A General Performance Model For Mobile Slotted ALOHA Networks With Capture

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Abstract -- A general analytical model is proposed to evaluate the performance of a mobile slotted ALOHA network. Near-far effects are incorporated in the model (in an approximate way) by considering a multi-group system. The model approximates the network behavior by an iterative solution between multiple one-dimensional Markov chains, one for each group. The model is quite broadly applicable since (1) different physical layer parameters, such as modulation and coding, are abstracted by a set of conditional success probabilities, and (2) different groups can have different link layer parameters, such as the probabilities of packet generation and retransmission. As an example, a 5-group system with fading is used to illustrate the usefulness of the proposed model. The agreement between the analytical and simulation results is good. It is found that the impact of the fading effect on network performance depends on the significance of the near-far effect. The problem of unbalanced throughput between near and far groups is solved by assigning different retransmission probabilities to different groups.

I. Introduction

ALOHA is an attractive protocol due to its simplicity [1], but its limited capacity ($1/2e$) makes it unacceptable for many applications. So there have been many efforts devoted to increase performance. By confining the packet transmission within a fixed time interval, slotted ALOHA increases the capacity to $1/e$ [2]. In the ground radio environment, the propagation loss and multipath fading can make the received power of packets differ by an order of magnitude allowing advantage to be taken of the capture effect, the phenomenon that the packet with the strongest power is successfully received even in the presence of other interfering packets. The capture effect further improves the system capacity. Many different capture models have been proposed to evaluate the system performance [3], [4], [5]. Most models comprise accuracy to achieve analytical tractability. For example, the multi-level power capture effect can be shown to drive the system capacity close to 100% [6], but this is based on the unrealistic assumption that high power groups will never be affected by the transmission of low power groups.

The stability problem is another important issue with ALOHA system[7]. One approach to stabilize a system is by choosing an appropriate retransmission probability. The stability problem for a system with capture is harder to analyze. In [8], the stability issue of mobile slotted ALOHA with capture is discussed. The stability condition for a two-group slotted ALOHA system with capture effect can be found in [9].

As pointed out in [6], multi-group slotted ALOHA with capture can improve the system capacity (throughput). It is of interest to know how much improvement can be obtained under different capture models. In addition, the delay performance and stability are also important in a real system with finite population. To the best of our knowledge, there has been no work on the performance analysis of the multi-group slotted ALOHA with a general capture model. In the present study, we propose an analytical model to evaluate the performance of such a system. As expected, the higher power groups usually have higher throughput than the lower power groups due to capture. With the proposed model, we found that by assigning higher retransmission probabilities to low power groups, all groups can have more balanced throughputs. The impact of fading and near-far effects on network throughput is also investigated by the model. It is found that fading may or may not impact network throughput depending on the significance of the near-far effect.

In Section II, the model for the terminal (user) in the system is defined. In Section III, a general capture model which can be used for a variety of physical layer parameters such as modulation, coding, channel characteristics, etc., is introduced. In Section IV, the network model and the interaction between different groups are presented. In Section V, the performance of a 5-group system with particular channel and capture models is analyzed. The influence of system parameters on the network performance is discussed.

II. Terminal Model

Consider a slotted ALOHA (S-ALOHA) system with K groups of terminals, labeled as G_1, $G_2, \ldots, G_K$. For group i, $i = 1, 2, \ldots, K$, G_i consists of M_i single-buffered terminals that are identical and independent. Each terminal is either in the idle state or in the backlogged state. When a terminal is in the idle state, a packet will be generated and transmitted in the next slot with probability σ_i. If the transmission is successful, the terminal will receive a positive feedback right after the transmission and remain in the idle state. Whereas if the transmission is not successful, it will enter the backlogged state. When it is in the backlogged state, it will transmit the packet with probability q_i in each slot until it succeeds.

III. General Capture Model

In general, the probability that the central station can successfully receive a packet in a slot depends on the activities of all the K groups. Define the activity vector $\underline{a} = (a_1, a_2,..., a_K)$, where a_i, $i = 1, 2,..., K$, is the number of transmissions from group i. Given the activity vector in a slot, the probability $p_i(\underline{a})$ that one of terminals in group i successfully transmits a packet to the central station depends on the factors such as modulation, coding, propagation law, channel characteristics, etc. We can think of $p_i(\underline{a})$ as a parameter that depends on the operating environment and hardware implementation. $p_i(\underline{a})$ serves as input parameter when the network performance is evaluated. In Section V.B, an example is used to show the derivation of $p_i(\underline{a})$ given the specific channel and capture models.

IV. Network Model

With the terminal model and the capture model, the network behavior can be described as a discrete-time K-dimensional Markov chain with the state being the number of backlogged terminals in each group. Because the state space grows exponentially, the K-dimensional Markov chain is analytically intractable in general. To avoid solving the K-dimensional Markov chain, we model the system as K interactive one-dimensional Markov chains, one for each group. How the K subchains interact with each other is detailed as follows.

A. The Markovian Model for a Decoupled Group

For a particular group i, the state variable of the corresponding Markov chain is the number of backlogged terminals in the group. The state transition probabilities are governed by M_i, σ_i, q_i, and $\phi_i(k)$, where $\phi_i(k)$ is the conditional probability that one of terminals in group i succeeds given that k terminals in group i transmit. Although $\phi_i(k)$ depends on the activities of all other groups, we assume the interference on group i due to other groups is statistically the same and independent in every slot. We will show how to compute $\phi_i(k)$ after the following notations are defined. Let

n_i = the state variable of group i, $i = 1, 2, \ldots, K$.

$\pi_i(k)$ = the stationary probability that group i is in state k.

