 0.1 ($\rho = 0.1$, $c = 0.1$, $\alpha = 0.55$). We varied ρ and c for each input process to the parallel queues to cover a wide range of values for the mean delay and loss probability down to as low as 10^{-7}. While errors in the delay estimation were about 5%, errors up to 20% in the loss probability occurred. This maximum loss probability error can be acceptable in the analysis of highly loaded systems.

The second process used in the validation experiments was a D-BMAP the parameters of which (transition probabilities and distribution of batch size) were computed according to a matching procedure which took into account the underload/overload periods of a server. This procedure is quite flexible and one can easily incorporate data, voice and video sources into a stream.

Overall, the maximum observed error of the delay estimation was under 7% and the maximum observed error of the loss probability estimation was under 20%.

VI) CONCLUSIONS

In this paper, we introduce a procedure for matching the statistics of the output process of a B-DMAP/D/1/K queue with the statistics of a two-state Markov Modulated Bernoulli Process. This procedure was shown to be accurate. Errors were less than 7% for the delay estimation and 20% for the loss probability, respectively. Moreover, we also describe a framework for the analysis of queueing networks with Markov Modulated flow. We are currently validating the queueing network framework for more generally connected networks.

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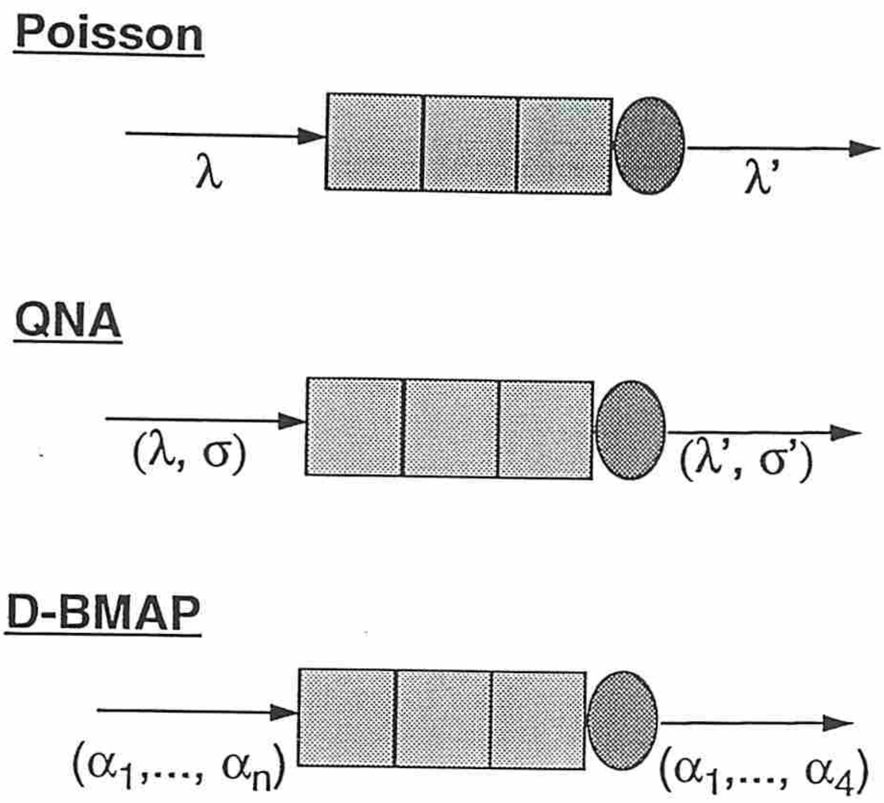

Figure 1: In product form networks the flow is characterized by the mean of a Poisson process. In queueing networks with renewal flow by the mean and the variance of the renewal process. In queueing networks with Markov modulated flow, the number of parameters depends on the size of the underlying Markov Chain. If the MC is a two state, we need four parameters.

