An Analytical Model For Multi-Group Slotted ALOHA With Capture

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Abstract -- The performance of multi-group slotted ALOHA with capture improves dramatically over single-group slotted ALOHA. An analytical model is proposed to evaluate the throughput, delay, and stability of a multi-group slotted ALOHA system with capture. The model approximates a K-group system as K interactive slotted ALOHA systems with independent Bernoulli jamming sources. An iterative procedure is used to find the parameters of the Bernoulli jamming sources. The fairness among groups is improved by assigning different retransmission probabilities to different groups. More balanced throughput and delay can be achieved. In the case that the total input traffic exceeds the system capacity, it might not necessarily lead to the total collapse of the whole system. High-power groups may have acceptable throughput due to the capture effect.

Keywords: Analytical modeling, slotted ALOHA, capture, heterogeneous networks

I. Introduction

In a radio ALOHA system, the received power of packets may be different due to the difference in transmission power, distance, or shadowing. The difference in the received power of colliding packets gives a chance for the packet with the strongest power to be successfully detected.

This phenomenon is called the capture effect [1], [2]. It is well known that the capture effect can increase the capacity of ALOHA systems [3], [4], [5]. Metzner shows that by increasing the number of different power levels, the utilization can approach one [6], but his model cannot be used to evaluate the delay performance. Goodman and Saleh found that under the near-far effect even the user with the weakest signal benefits from the capture effect [7]. But their model cannot be used to study the issue of stability, which is inherent in ALOHA systems [8]. Onozato, Liu, and Noguchi found the stability conditions for a two-group slotted ALOHA system with capture [9]. Plas and Linnartz considered the stability of a slotted ALOHA system with fading, shadowing, and near-far effect [10]. In [10], the stability of the whole system is treated as the stability in one group.

In the present study, we consider a multi-group slotted ALOHA system where different groups reach the central station with different power levels resulting in a capture effect. The throughput-delay performance and the stability of each group will be evaluated by the proposed model. Though the near-far effect can improve the capacity of ALOHA systems, it introduces unfairness between users in the system. The fairness issue will also be examined.

In Section II, an analytical model for a two-group slotted ALOHA system is introduced. A decoupling approximation is used to find the performance of each group. In Section III, the two-group model is extended for a multi-group system. In Section IV, examples of the application of the model are presented and the fairness and stability issues are discussed.

II. Model for 2-group Slotted ALOHA

A. Capture Model and Terminal Model

Consider a noise-free slotted ALOHA (S-ALOHA) system with two groups of terminals (group 1 and group 2). The terminals in group 1 can reach the central station with much stronger power than the terminals in group 2. The two levels of power are assumed to differ substantially such that the central station can always receive the packet from group 1 if exactly one terminal in group 1 transmits and less than C (called the threshold of backward interference) terminals in group 2 transmit. In other words, if C or more terminals in group 2 transmit, the total interference is assumed to be large enough to prevent group 1 from capturing the receiver at the central station. A terminal in group 2 can make a successful transmission if none of terminals in group 1 transmits and exactly one terminal in group 2 transmits.

Assume group 1 consists of M_1 single-buffered terminals that are identical and independent. Each terminal is either in the idle state or in the backlogged state. When a terminal is in the idle state, a packet will be generated and transmitted in the next slot with probability σ_1. If the transmission is successful, the terminal will receive a positive feedback right after the transmission and remain in the idle state. Whereas if the transmission is not successful, it will enter the backlogged state. When it is in the backlogged state, it will transmit the packet with probability q_1 in each slot until it succeeds. Similarly, there are M_2 terminals in group 2, and the probabilities of packet transmission when a terminal is in the idle state and in the backlogged state are σ_2 and q_2, respectively.

B. Network Model

Because the terminals within a group are independent and identical, a group can be characterized by the number of terminals in the backlogged state. Thus the behavior of the entire system can be modelled as a finite two-dimensional Markov chain with system state (n_1, n_2)

being the number of backlogged terminals in the two groups. For a system with only a small number of terminals, the Markov chain can be solved numerically and the network performance can be determined accordingly. For a large system with hundreds of terminals, the state space of the Markov chain will become intractably large. To avoid solving the two-dimensional Markov chain exactly, we model the system as two interactive one-dimensional Markov chains, one for each group.

Because of the two power levels, the terminals in group 1 are relatively immune to the transmissions from the terminals in group 2. Therefore it is a reasonable approximation to solve the Markov chain for group 1 independently of group 2.

Group 1 brings a jamming effect to terminals in group 2 since whenever terminals in group 1 transmit in a slot, it is impossible for terminals in group 2 to make a successful transmission. As an approximation, we represent group 1 as an independent Bernoulli jamming source with the parameter determined by the solution to the Markov chain for group 1. The behavior of group 2 then can be modelled by a S-ALOHA system with a Bernoulli jamming source (detailed in the next two sections). After the Markov chain for group 2 is solved, we represent group 2 as a Bernoulli jamming source and solve the chain for group 1. The procedure is repeated until the state occupancy probabilities of these two Markov chains do not change significantly.

Fig. 1 illustrates the interaction between group 1 and group 2. Before the decoupling approximation, the behavior of the 2-group system is described by a 2-dimensional Markov chain. After the decoupling approximation, the 2 groups interact with each other via the interface of two Bernoulli jamming sources. We will show how to find the interface parameters (J_{21} and J_{12}) in Section II.D. In the next section, we introduce the model used to evaluate the performance of a group with known parameters.

C. Slotted ALOHA with an Independent Bernoulli Jamming Source

A slotted ALOHA system with an independent Bernoulli jamming source (S-ALOHA/IBJS) is characterized by four parameters. The first three parameters (M, σ, and q) are the same as those of finite population slotted ALOHA system. The last parameter, J, is the probability that the jamming source will be active in a slot. The behavior of the system can be described as a discrete-time Markov chain with state being the number of backlogged terminals.