$\underline{\pi}_i = (\pi_i(0), \pi_i(1), \ldots, \pi_i(M_i))$, i.e., the stationary state vector of group i.

a_i = the number of transmissions in group i, where $0 \le a_i \le M_i$.

$\min(x, y)$ = the smaller of x and y.

$$A_i(j, k) = \Pr[j \text{ unbacklogged terminals in group } i \text{ transmit} \mid n_i = k]$$
$$= \binom{M_i - k}{j} \sigma_i^j (1 - \sigma_i)^{M_i - k - j}, \text{ where } 0 \le j \le M_i - k.$$

$$B_i(j, k) = \Pr[j \text{ backlogged terminals in group } i \text{ transmit} \mid n_i = k]$$
$$= \binom{k}{j} q_i^j (1 - q_i)^{k - j}, \text{ where } 0 \le j \le k.$$

We have

$$\phi_i(k) = \sum_{j_1}^{M_1} \cdots \sum_{j_{i-1}}^{M_{i-1}} \sum_{j_{i+1}}^{M_{i+1}} \cdots \sum_{j_K}^{M_K} p_i(j_1, \ldots, j_{i-1}, k, j_{i+1}, \ldots, j_K)$$
$$\cdot \Pr[a_1 = j_1, \ldots, a_{i-1} = j_{i-1}, a_{i+1} = j_{i+1}, \ldots, a_K = j_K | a_i = k],$$

where $p_i(j_1, \ldots, j_{i-1}, k, j_{i+1}, \ldots, j_K)$ is the probability that one terminal in group i succeeds given the activity vector $(j_1, ..., j_{i-1}, k, j_{i+1}, ..., j_K)$. Further assuming that the activities of different groups are independent, we have

$$\phi_i(k) = \sum_{j_1}^{M_1} \cdots \sum_{j_{i-1}}^{M_{i-1}} \sum_{j_{i+1}}^{M_{i+1}} \cdots \sum_{j_K}^{M_K} p_i(j_1, ..., j_{i-1}, k, j_{i+1}, ..., j_K) \cdot \prod_{l=1, l \neq i}^{K} \Pr[a_l = j_l] .$$

By the law of total probability,

$$\Pr[\, a_l = j_l \,] = \sum_{k=0}^{M_l} \pi_l^k \Pr[\, a_l = j_l \mid n_l = k \,]$$

$$= \sum_{k=0}^{M_l} \pi_l^k \sum_{b=0}^{min(k, j_l)} \Pr[\, b \text{ backlogged terminals}$$

$$\text{and } (j_l - b) \text{ unbacklogged terminals transmit} \mid n_l = k \,]$$

$$= \sum_{k=0}^{M_l} \pi_l^k \sum_{b=0}^{min(k, j_l)} B_l(b, k) A_l(j_l - b, k) .$$

We have shown in the above how to compute $\phi_i(k)$ given the stationary state occupancy probabilities of all interfering groups. Namislo [11] expressed the state transition probabilities for a slotted ALOHA network with capture in terms of the number of terminals, the probability of packet generation, the retransmission probabilities, and the conditional success probability given the number of active transmissions in a slot. The state occupancy probabilities can then be obtained recursively [10]. Now we can compute $\underline{\pi}_i$ in terms of M_i, σ_i, q_i, and $\phi_i(k)$, $k = 1,2,..., M_i$. From $\underline{\pi}_i$, the average throughput and delay can be obtained. The drift analysis can be used to determine whether group i is stable or bistable.

B. The Interaction of the Decoupled Markov Chains

In Section IV.A, we showed how to compute the state probabilities $\underline{\pi}_i$ given the state probabilities of the other groups $\{\underline{\pi}_j\}_{j=1, j\neq i}^{j=K}$. We now introduce the iterative procedure that can be used to obtained the state probabilities of all groups. The procedure consists of 3 steps.

Step 1: Set $\underline{\pi}_i = (1, 0,..., 0)$, $i = 1, 2,..., K$.

Step 2: For $i = 1, 2,..., K$, do Step 2.1-2.

Step 2.1: Compute $\phi_i(k)$, $k = 1, 2,..., M_i$.

Step 2.2: Solve for $\underline{\pi}_i$ in terms of M_i, σ_i, q_i, and $\phi_i(k)$, $k = 1,2,..., M_i$, and $\{\underline{\pi}_j\}_{j=1, j\neq i}^{j=K}$.

Step 3: Stop if no significant change in $\{\underline{\pi}_j\}_{j=1}^{j=K}$ is observed. Otherwise, go to Step 2.

After the iterative procedure stops, the state occupancy probabilities can be used to determined the throughput, delay, and stability of each group.

V. Application

The model just presented is generic in the sense that it can be used to analyze the performance of a multi-group slotted ALOHA system with an arbitrary choice of physical layer parameters such as modulation, coding, and channel characteristics. All the complexity of the physical layer is summarized in the set of conditional success probabilities. $\{p_i(\underline{a})\}_{i=1}^{i=K}$ for all possible activity vectors $\underline{a}$. The size of the set of conditional success probabilities is not trivial. They can be either implemented as a subroutine or precomputed and stored in a database to shorten the computational time. Note that the set of conditional success probabilities is also needed if we want to evaluate the system performance by simulation.

As an example, we evaluate the throughput performance for a K-group slotted ALOHA with the fading and near-far effects described in the following.