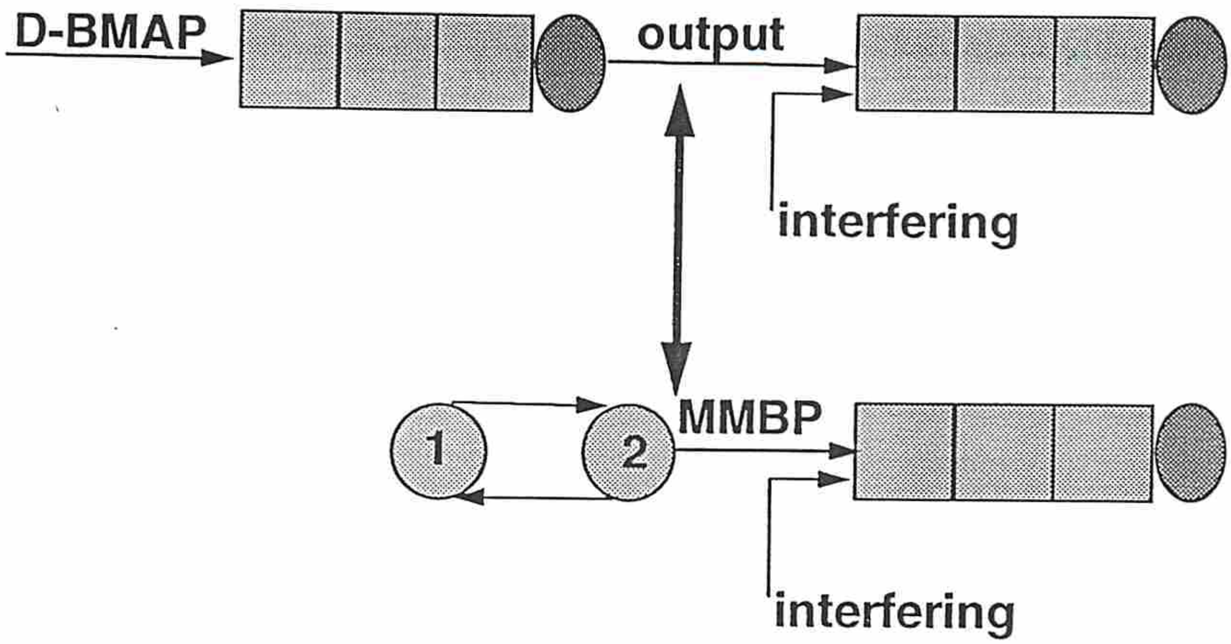

Figure 2: Scheme of the first set of experiments - the output process of the first queue was substituted by a two-state MMBP