Let $A(k,n)$ be the probability that k unbacklogged terminals transmit in a slot given that the system is in state n and $B(k,n)$ be the probability that k backlogged terminals transmit given that the system is in state n. We have

$$A(k, n) = \binom{M-n}{k} \sigma^k (1-\sigma)^{M-n-k},$$

and

$$B(k, n) = \binom{n}{k} q^k (1-q)^{n-k}.$$

The state transition probability of going from state n to $(n+k)$ is given by

$$P_{n,n+k} = \begin{cases} A(k, n) & \text{if } 2 \le k \le (M-n) \\ A(1, n) [1 - (1-J) B(0, n)] & \text{if } k = 1 \\ A(1, n) (1-J) B(0, n) + A(0, n) [1 - (1-J) B(1, n)] & \text{if } k = 0 \\ A(0, n) (1-J) B(1, n) & \text{if } k = -1. \end{cases}$$

The state occupancy probabilities π_n can be obtained iteratively [11]. Given that n terminals are backlogged, the conditional throughput S_n is the probability that a packet is successfully received in a slot. We have $S_n = (1-J) [A(1, n) B(0, n) + A(0, n) B(1, n)]$. The average network throughput can then be found by:

$$S = \sum_{n=0}^{M} \pi_n S_n.$$

By Little's Formula the average delay D is given by $D = N/S$, where N is the average backlog size defined as

$$N = \sum_{n=0}^{M} n\pi_n .$$

To decide whether the system is stable or bistable, we study the drift [8]. Define the drift in state n (D_n) as the expected change in backlog over one time slot, starting from state n. Thus D_n is the expected number of new packets generated, less than the expected number of packets that are successfully transmitted, i.e., $D_n = (M-n)\sigma - S_n$. The system is stable if the equation $D_n = 0$ has only one solution whereas the system is bistable if D_n has 3 zeros. Fig. 2 shows the drift for a system with $M = 20$, $\sigma = 0.015$, and $q = 0.2$. For $J = 0.3$, 0.4, and 0.5, the system is stable (high throughput), bistable, and stable (low throughput), respectively.

D. The Interaction between Two Coupled Markov Chains

In this section, we will introduce our method to find the parameter of the Bernoulli jamming source. Define

n_i = the state of group i.

n_i^k = the event that group i is in state k, where $0 \le k \le M_i$.

π_i^k = the stationary probability that group i is in state k, where $0 \le k \le M_i$.

C = the threshold of backward interference (i.e., C or more simultaneous transmission in group 2 will jam group 1).

$A_i(j, k)$ = Pr[j unbacklogged terminals in group i transmit | n_i^k]

$$= \binom{M_i - k}{j} \sigma_i^j (1 - \sigma_i)^{M_i - k - j}, \text{ where } 0 \le j \le M_i - k.$$

$B_i(j, k)$ = Pr[j backlogged terminals in group i transmit | n_i^k]

$$= \binom{k}{j} q_i^j (1 - q_i)^{k-j}, \text{ where } 0 \le j \le k.$$

$J_{i,k}$ = Pr[group i jams group k in a slot].

min(x, y) = the smaller of x and y.

Initially, J_{21} is set to zero. π_1^k can be obtained by solving S-ALOHA/IBJS with parameters M_1, σ_1, q_1, and J_{21}. J_{12} is then computed by

$$J_{12} = \sum_{k=0}^{M_1} \pi_1^k \text{Pr[group 1 jams group 2} \mid n_1^k]$$

$$= \sum_{k=0}^{M_1} \pi_1^k (1\text{-Pr[No terminal in group 1 transmit} \mid n_1^k])$$

$$= \sum_{k=0}^{M_1} \pi_1^k [1 - A_1(0, k) B_1(0, k)].$$

Similarly, π_2^k can be obtained by solving S-ALOHA/IBJS with parameters M_2, σ_2, q_2, and J_{12}.

With π_2^k, J_{21} can be updated by

$$J_{21} = \sum_{k=0}^{M_2} \pi_2^k \text{Pr[group 2 jams group 1} \mid n_2^k]$$

$$= \sum_{k=0}^{M_2} \pi_2^k \text{Pr[}C\text{ or more terminals in group 2 transmit} \mid n_2^k]$$

$$= \sum_{k=0}^{M_2} \pi_2^k \sum_{j=C}^{M_2} \Pr[j \text{ terminals in group 2 transmit} \mid n_2^k]$$

$$= \sum_{k=0}^{M_2} \pi_2^k \sum_{j=C}^{M_2} \sum_{l=0}^{min(j,k)} B_2(l,k) A_2(j-l,k) .$$

The above procedure is repeated until the state occupancy probabilities of the 2 chains (π_1^k and π_2^k) do not change substantially. With the state occupancy probabilities, the throughput, delay, and stability of the two groups can be determined.

III. Model for Multi-group Slotted ALOHA

In this section, we extend the 2-group model to a general K-group model.

A. Capture Model and Terminal Model

Consider a S-ALOHA system with K groups of terminals, labeled as G_1, G_2,..., G_K. The levels of received power at the central station from different groups are assumed to be different such that G_1 dominates G_2, G_2 dominates G_3,..., and G_{K-1} dominates G_K. In other words, for $i < j$, G_i will jam G_j whenever one or more terminals in G_i transmit. On the other hand, we assume that a single transmission in G_j can succeed if and only if there is no transmission in G_i (for all $i < j$) and less than C_{j+1} transmissions occur in G_{j+1}, where C_{j+1} is the threshold of backward interference of G_{j+1}. For $i > (j+1)$, G_i is assumed to have no impact on G_j due to the large difference in the levels of received power. For example, in a 5-group system, G_3 will potentially be jammed by G_1, G_2, and G_4 (if C_4 or more simultaneous transmissions occur in G_4). G_5 can never jam G_3.

The terminal model is the same as that in Section II.A. For G_i, $i = 1, 2, \ldots, K$, all the M_i terminals are identical, mutually independent, single-buffered, and with parameter σ_i and q_i.

B. Network Model

The complete system state is the state of each group, which can be characterized by the number of backlogged terminals in each group, denoted by $n_1, n_2, \ldots, n_K$. The exact behavior can be described as a K-dimensional Markov chain, which is analytically intractable. Therefore we approximate the entire system as K interactive one-dimensional Markov chains, one for each group. For a particular group G_k, the interference due to the rest of the groups in the system is approximated as due to an independent Bernoulli jamming source with parameter J_k. We show how to find the parameter J_k in the following.