A. Terminal Model and Channel Model

Fig. 1 shows the topology of the system of interest. The system consists of K groups of terminals, denoted by $G_1, G_2, \ldots, G_K$. The terminal model is the same as that in Section II, i.e., G_i is characterized by (M_i, σ_i, q_i). It is assumed that all terminals use the same power to transmit the packets to the central station. Because of the fading and near-far effects, the received power of a packet from G_i at central station is assumed to be constant over the entire slot duration and is distributed as a truncated normal random variable, denoted by $TN(\mu_i, \omega_i)$, with the probability density function (pdf) given by

$$f(x) = \begin{cases} \dfrac{C}{\sqrt{2\pi}\omega_i} exp\left(-(x-\mu_i)^2/2\omega_i^2\right) & \text{if } \mu_i - \omega_i < x < \mu_i + \omega_i \\ 0 & \text{elsewhere.} \end{cases}$$

The normalization constant C in the pdf is given by

$$C = \left[\int_{\mu_i - \omega_i}^{\mu_i + \omega_i} \frac{1}{\sqrt{2\pi}\omega_i} exp\left(-(x-\mu_i)^2/2\omega_i^2\right)dx\right]^{-1}$$

$$= \left[\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} exp\left(-x^2/2\right)dx\right]^{-1}.$$

The variance of $TN(\mu, \omega)$ is equal to $(\beta\omega)^2$, where $\beta \approx 0.54$.

Although the truncated normal density function is just a hypothetical channel model, one can see how the range of power fluctuation influence the system performance by using different values of ω_i.

B. Capture Model

It is assumed that the central station can successfully receive a packet if the power of the strongest packet is larger than the sum of all interfering packets by at least a factor of R (called the capture ratio.)

Given the activity vector $\underline{a} = (a_1, a_2, ... a_K)$, the conditional success probability

$$p_i(\underline{a}) = a_i \cdot \Pr[X > RY] ,$$

where X is the received power of a particular packet of group i and Y is the received power of all other packets. Given $a_i \neq 0$, X is distributed as $TN(\mu_i, \omega_i)$ and Y has mean and variance given by

$$E[Y] = \sum_{j=1}^{K} a_j \mu_j - \mu_i ,$$

and

$$VAR[Y] = \sum_{j=1}^{K} a_j (\beta\omega_j)^2 - (\beta\omega_i)^2 .$$

The exact pdf of Y is the convolution of the pdf's of the received power of all other packets. To simplify the computation of $p_i(\underline{a})$, we first approximate the pdf of Y by a truncated normal density with parameters μ_Y and ω_Y where $\mu_Y = E[Y]$ and $\omega_Y = \sqrt{VAR[Y]}/\beta$. Then we approximate RY by $TN(R\mu_Y, R\omega_Y)$. With these approximations, $p_i(\underline{a})$ can be further simplified as (assuming $a_i \neq 0$)

$$p_i(\underline{a}) = \Pr[X > RY]$$

$$\approx \Pr\left[TN(\mu_i, \omega_i) > R \cdot TN(\mu_i, \omega_i) \right]$$

$$\approx \Pr\left[TN(\mu_i, \omega_i) > TN(R\mu_i, R\omega_i) \right]$$

$$= \int_{\mu_i - \omega_i}^{\mu_i + \omega_i} \frac{C}{\sqrt{2\pi}\omega_i} exp\left(-(x-\mu_i)^2 / 2\omega_i^2 \right)$$

$$\cdot \int_{R\mu_Y - R\omega_Y}^{x} \frac{C}{\sqrt{2\pi}R\omega_Y} exp\left(-\frac{1}{2}\left(\frac{y - R\mu_Y}{R\omega_Y} \right)^2 \right) dy dx.$$

Now $p_i(\underline{a})$ can be evaluated numerically.

C. Numerical Results

We consider a 5-group slotted ALOHA system with 5 terminals in each group. We assume $\mu_i = \gamma \cdot \mu_{i-1}$, $i = 2, 3, 4, 5$, and $\omega_i = \delta \cdot \mu_i$, $i = 1, 2, 3, 4, 5$. γ ($0 < \gamma < 1$), the ratio of the mean power of the adjacent groups, represents the significance of near-far effect. The smaller γ is, the more likely the capture effect will occur between adjacent groups. δ ($0 \le \delta \le 1$) is used to model different fading characteristics. The larger δ is, the larger the dynamic range of received power is. The capture ratio R is either 4 (representing stronger capture capability of the receiver) or 10 (representing weaker capture capability.) To check the validity of the decoupling approximation of the proposed model, both the analytical model and simulation use the same set of conditional success probabilities, which are computed using the truncated normal approximation presented in Section V.B.

Fig. 2 and Fig. 3 shows the throughput of each group for various γ and R. As expected, the throughput is larger when γ is smaller given the rest of the parameters fixed. Similarly, given the rest of the parameters fixed, throughput improves as R decreases. The results show that capture effect improves the performance.

Fig. 4 gives an example demonstrating that the fading effect may improve the throughput performance. This is because when the near-far effect is not significant ($\gamma = 0.5$) and the capture ratio is high ($R = 10$), the fluctuating received power due to fading increases the possibility of a packet being captured.

Fig. 5 gives an example showing that fading effect may not improve the throughput. This is because when the near-far effect is significant ($\gamma = 0.125$) and the capture ratio is low ($R = 4$), the fading effect may destroy the advantage of near groups over far groups and thus effectively prevent capture from occurring. But the degradation is compensated by the fact that the fading effect does help the terminals in the same group capture the receiver.

Fig. 6 shows the throughput under different data traffic loads. When the load is light, each group has approximately the same throughput. As the load increases, the imbalance of throughput between near and far groups becomes more and more serious. The unfairness between near and far groups can be mitigated by assigning higher retransmission probability to more distant groups. Fig. 7 shows the balanced throughput with a particular choice of retransmission probabilities.

In all the examples presented, one can see that due to the decoupling approximation, the analytical model always slightly overestimates the performance. Note that in Fig. 4, although the bias of the model is observed, the relative error is only about 2.6%. The good agreement between the analytical and simulation results validates the decoupling approximation made by the model.