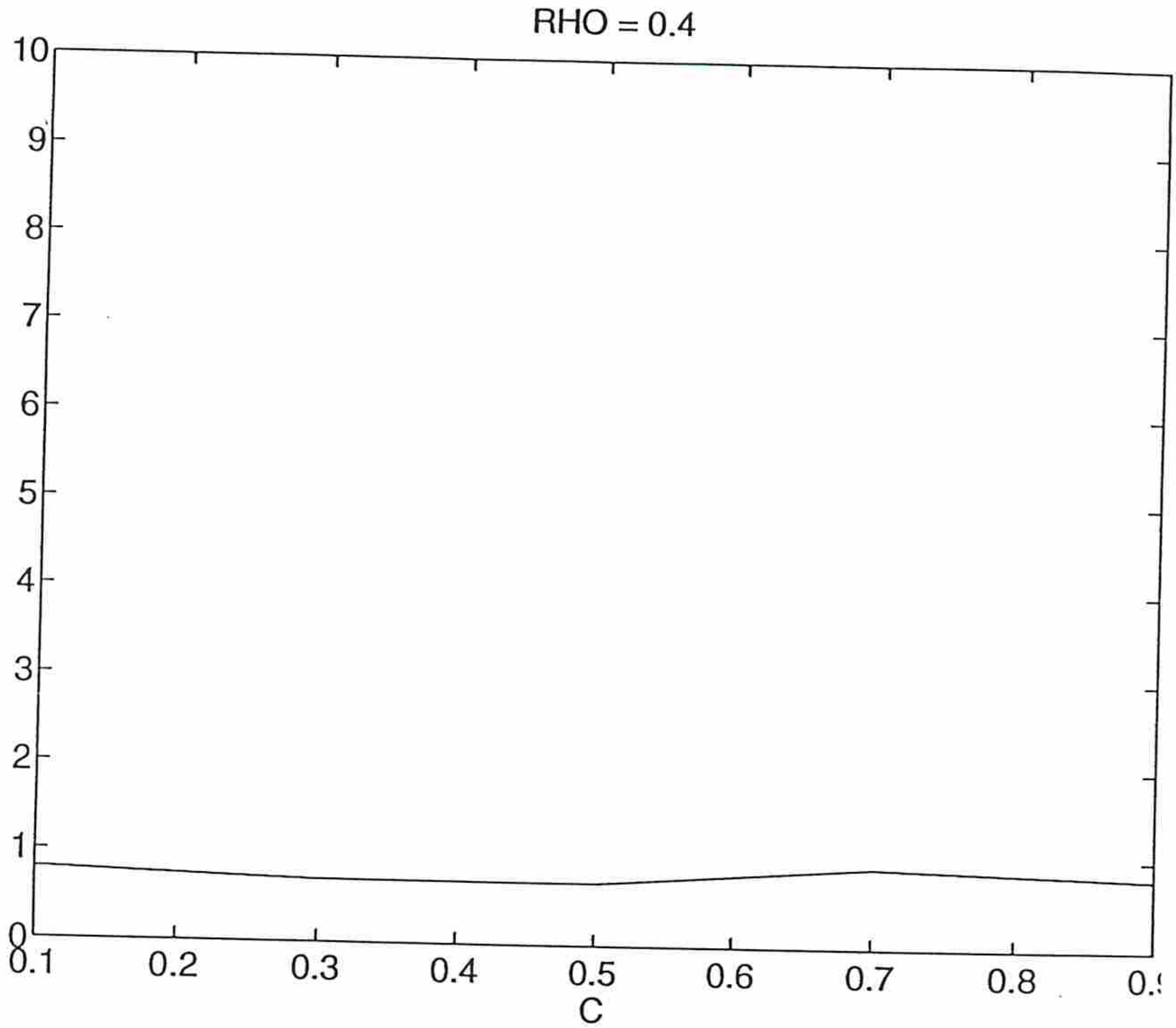

Figure 3: Percentual error of the delay estimation as a function of *c* (system load = 0.4)

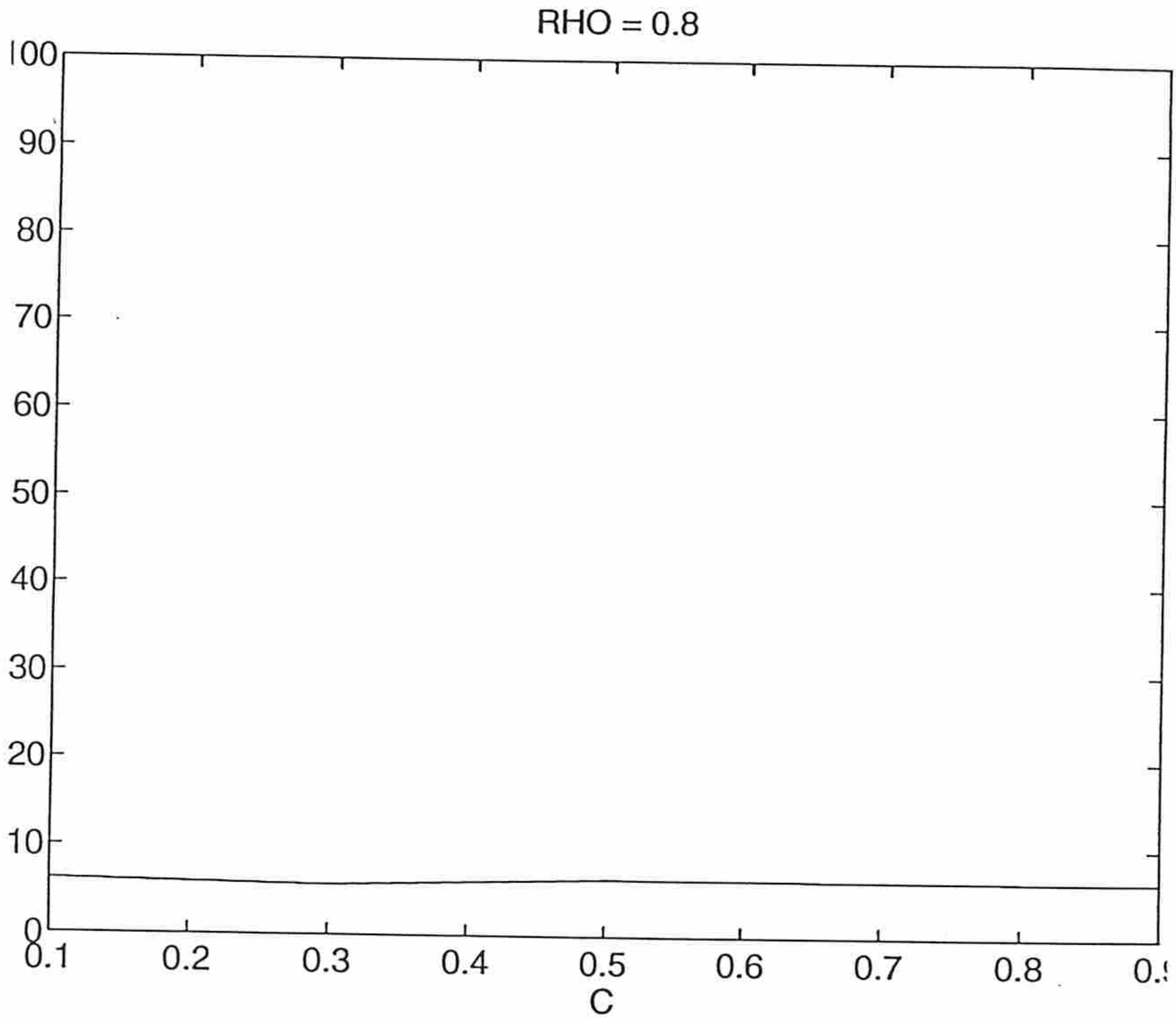

Figure 4: Percentual error of the delay estimation as a function of *c* (system load= 0.8)