Fig. 3 shows the process to find the parameter of the jamming source for a particular group G_k. Under the capture model, only G_1, G_2,..., G_{k-1}, and G_{k+1} are potential jamming sources. Suppose the stationary state occupancy probabilities of the potential jamming sources are known. $J_{i,k}$ can be computed by

$$J_{i,k} = \sum_{m=0}^{M_i} \pi_i^m \Pr[G_i \text{ jams } G_k \mid n_i^m].$$

For $i < k$,

$$\Pr[G_i \text{ jams } G_k \mid n_i^m] = 1 - \Pr[\text{No terminal in } G_i \text{ transmits} \mid n_i^m]$$

$$= 1 - A_i(0, m)\, B_i(0, m)\ .$$

For $i = k + 1$,

$$\Pr[G_i \text{ jams } G_k \mid n_i^m]$$

$$= \Pr[C_i \text{ or more terminals in } G_i \text{ transmit} \mid n_i^m]$$

$$= \sum_{j = C_i}^{M_i} \Pr[j \text{ terminals in } G_i \text{ transmit} \mid n_i^m]$$

$$= \sum_{j = C_i}^{M_i} \sum_{l = 0}^{min(j, m)} B_i(l, m) A_i(j - l, m) .$$

Now assuming that all the potential jamming sources are independent, we can aggregate them to be a single Bernoulli source with parameter J_k given by:

$$J_k = \begin{cases} J_{2,k} & \text{if } k = 1 \\ 1 - (1 - J_{1,k})(1 - J_{2,k}) \ldots (1 - J_{k-1,k}) & \text{if } k = K \\ 1 - (1 - J_{1,k})(1 - J_{2,k}) \ldots (1 - J_{k-1,k})(1 - J_{k+1,k}) & \text{elsewhere.} \end{cases} \quad (1)$$

Then we solve the Markov chain for G_k by S-ALOHA/IBJS with parameters M_k, σ_k, q_k, and J_k (detailed in Section II.C). The state occupancy probabilities for G_k can be updated accordingly.

The complete procedure to find J_k for all groups is:

Step 1: Set $J_{i,k} = 0$ for all i and k.

Step 2: For $k = 1$ to K

{

Step 2.1: Compute J_k by (1).

Step 2.2: Solve for the state occupancy probabilities for G_k by S-ALOHA/IBJS with parameters M_k, σ_k, q_k, and J_k.

Step 2.3: Update $J_{k,l}$ for $l = k$-1,k+1,..., K.

}

Step 3: Stop if no significant change is observed. Otherwise go to Step 2.

After the iterative procedure stops, the state occupancy probabilities can be used to determine the throughput, delay, and stability for each group.

IV. Applications

The proposed model is useful in evaluating the throughput, delay and stability of a class of a S-ALOHA system where several heterogeneous groups of terminals exist. For instance, in a S-ALOHA network where the near-far effect is pronounced, users can be divided into several groups such that the power level of the received packets from the same group is approximately the same. The model assumes that terminals in a group are identical but different groups can have totally different values of parameters, e.g., the number of terminals in a group, the retransmission probability, and the threshold of backward interference. Throughout this section, all the thresholds of backward interference are fixed to be 3.

Consider a 5-group system (called system *A*) with $M = 20$, $\sigma = 0.005$, and $q = 0.05$ for each group. Fig. 4 and 5 show the throughput and delay performance of each group in system *A*. We can see that the groups with higher power have higher throughput and lower delay as expected.

In practice, it might be desirable to design a system which provides equal throughput for all terminals, thus eliminating the unfairness between high-power and low-power terminals. To achieve this, we can set the retransmission probability of a low-power group higher than that of a high-power group so that the throughput is balanced for all terminals in the system. Consider a system (called system *B*), which is almost the same as system *A* except that $q_1 = 0.05$,

$q_2 = 0.06$, $q_3 = 0.08$, $q_4 = 0.12$, and $q_5 = 0.22$. Fig. 6 shows that all the five groups in system B have roughly the same throughput. Fig. 7 shows the average delay for the five groups in system B. From Fig. 5 and 7, one can easily see that system B is more fair.

Fig. 8 and 9 show the throughput and delay performance for a 5-group system with $M = 20$, $\sigma = 0.01$, and $q = 0.1$ for each group. Note that although groups 4 and 5 suffer low throughput and high delay, group 1, 2, and 3 still have acceptable throughput. This occurs because when total input traffic exceeds the system capacity, high-power groups have the priority to grab the system capacity whereas low-power groups can only use the rest of the system capacity, which is very small when the system is overloaded.

V. Conclusion

An analytical model, S-ALOHA/IBJS, has been developed to analyze the performance of a multi-group slotted ALOHA system with capture. By approximating the interference from other groups as an independent Bernoulli jamming source, this model gives us a way to evaluate the throughput, delay, and stability of each group. When the system is not overloaded, the fairness among the groups can be improved by choosing proper retransmission probabilities for each group. Thus, the throughput is balanced and the discrepancy of delays among the groups is mitigated. Whereas when the system is overloaded, it might not necessarily result in avalanche of the performance of all groups. High-power groups might have acceptable throughput due to the capture effect. Future work is considering a more general capture model that can take into account effects such as fading.

References

[1] L. Fratta and D. Sant, "Some models of packet radio networks with capture," in *Proc. Fifth Int. Conf. Computer Comm.*, 1980, pp. 155-161.

[2] N. Shacham, "Throughput-delay performance of packet switching multiple-access channel with power capture," *Performance Evaluation*, vol. 4, pp. 153-170, 1984.

[3] C. Namislo, "Analysis of mobile radio slotted ALOHA networks," *IEEE Trans. Veh. Technol.*, vol. VT-83, no.3, pp. 199-204, Aug. 1984.

[4] R. Du, H. Okada, H. Sanada, and Y. Tezuka, "Performance evaluation and optimization of ALOHA scheme with capture effect," in *Proc. Globecom'87*, Nov. 1987.

[5] N. Abramson, "The throughput of packet broadcasting channels," *IEEE Trans. Commun.*, vol. COM-25, pp. 117-128, Jan. 1977.

[6] J. J. Metzner, "On improving utilization of ALOHA networks," *IEEE Trans. Commun.*, vol. COM-24, no. 4, pp. 447-448, Apr. 1976.

[7] D. Goodman and A. A. M. Saleh, "The near/far effect in local ALOHA radio communications," *IEEE Trans. Veh. Technol.*, vol. VT-36, no. 1, pp. 19-27, Feb. 1987.

[8] A. B. Carleial and M. E. Hellman, "Bistable behavior of ALOHA-type systems," *IEEE Trans. Commun.*, vol. COM-23, pp. 401-410, Apr. 1975.

[9] Y. Onozato, J. Liu, and S. Noguchi, "Stability of a slotted ALOHA system with capture effect," *IEEE Trans. Veh. Technol.*, vol. 38, no. 1, pp. 31-36, Feb. 1989.

[10] C. Plas and J. M. G. Linnartz, "Stability of mobile slotted ALOHA network with Rayleigh fading, shadowing, and near-far effect," *IEEE Trans. Veh. Technol.*, vol. 39, no. 4, pp. 359-366, Nov. 1990.