Conclusion

A general analytical model is proposed for a slotted ALOHA network with multiple heterogeneous groups, which can be used to evaluate the performance of a mobile slotted ALOHA network. The model, which approximates the network by multiple interactive one-dimensional Markov chains, can be used to find the throughput and delay as well as the stability for each group. A rich class of slotted ALOHA systems can be modelled by varying the conditional success probabilities which summarize the physical layer parameters (such as modulation, capture ratio, channel characteristics, etc.).

As an example, a 5-group system is studied. With the assumed channel and capture model, it is found that the impact of the fading effect on the network performance depends on the significance of the near-far effect. The problem of unbalanced throughput between near and far groups can be solved by assigning different retransmission probabilities to different groups. The good correspondence between the analytical and simulation results verifies the decoupling approximation used by our model.

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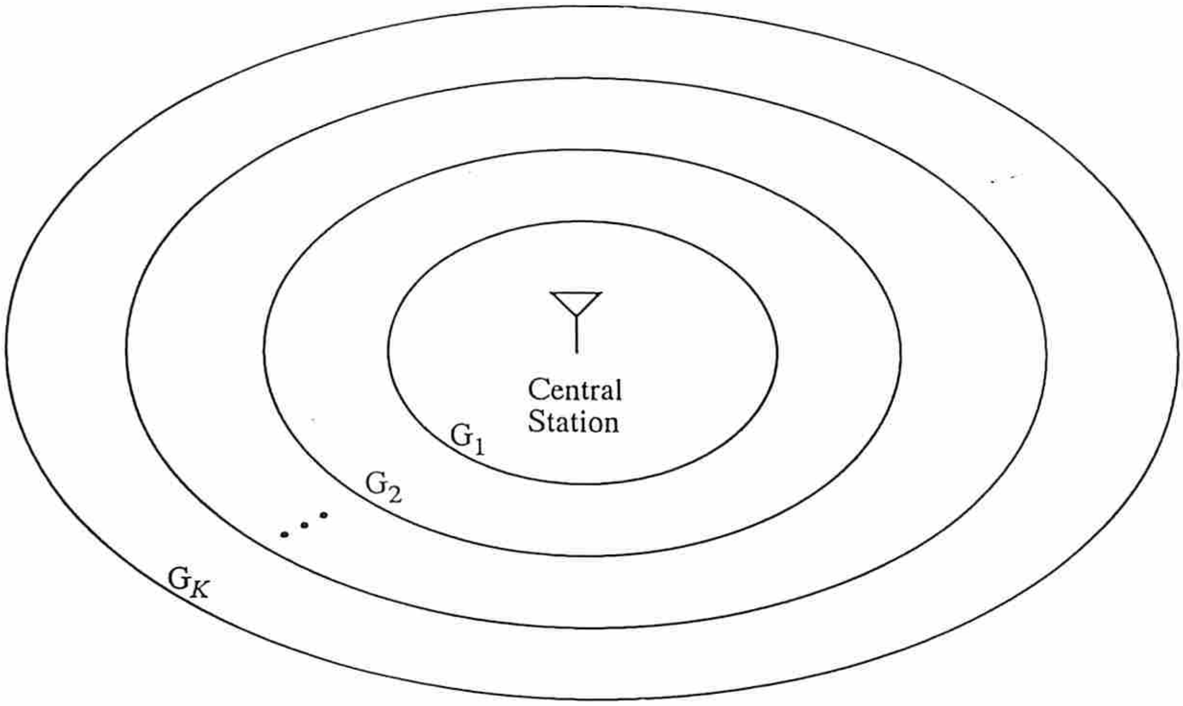

Fig. 1. The topology of a K-group slotted ALOHA system. All groups use the same transmission power. But due to the near-far effect, the received power at the central station is different.

