Vector Compaction Using Dynamic Markov Models

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Abstract

Evaluation of power dissipation is a critical step in the design of today's ICs. Power disssipation is strongly input pattern dependent and hence, to obtain accurate power values, one must simulate the circuit with a large number of input vectors that typify the application data. The goal of this paper is to present an effective and robust technique for compacting a large sequence of input vectors into a much smaller input sequence so as to reduce the circuit/ gate level simulation time by orders of magnitude and mantain the accuracy of the power estimates. In particular, this paper introduces and characterizes a family of dynamic Markov trees that can model complex spatiotemporal correlations which occur during power estimation both in combinational and sequential circuits. This new framework is very effective and flexible: the Markov model itself is derived through a one-pass traversal of the initial sequence and it can be used after that with any available simulator to derive power consumption. As the results demonstrate, large compaction ratios of 1-2 orders of magnitude can be obtained without significant loss (less than 3% on average) in the accuracy of power estimates.

1. Introduction

CAD tools have played a significant role in the efficient design of the high-performance digital systems. In the past, time and area were the primary concerns of the CAD community during the optimization phase. More recently, circuit testability was added as yet another important consideration during the design process. With the growing need for low-power electronic circuits and systems, power analysis and low-power synthesis have become crucial tasks that must also be addressed. It is expected that, in the forthcoming years, power issues will receive increasing attention due to the widespread use of portable applications and the desire to reduce packaging and cooling costs of high-end systems.

Power estimation is in general a difficult problem; the key task in this process is the accurate and fast estimation of average switching activity. To date, both simulative [1]-[4] and nonsimulative approaches [5]-[10] have been tried, each one having its own advantages and limitations [11]. More specifically, general simulation techniques provide sufficient accuracy, but at high computational cost; it is simply expensive to simulate thousands of vectors. On the other hand, nonsimulative approaches (best represented by probabilistic power estimation techniques) are in general faster, but less accurate than those based simulation; usually, the input correlations and the reconvergent fan-out in the target circuit make things very complicated and simplifying assumptions (like input independence) become mandatory. During the last years, the gap between simulative and nonsimulative approaches remained basically the same, despite remarcable advances made in both directions.

As a conclusion, a number of issues appear to be important for power estimation and low-power synthesis. The *input statistics* which must be properly captured and the *length of the input sequences* which must be applied are two such issues. Generating a minimal-length sequence of input vectors that satisfies these statistics in not trivial. More precisely, LFSRs which have traditionally found use in testing or functional verification [12], are of little or no help

here. The reason is that more elaborate set of input statistics must be preserved or reproduced during sequence generation for use by power simulators. One such attempt is [13] where authors use deterministic FSMs to model user-specified input sequences. Since the number of states in the FSM is equal to the length of the sequence to be modeled, the ability to characterize anything else but short input sequences is severely limited. A more elaborate and effective technique was presented in [14] where, based on stochastic sequential machines, the authors succeed in compacting large sequences without significant loss in accuracy. However, in the present research, the limitations of that approach are pointed out and overcome by the proposed technique.

The present paper improves the-state-of the art by providing an original solution for vector compaction problem which potentially reduces the gap between simulative and nonsimulative approaches. Traditionally, data compression techniques were aimed to lossless compression, that is the ability to encode a body of data, transmit it eventually over a communication line and finally, decode it uniquely, without any loss of information during this process. Our objective in this paper is slightly different: having an initial sequence (assumed representative for some target circuit), we target *lossy compression* [15], that is the process of transforming an input sequence into a smaller one, such that the new body of data represents a *good approximation* as far as total power consumption is concerned. We use throughout the paper the term 'compaction' instead of 'compression', because we think it better suits our intentions.

The foundation of our approach is probabilistic in nature; it relies on *adaptive* (*dynamic*) *modeling* of binary input streams as first-order Markov sources of information and is applicable both to combinational and sequential circuits. The adaptive modeling technique itself (best known as Dynamic Markov Chain or DMC modeling) was introduced very recently in the literature on data compression [16] as a candidate to solve various compression problems. From the very begining, this technique looked very promising and indeed, in most practical situations, has been more effective than any other compression technique available to date. However, the original model introduced in [17] is not completely satisfactory for our purpose. In this paper, we thus extend the initial formulation to manage not only correlations among adjacent bits that belong to the same input vector, but also correlations between successive input patterns.

As demonstrated and supported by practical evidence, this new framework is extremely effective in power estimation. The basic idea is illustrated in Fig.1. To evaluate the total power consumption of a target circuit for a given input sequence L_0 (Fig.1a), we derive first the Markov model of the input sequence through a one-pass traversal technique and after that, having this compact representation, we generate a much shorter sequence L, equivalent with L_0, which can be used with any available simulator to derive accurate power estimates (Fig.1b).