[11] D. Bertsekas and R. Gallager, *Data networks.* 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1992, pp. 353.

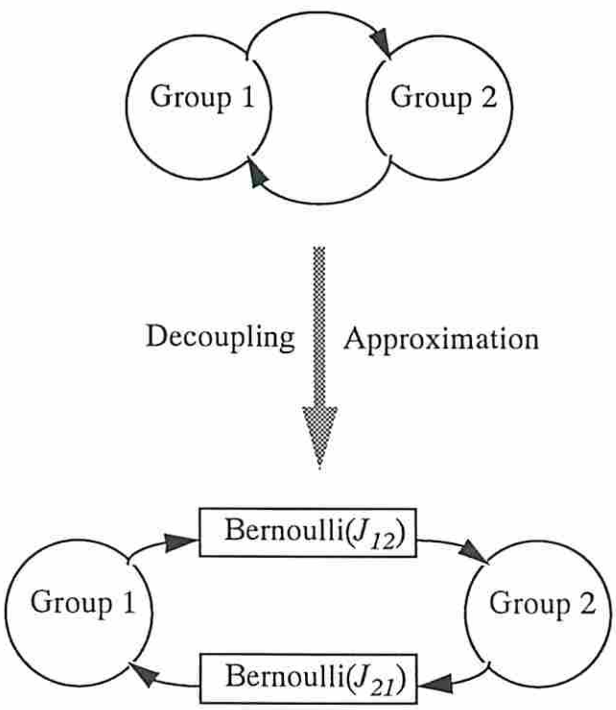

Fig. 1. The approximated interaction between 2 groups.

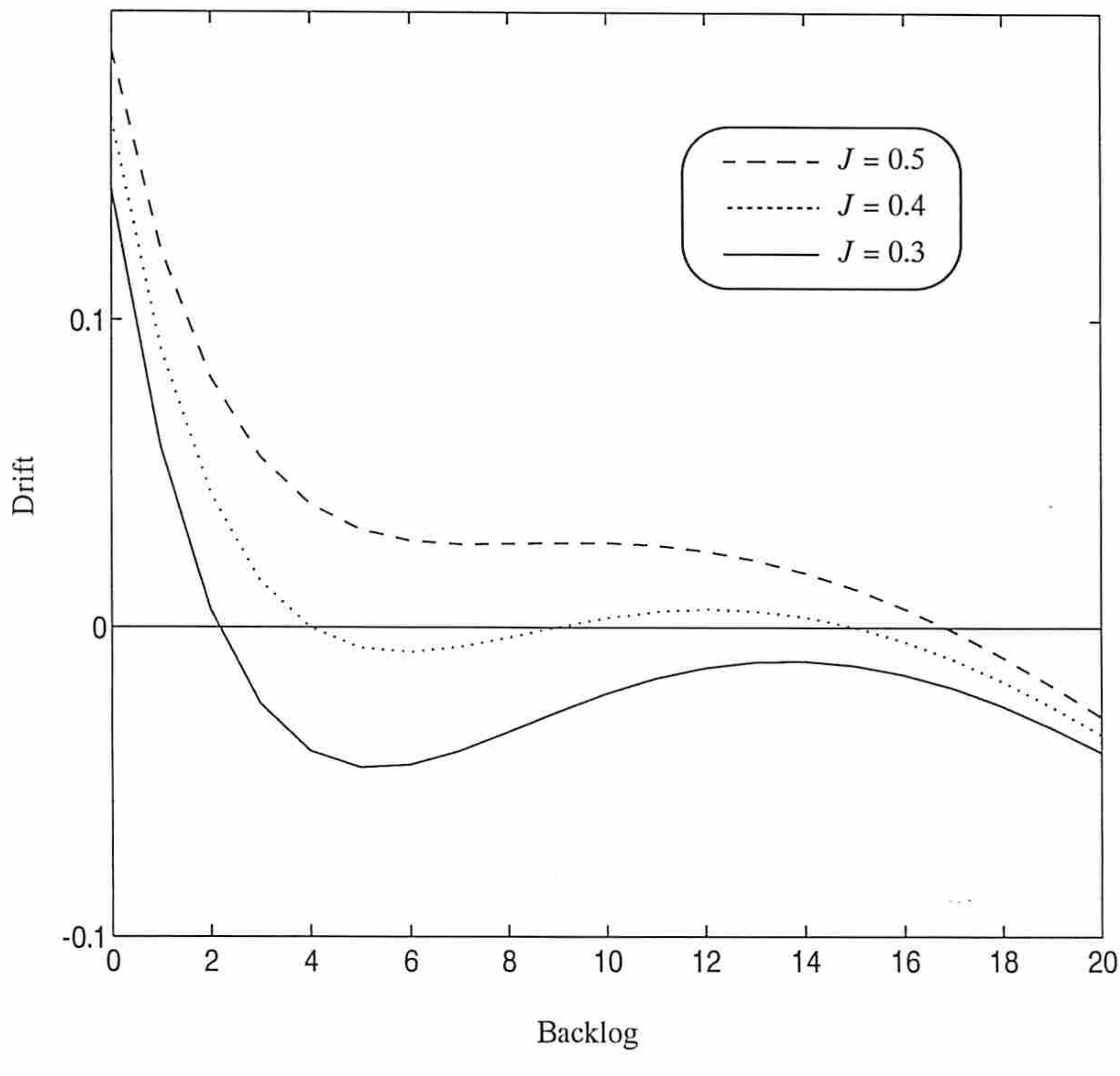

Fig. 2. Drift for S-ALOHA/IBJS with M = 20, σ = 0.015, and q = 0.2.