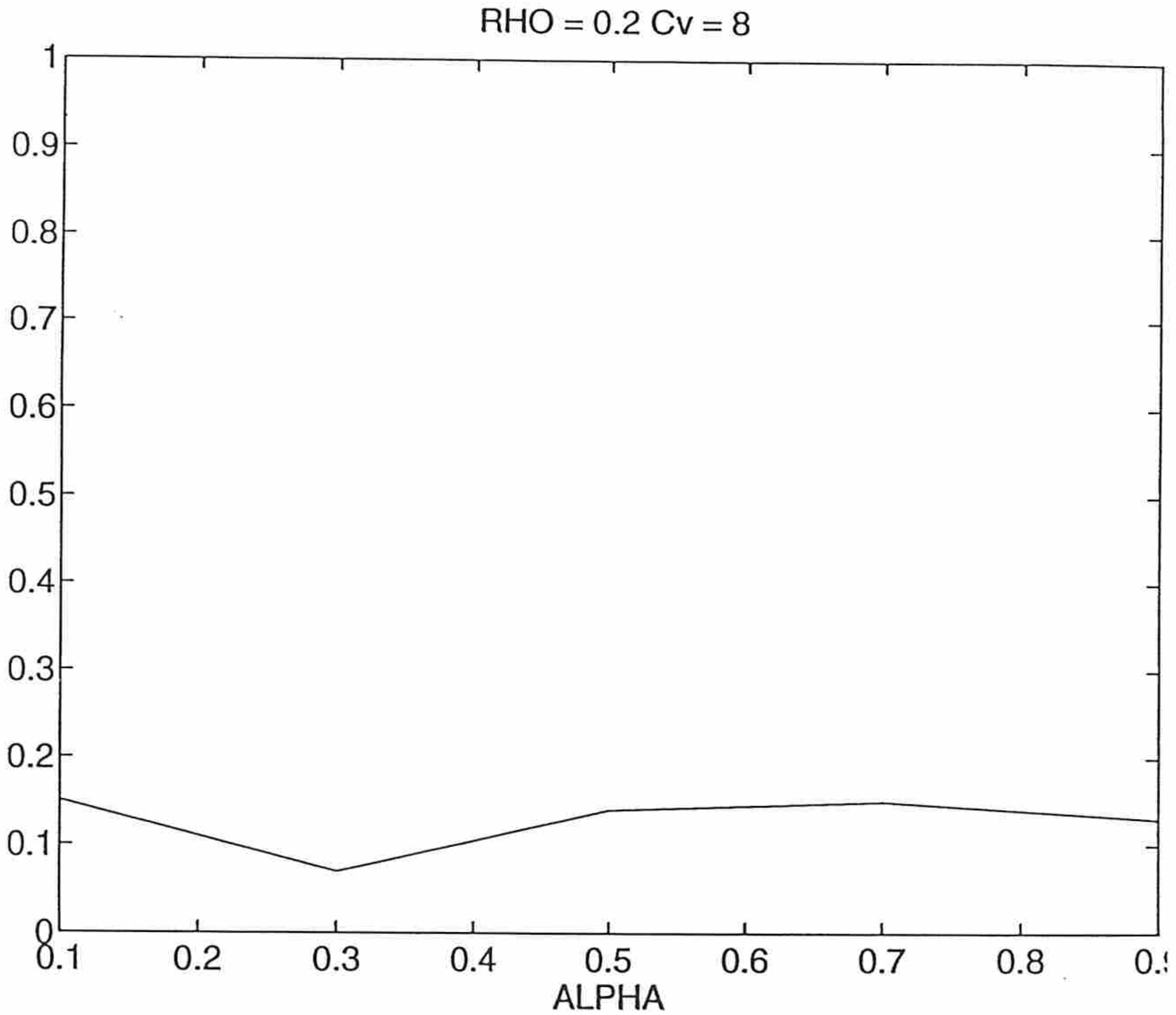

Figure 5: Percentual error of the delay estimation as a function of $\alpha(\rho = 0.2, C_v = 8)$