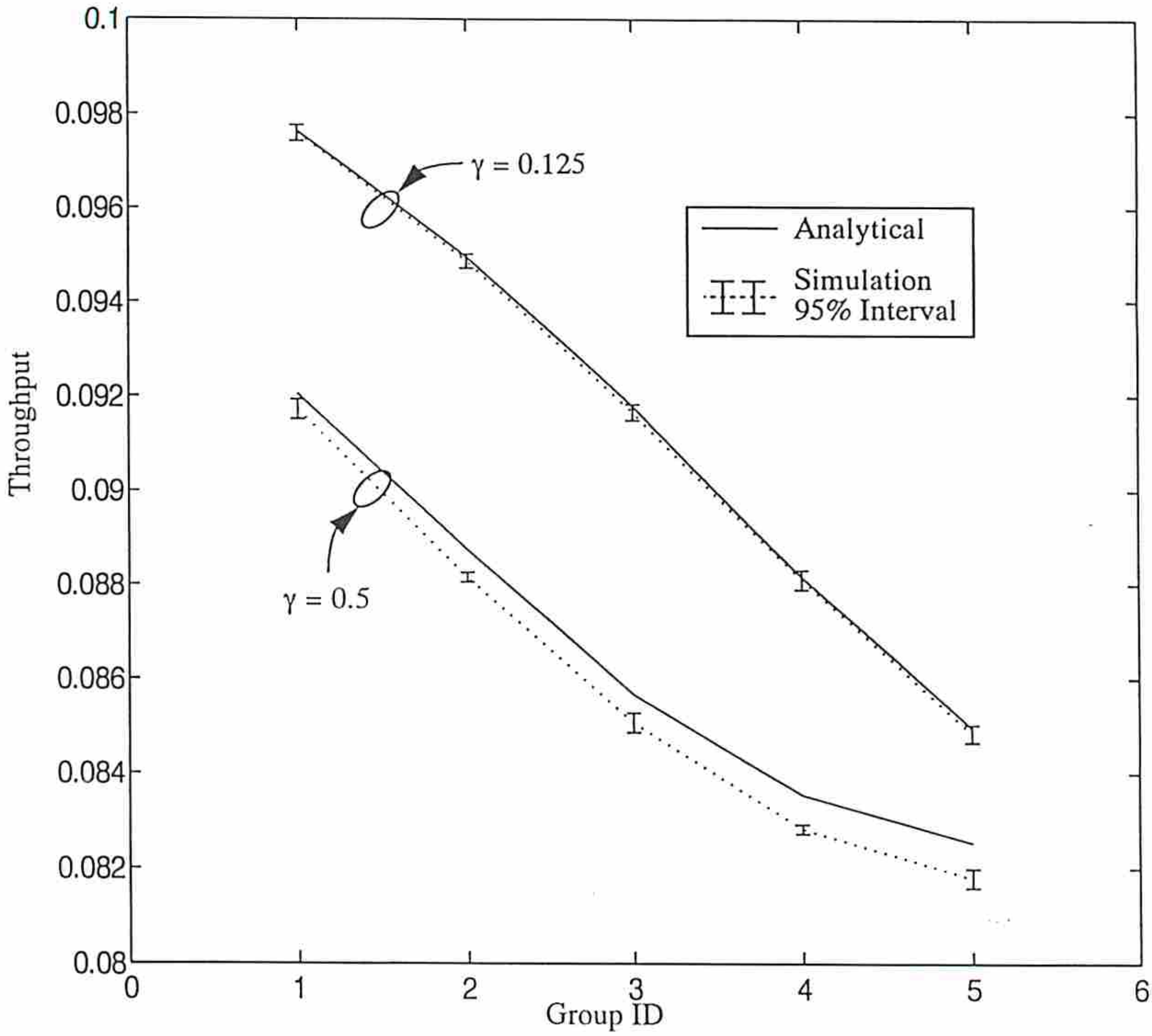

Fig. 2. Throughput for a 5-group system with $\sigma = 0.02$, $q = 0.1$, $\delta = 1.0$, and $R = 4$. Note that as γ decreases, throughput increases.

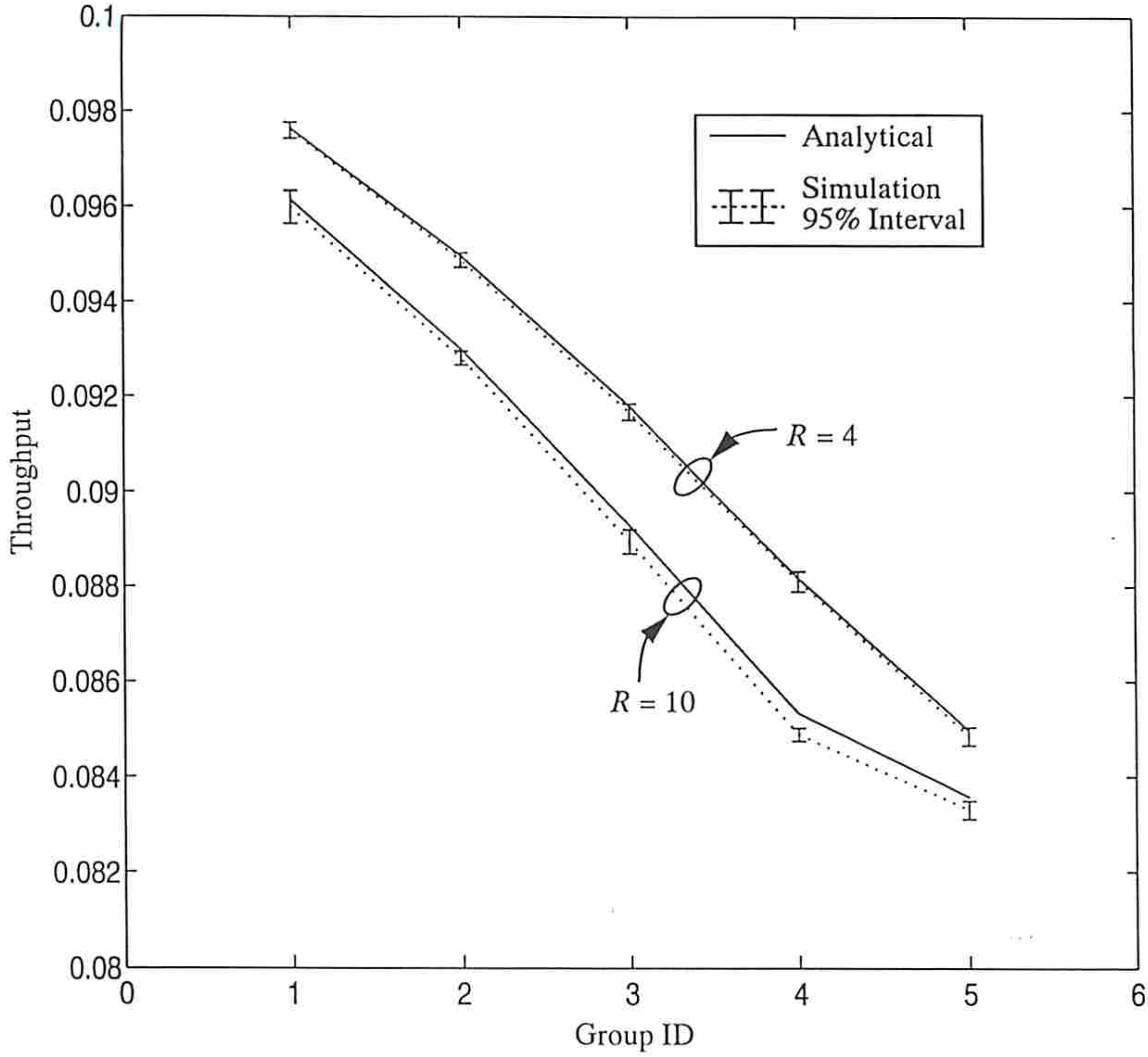

Fig. 3. Throughput for a 5-group system with $\sigma = 0.02$, $q = 0.1$, $\gamma = 0.125$, and $\delta = 1.0$. Note that as R decreases throughput increases.