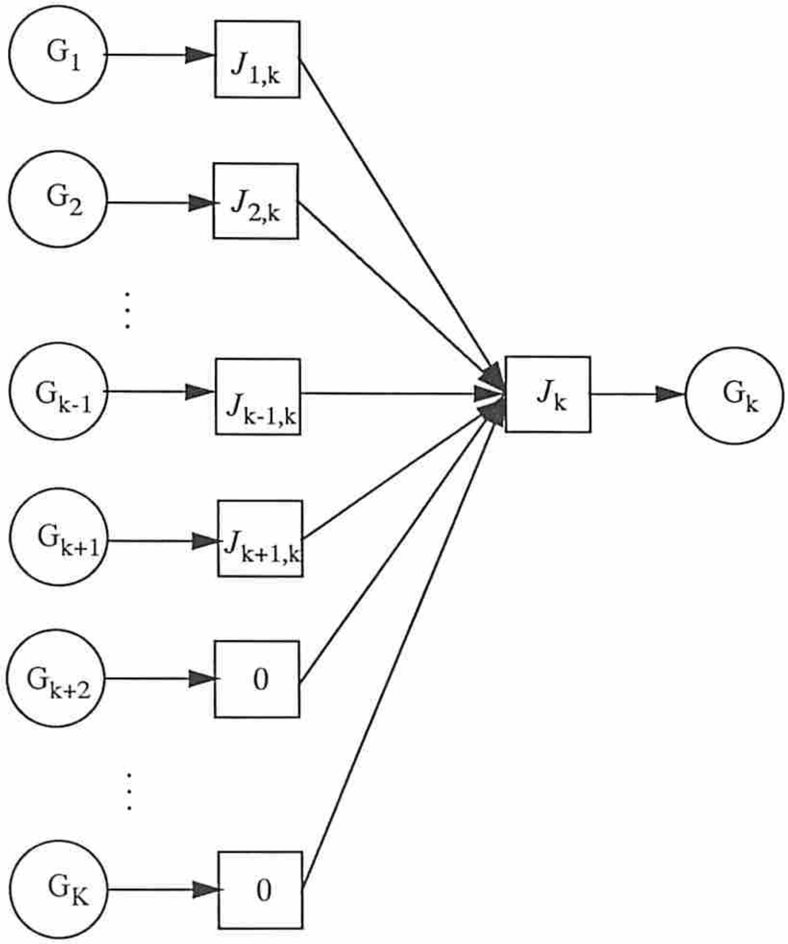

Fig. 3. The flow chart to compute the parameter of the jamming source for group k. $J_{i,k}$ is the probability that G_i jams G_k.

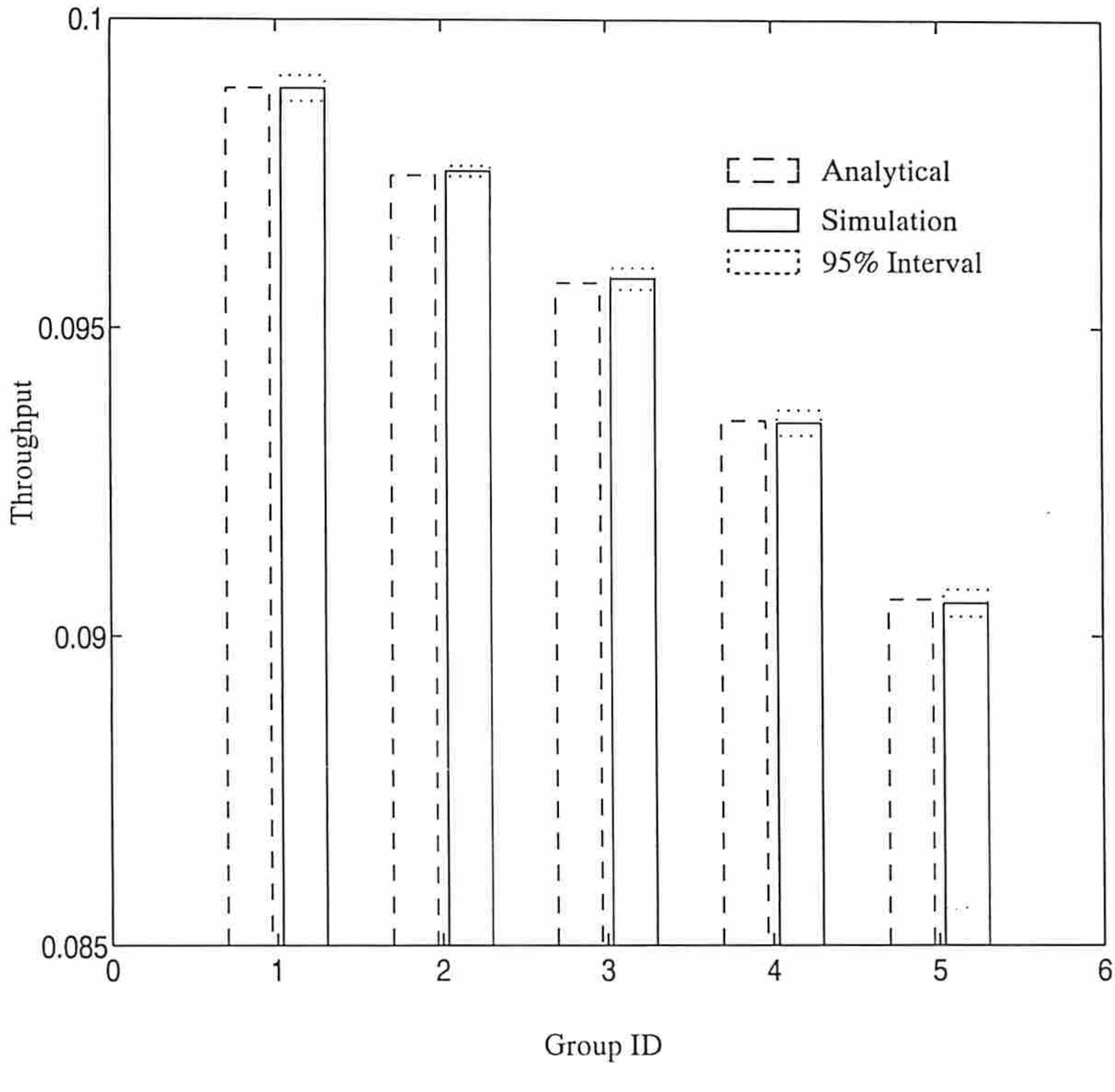

Fig. 4. Throughput for a 5-group S-ALOHA system with $M = 20$, $\sigma = 0.005$, and $q = 0.05$ for each group.