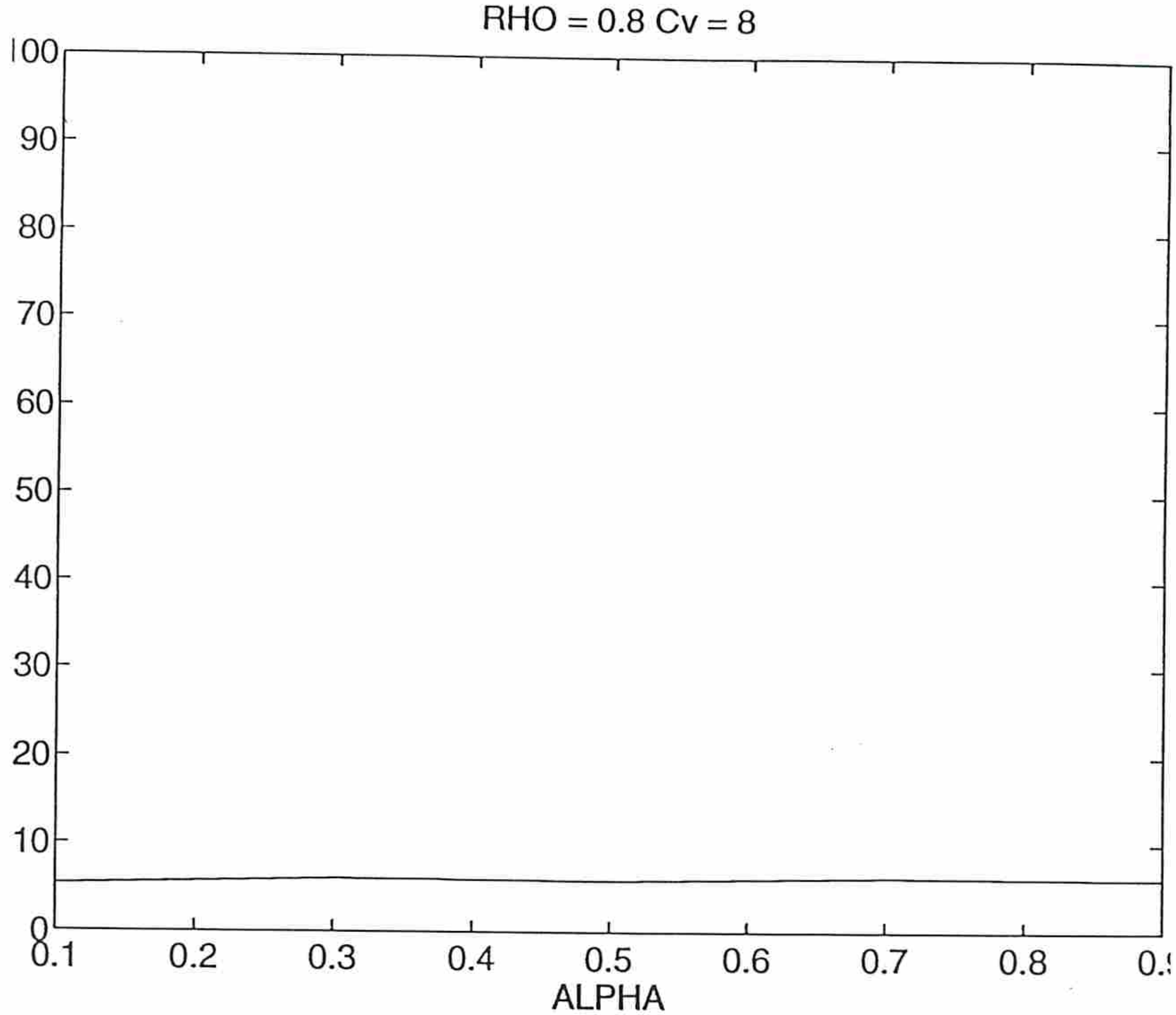

Figure 6: Percentual error of the delay estimation as a function of $\alpha(\rho = 0.8, C_v = 8)$