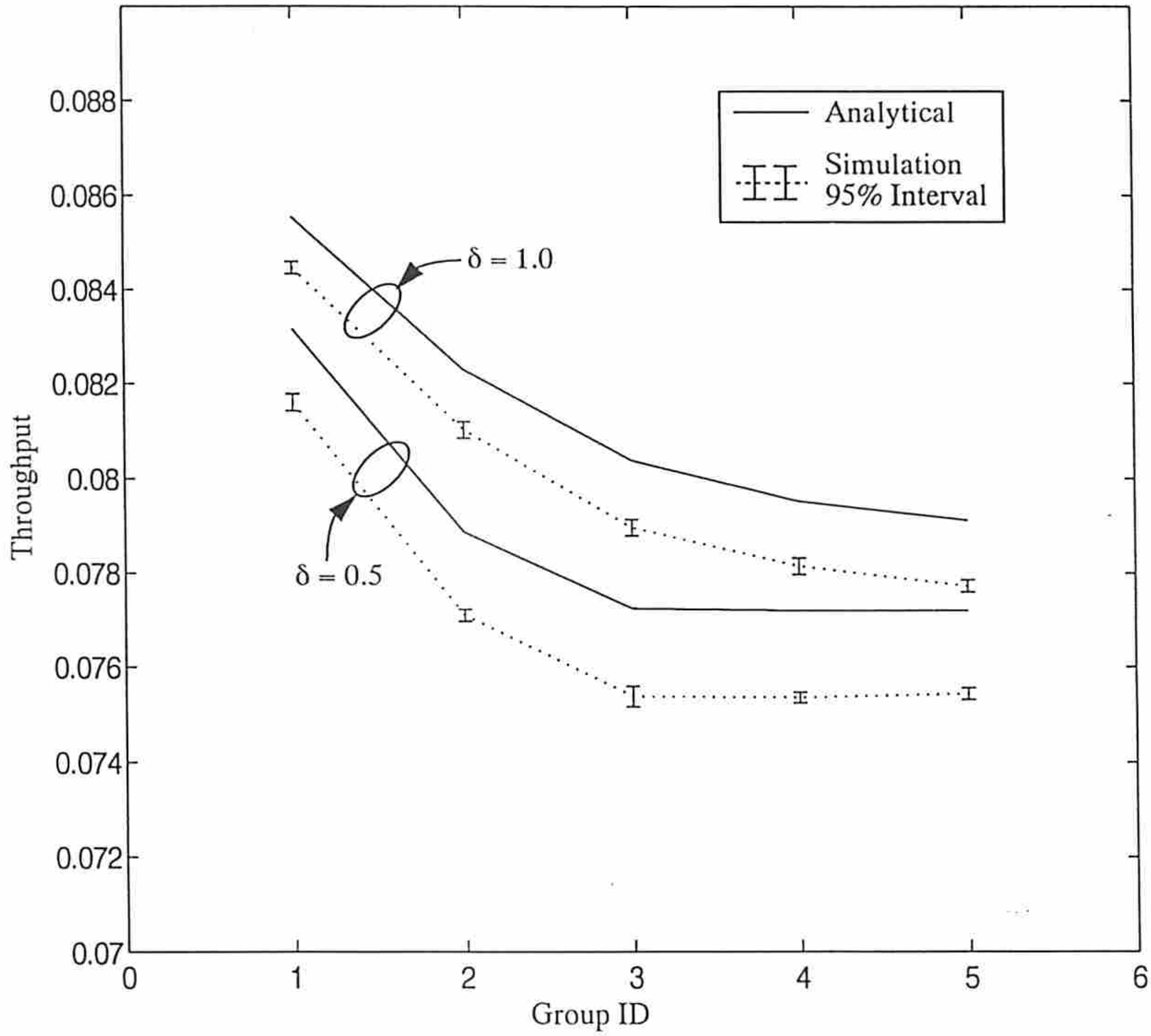

Fig. 4. Throughput for a 5-group system with different fading parameters (δ). In the cases where the near-far effect is not significant (e.g., $\gamma = 0.5$, $R = 10$ in the example), throughput increases as δ increases. (For other parameters, $\sigma = 0.02$ and $q = 0.1$ for all groups.)