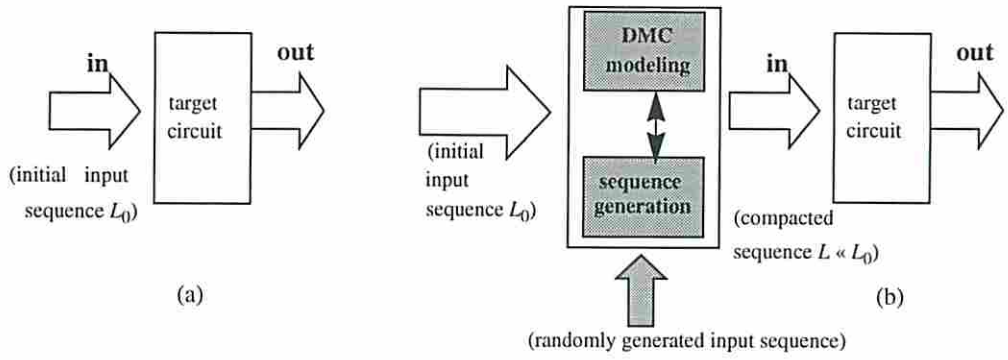

Fig.1

We point out here that the present approach can be used without any difficulty to generate benchmark data for power estimation, that is, input sequences with different lengths that satisfy a set of user-prescribed characteristics in terms of word-level transitions or conditional probabilities.

To conclude, both simulation-based approaches and probabilistic techniques for power estimation may benefit from this research. The issues brought into attention in this paper are new and represent an important step toward reducing the gap between the simulative and probabilistic techniques commonly used in power estimation. Finally, the concept of DMC modeling itself may find useful applications in other CAD fields.

The paper is organized as follows: Section 2 reviews the basic concepts of DMC modeling technique. Section 3 formalizes the power-oriented vector compaction problem and discusses parameters which makes this approach effective in practice. Section 4 presents a DMC-based procedure for vector compaction. In sections 5 and 6, we give some practical considerations and experimental results, respectively. Finally, we conclude by summarizing our main contribution.

2. Background on Dynamic Markov models

Modeling for data compaction in digital systems involves essentially the derivation of certain source-string events and their contexts (the set of bits or words surrounding some bit or word under consideration), which uniquely describe the original source string. We consider therefore the *model* as having two parts: 1) the *structure* which is the set of events and their contexts and 2) the *parameters* which are probabilities assigned to the events. The structure is intended to capture the characteristics of the entire set of sequences under consideration while the parameters are tailored to each individual sequence.

Without loss of generality, in what follows we restrict ourselves to finite binary strings, that is, finite sequences consisting only of 0's and 1's. The set of events of interest is the set S of all finite binary sequences on k bits. A particular sequence S_1 in S consists of vectors v_1, v_2,..., v_n (which may be distinct or not), each having a positive occurence probability[1]. Indices 1, 2,..., n represent the discrete time steps when a particular vector is applied to a

1. Throughout the paper, we may refer occasionally to vectors v_1, v_2,..., v_n as 'symbols' or 'states'.

target circuit. Imposing a total ordering among bits, such a sequence may be conveniently viewed as a binary tree (for reasons that will become obvious later on, let's call it DMT_0 from *Dynamic Markov Tree of order zero*) where nodes at level j correspond to bit j ($1 \leq j \leq k$) in the original sequence; each edge that emerges from a node is labelled with a positive count (and therefore with a positive probability) that indicates how many times the substring from the root to that particular node, occured in the original sequence. For clarity, let's consider the following example.

Example 1: For the following 4-bit sequence consisting of 8 non-distinct vectors: (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8) = (0000, 0001, 1001, 1100, 1001, 1100, 1001, 1100) the construction of the tree DMT_0 is shown step-by-step in Fig.2a. Obviously, the whole Markov tree that models this sequence must have four levels because the original sequence is a 4-bit sequence. Without any loss in generality, we assume a left-to-right order among bits that is, the leftmost bit in any vector v_1 to v_8 is considered as being bit number one (and consequently represented at level one in DMT_0 as shown in Fig.2a), the next bit is considered as being bit number two and so on. Every time when a vector is completely scanned (that corresponds to reaching the level four in the tree), we come back to the root and start again with the next vector in the sequence. While the input sequence is scanned, the actual counts on the edges are dynamically updated (as shown in Fig.2a for the first three vectors) such that, for this particular example, they finally become as indicated in Fig.2b. The Markov tree in Fig.2b contains in a compact form all the spatial information about the original sequence v_1, v_2,..., v_8. We point out that this sparse structure is possible only by using the *dynamic (adaptive)* fashion of growing the tree DMT_0 just illustrated. Another approach would have been to consider a static binary tree capable to model any 4-bit sequence and just to update the counts on the edges while scanning the original sequence (Fig.3). By doing so, we would end up with the obvious disadvantage of having 15 instead of 9 nodes in the structure for the same amount of information; this reason alone is sufficient for considering from now on only dynamically grown models.