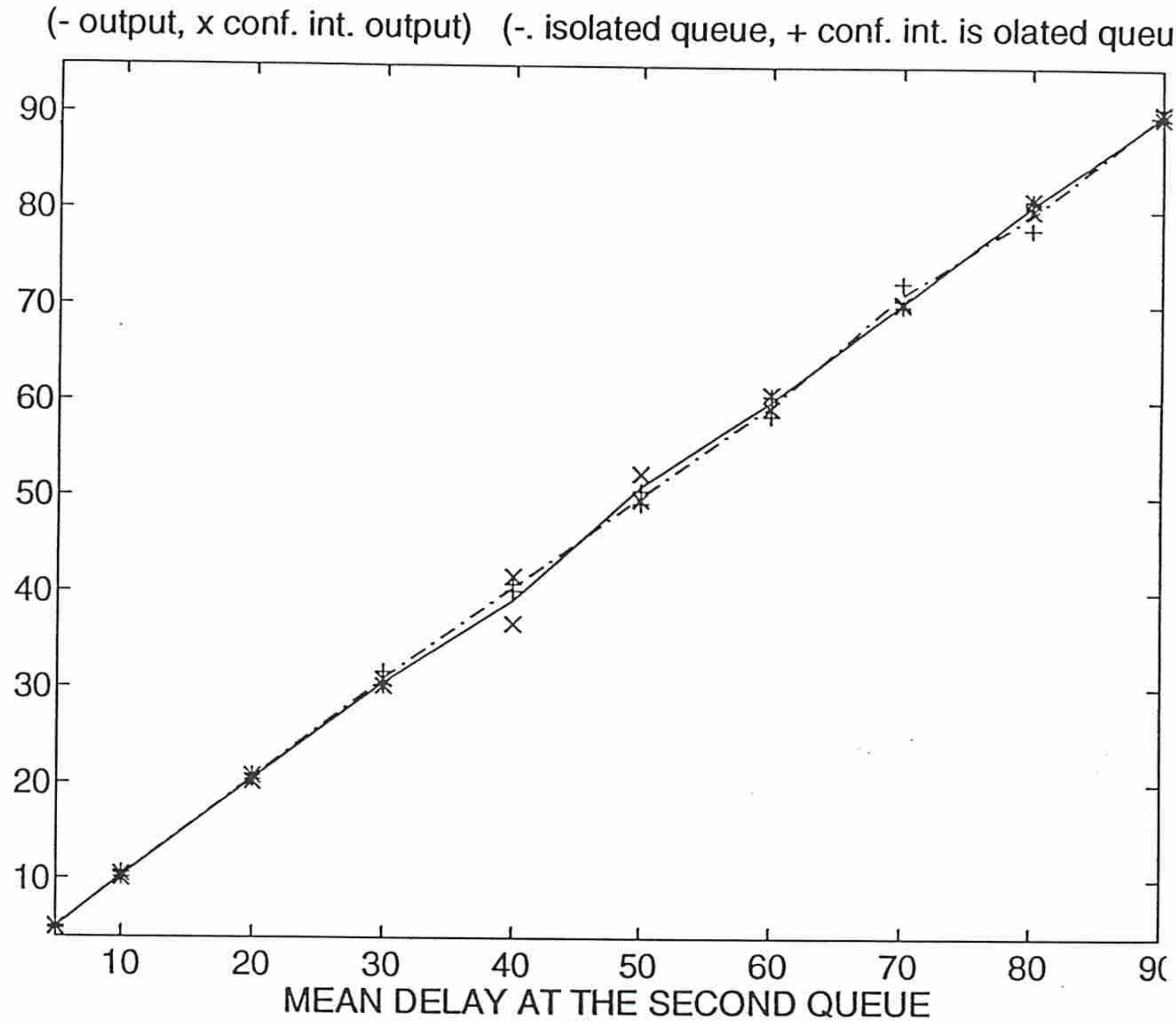

Figure 7: Estimated mean delay by using the matching procedure and mean delay at the second queue