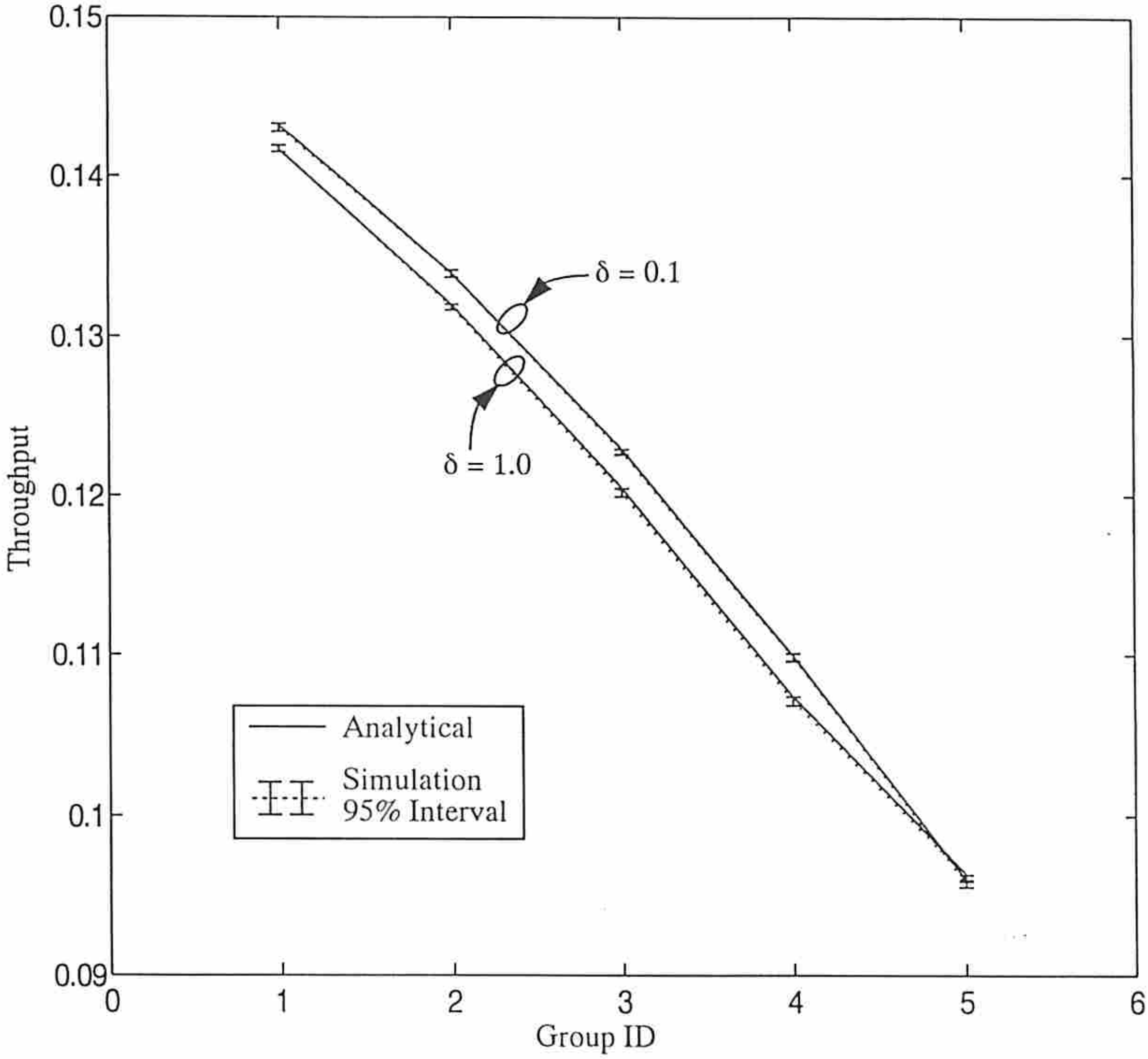

Fig. 5. In the cases where the near-far effect is significant (e.g., $\gamma = 0.125$ and $R = 4$ in the example), throughput decreases as δ increases. ($\sigma = 0.03$ and $q = 0.1$ for all groups.)