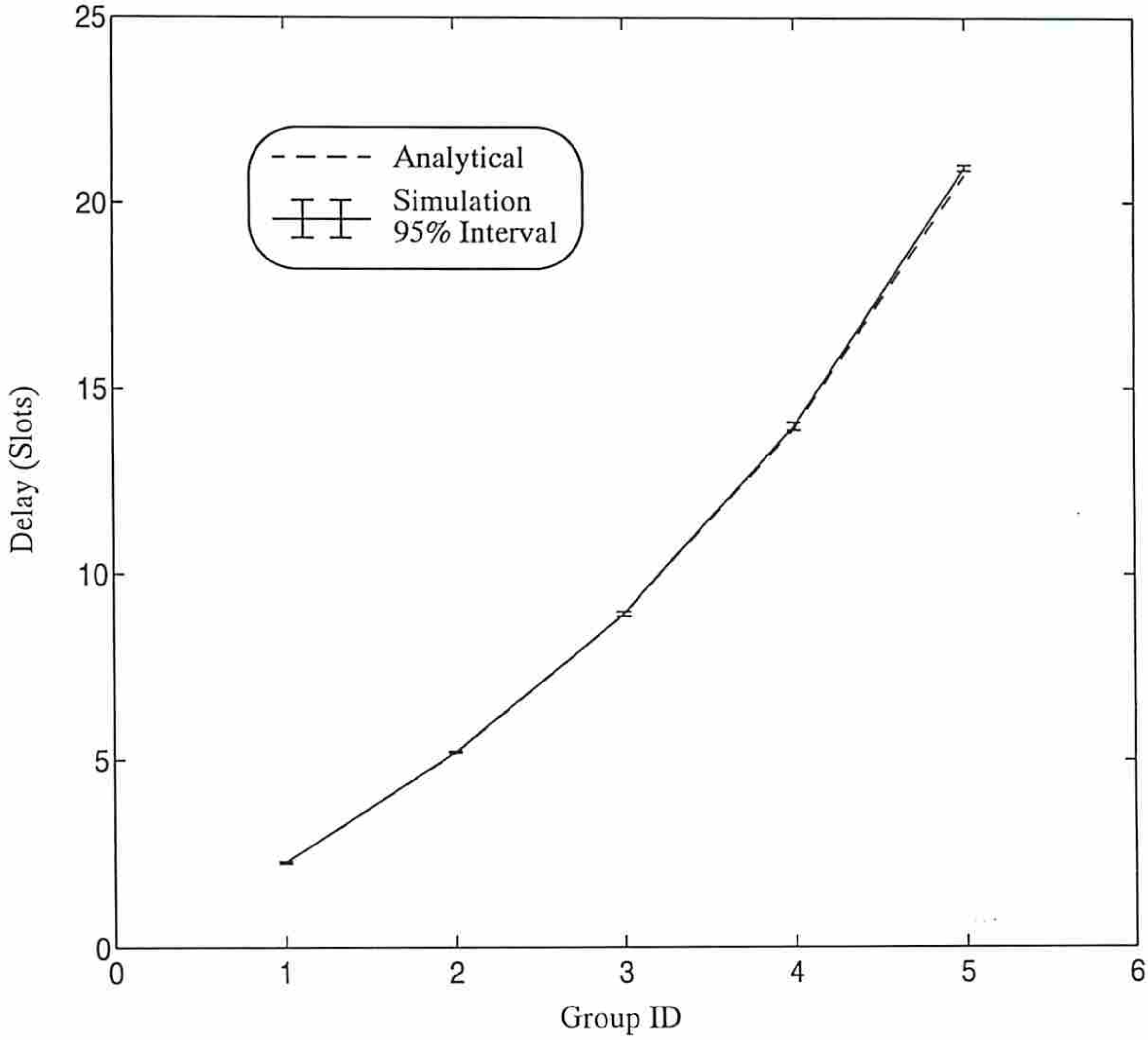

Fig. 5. Delay for a 5-group S-ALOHA system with $M = 20$, $\sigma = 0.005$, and $q = 0.05$ for each group.