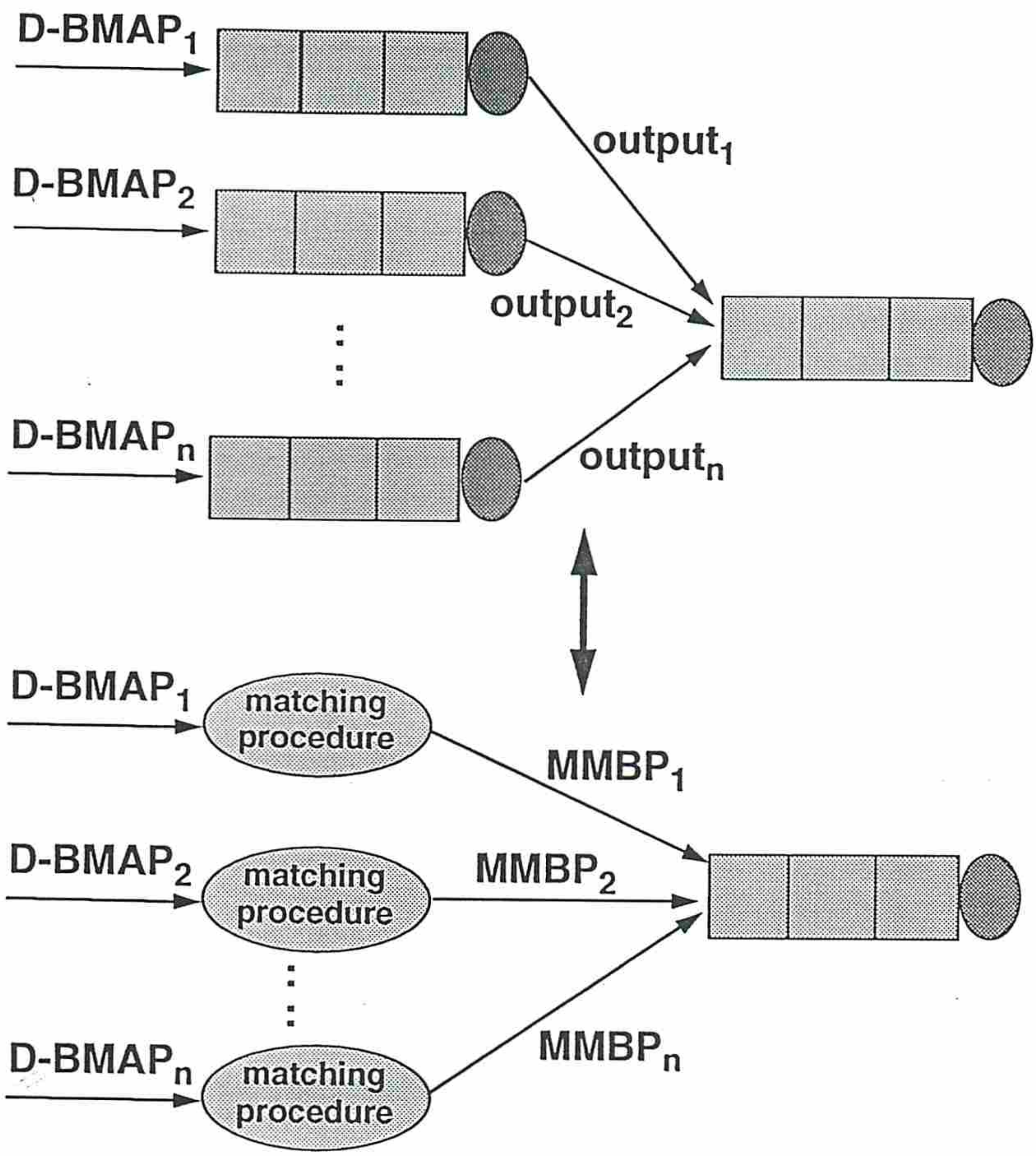

Figure 8: Scheme for the second set of experiments - the output of each queue was substituted by a two-sate MMBP