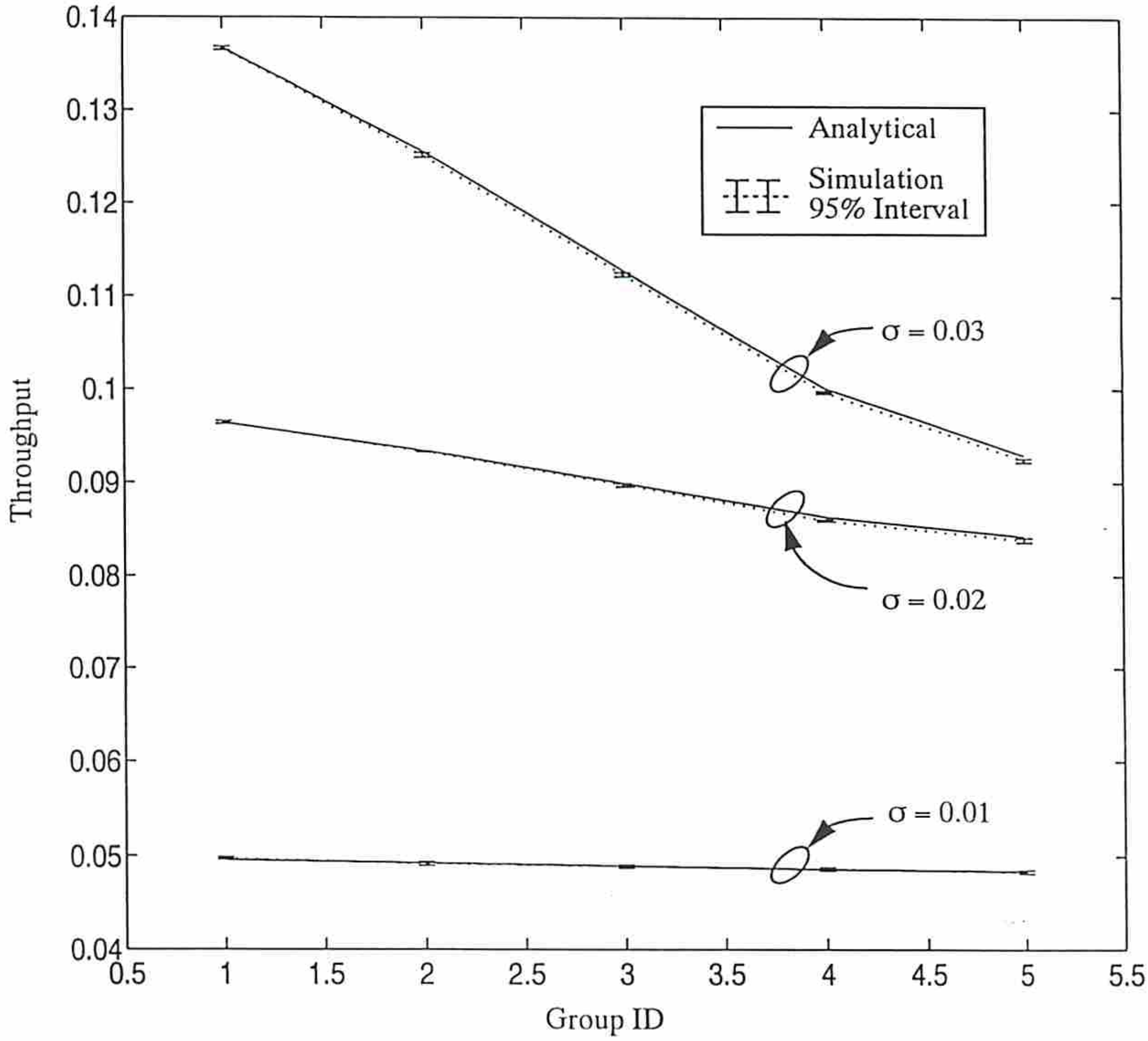

Fig. 6. Unbalanced throughput for a 5-group system with $q = 0.1$, $\gamma = 0.25$, $\delta = 1.0$, and $R = 4$ for all groups.

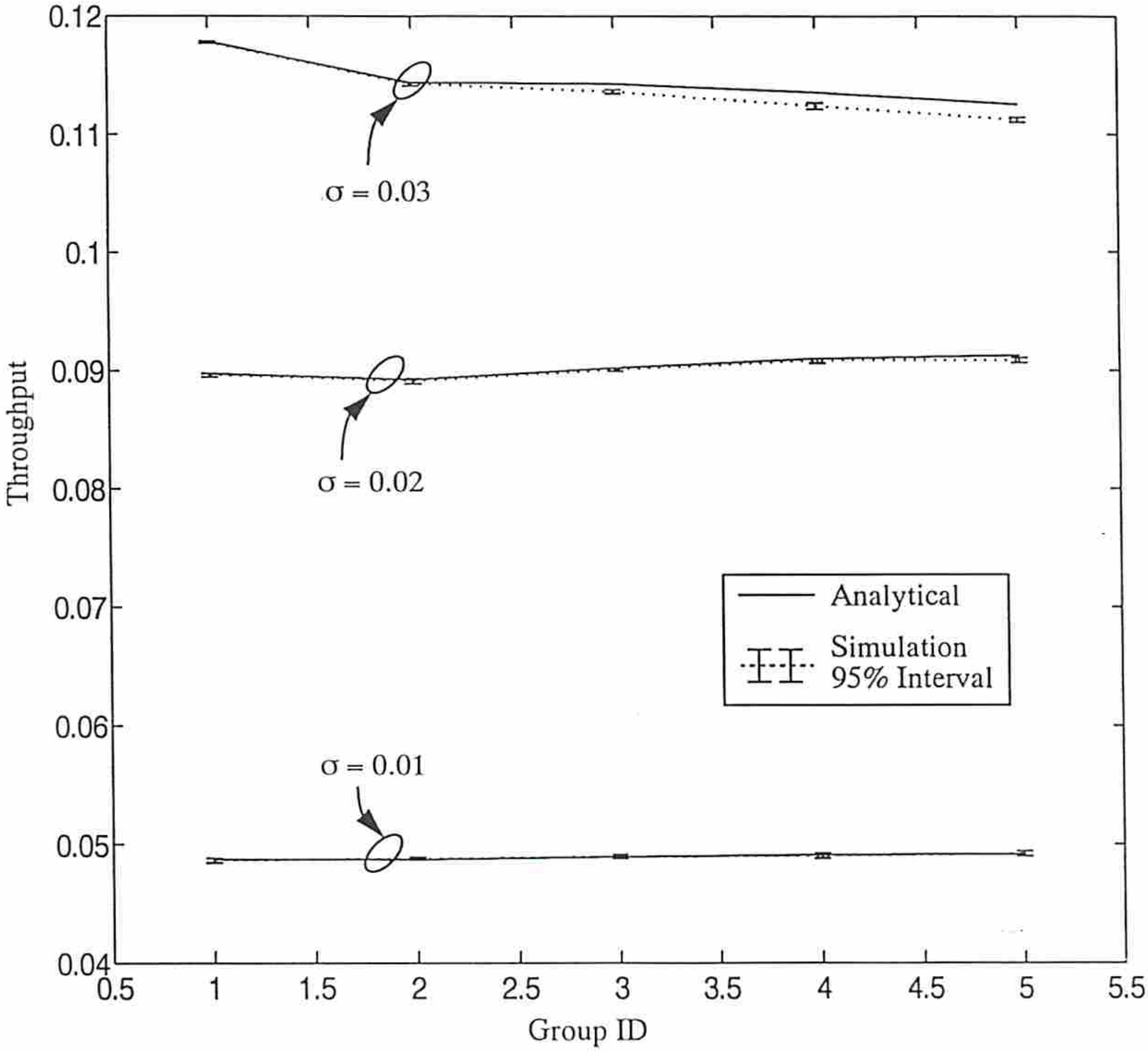

Fig. 7. Balanced throughput for a 5-group system with different restransimission probabilities for each group. Specifically, $\gamma = 0.25$, $\delta = 1.0$, and $R = 4$ for all groups; $q_1 = 0.03$, $q_2 = 0.055$, $q_3 = 0.1$, $q_4 = 0.16$, and $q_5 = 0.2$ for groups 1 through 5, respectively.