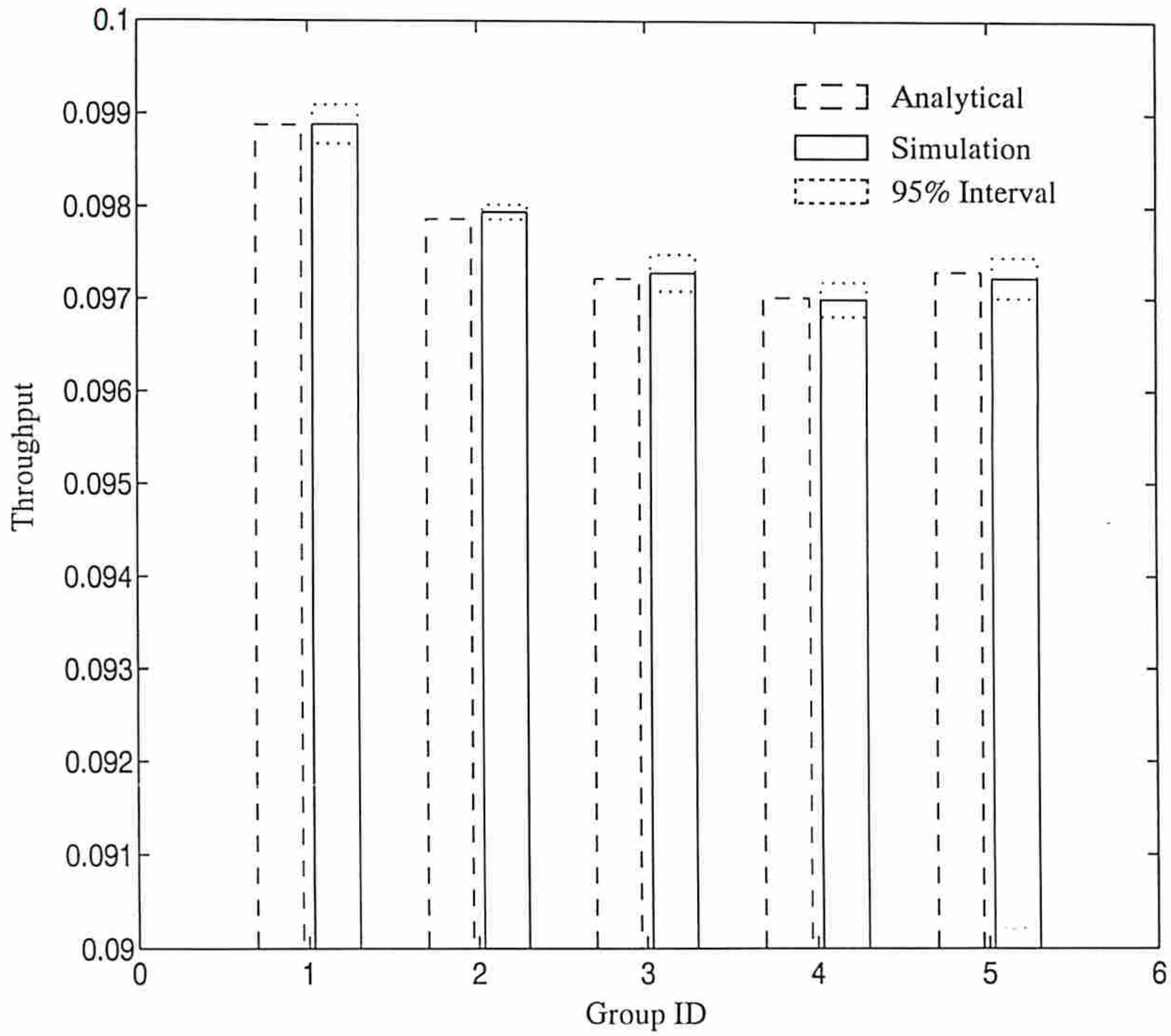

Fig. 6. Throughput for a 5-group S-ALOHA system with parameters for each group as follows: G_1: ($M = 20$, $\sigma = 0.005$, $q = 0.05$); G_2: ($M = 20$, $\sigma = 0.005$, $q = 0.06$); G_3: ($M = 20$, $\sigma = 0.005$, $q = 0.08$); G_4: ($M = 20$, $\sigma = 0.005$, $q = 0.12$); G_5: ($M = 20$, $\sigma = 0.005$, $q = 0.22$).

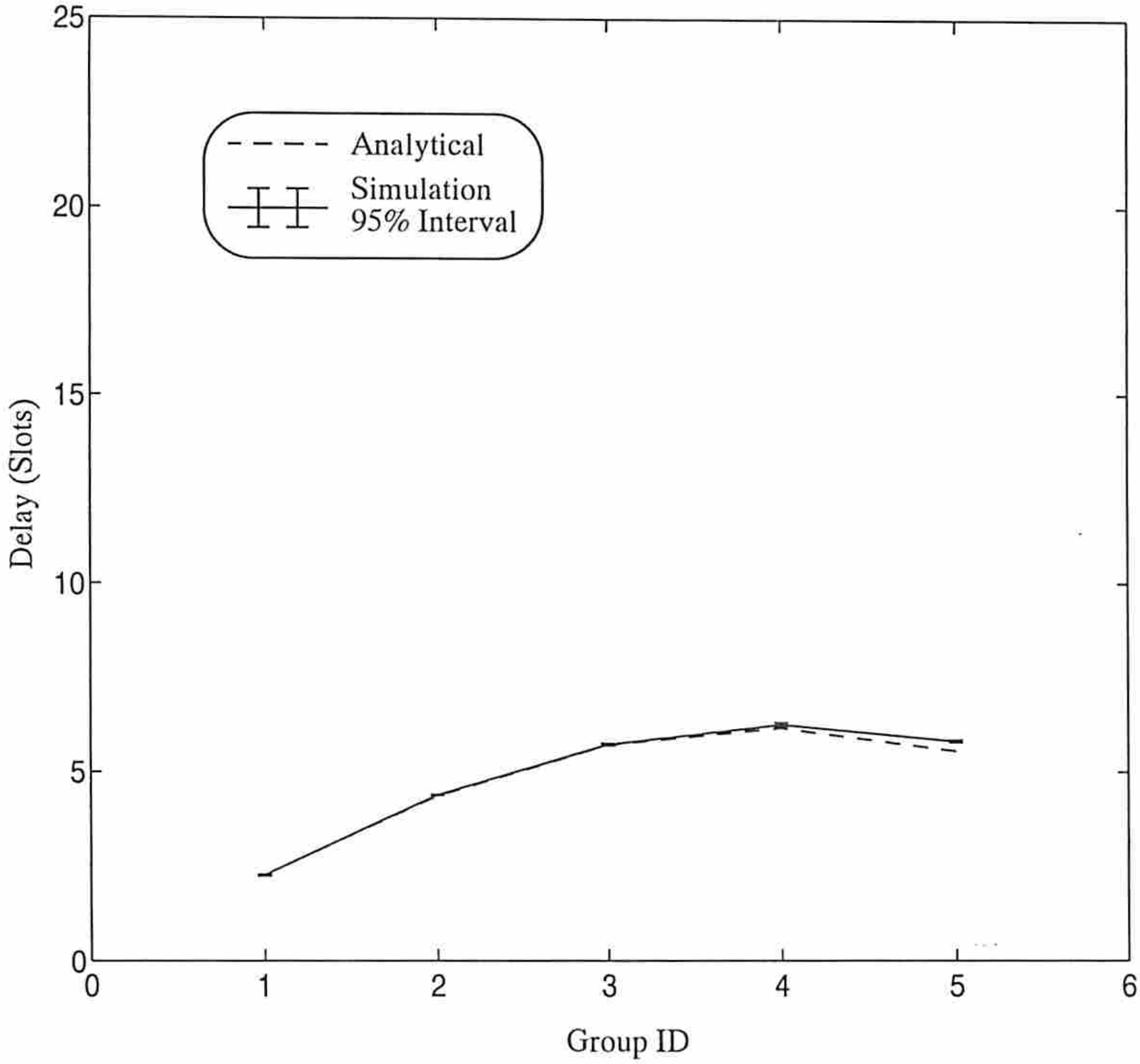

Fig. 7. Delay for a 5-group S-ALOHA system with parameters the same as in Fig. 5.