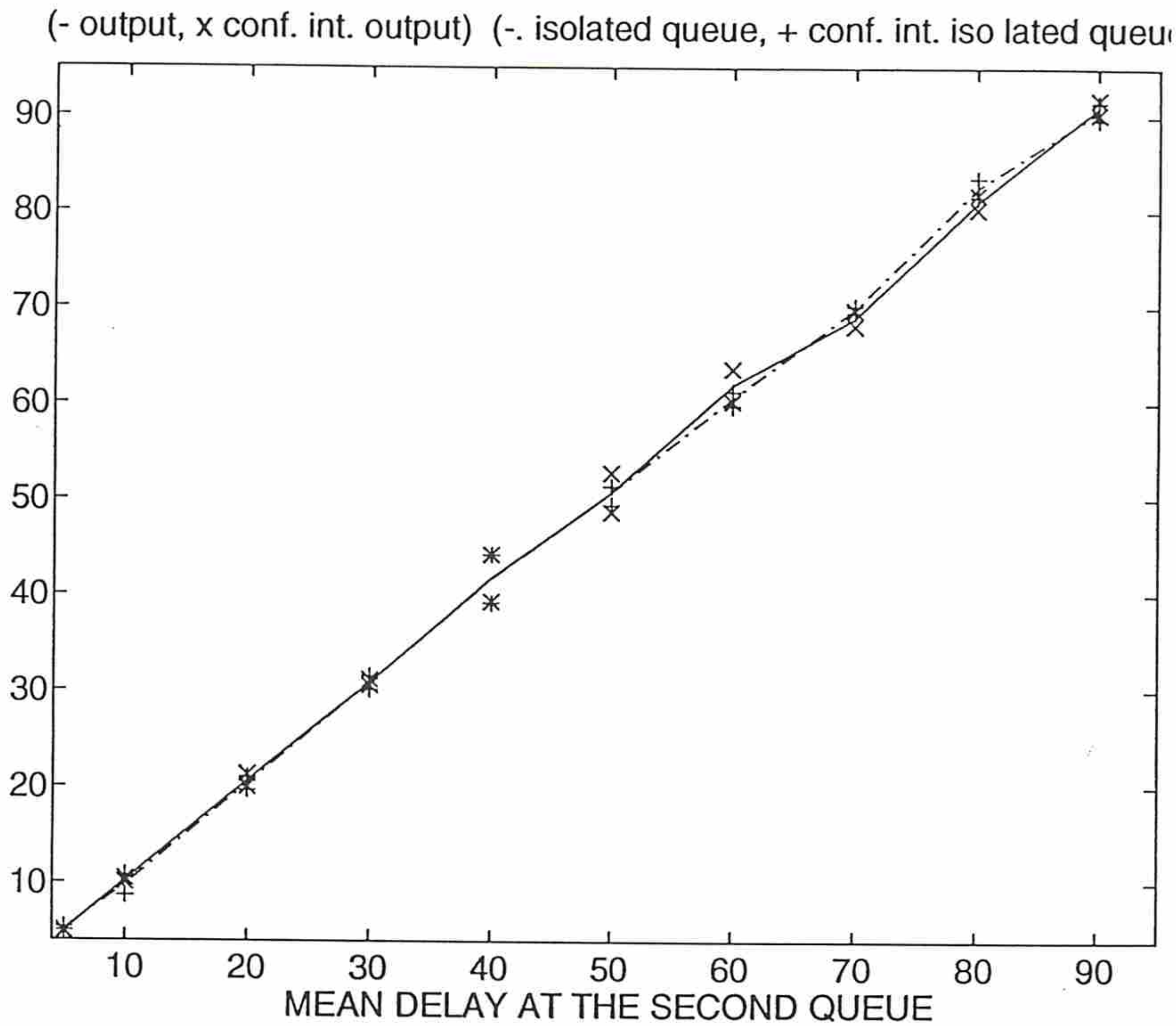

Figure 9: Estimated mean delay by using the matching procedure and mean delay at the second tandem queue

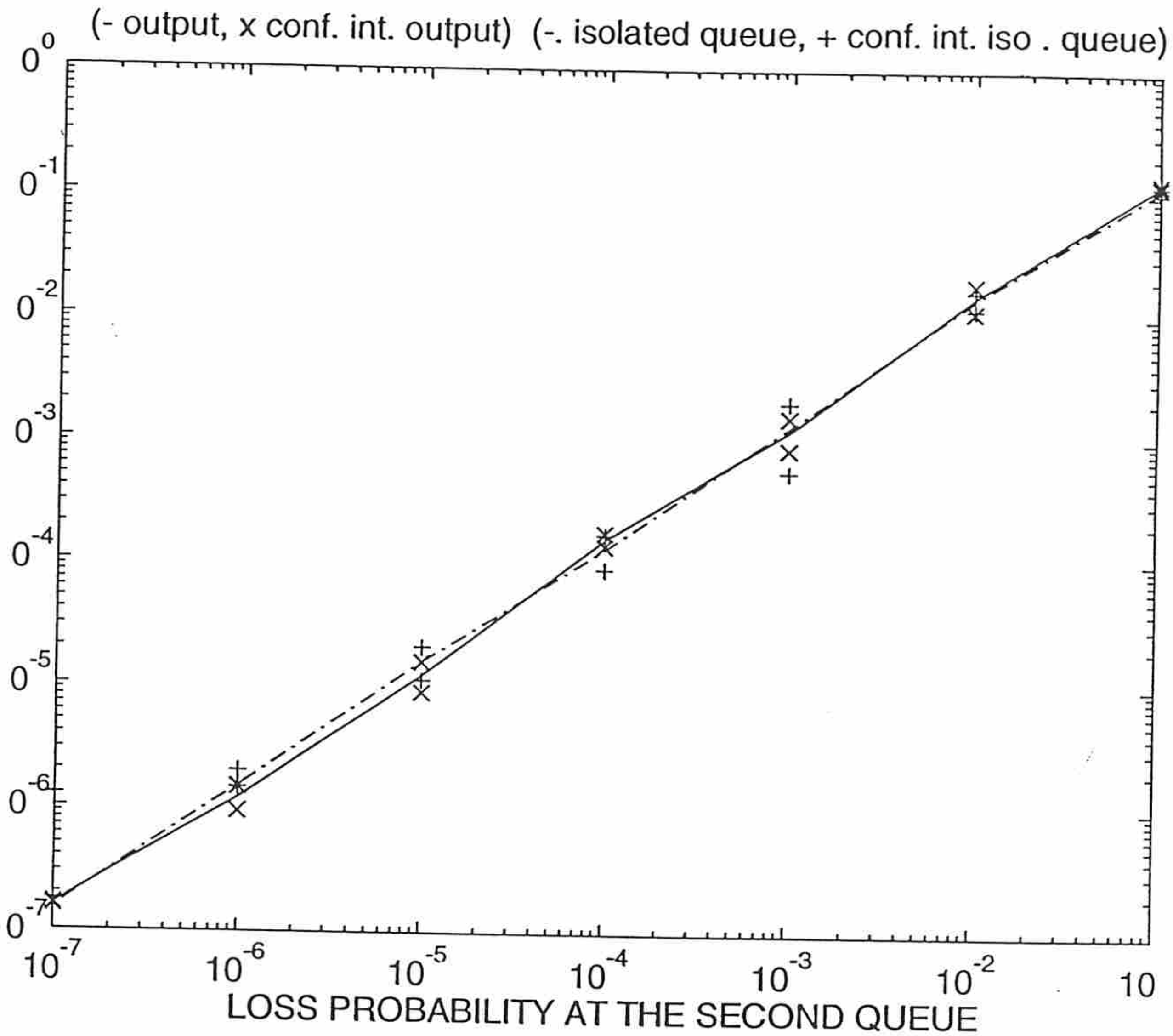

Figure 10: Estimated loss probability by using the matching procedure and loss probability at the second tandem queue