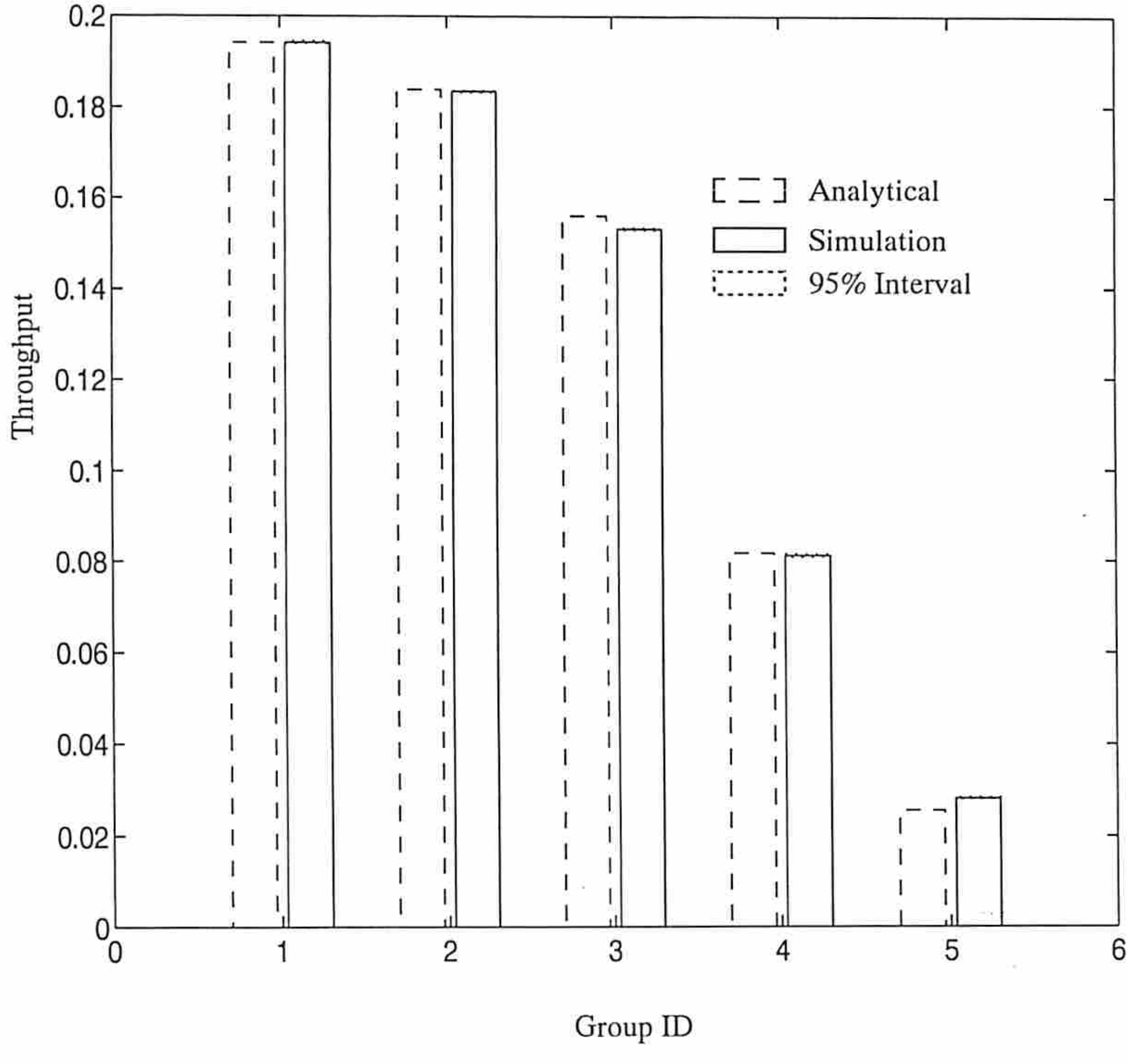

Fig. 8. Throughput for a 5-group S-ALOHA system with $M = 20$, $\sigma = 0.01$, and $q = 0.1$ for each group.

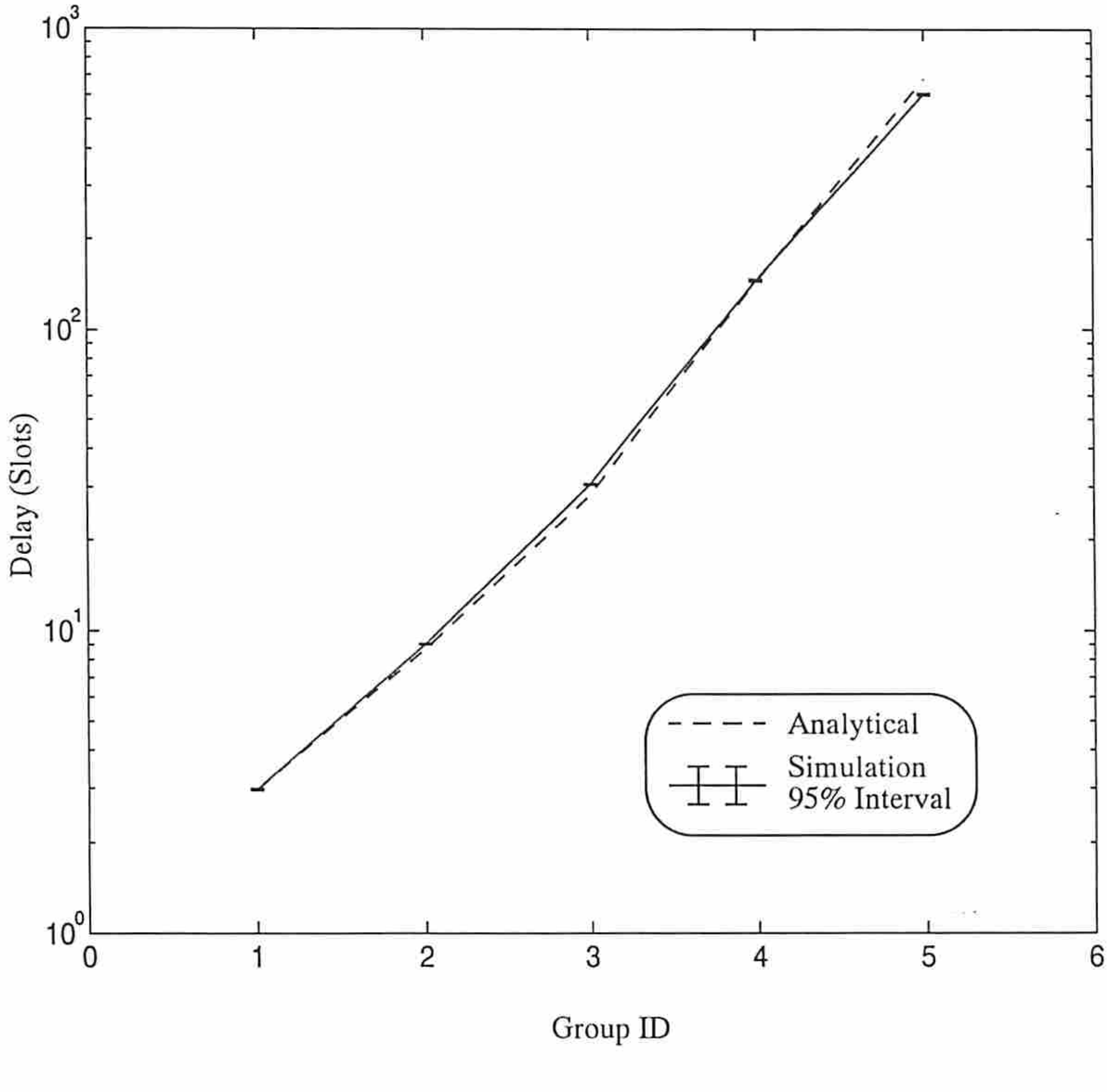

Fig. 9. Delay for a 5-group S-ALOHA system with $M = 20$, $\sigma = 0.01$, and $q = 0.1$ for each group.