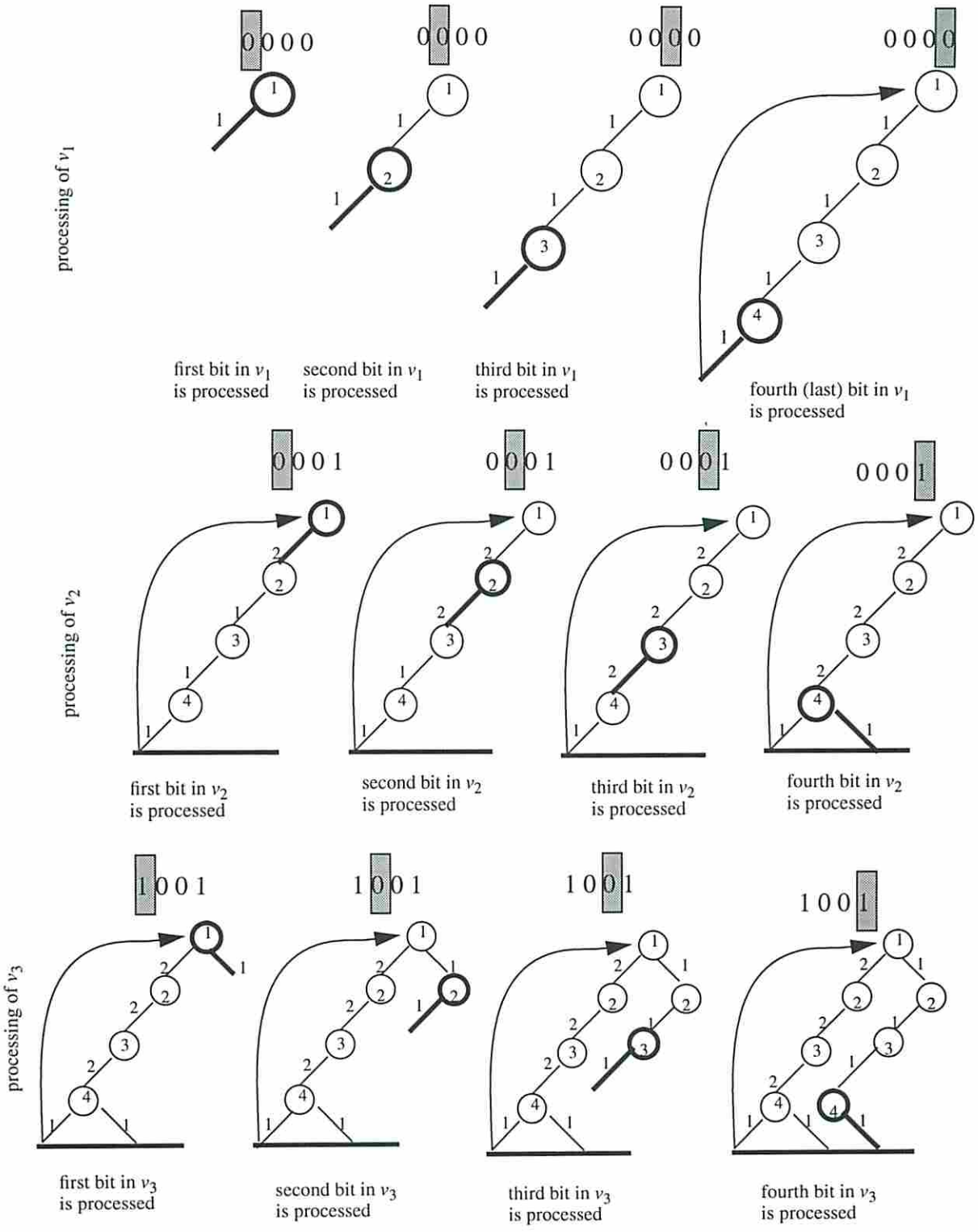
processing of v_1
0 0 0 0
first bit in v_1 is processed
second bit in v_1 is processed
third bit in v_1 is processed
fourth (last) bit in v_1 is processed
processing of v_2
0 0 0 1
first bit in v_2 is processed
second bit in v_2 is processed
third bit in v_2 is processed
fourth bit in v_2 is processed
processing of v_3
1 0 0 1
first bit in v_3 is processed
second bit in v_3 is processed
third bit in v_3 is processed
fourth bit in v_3 is processed

Fig.2 (a)

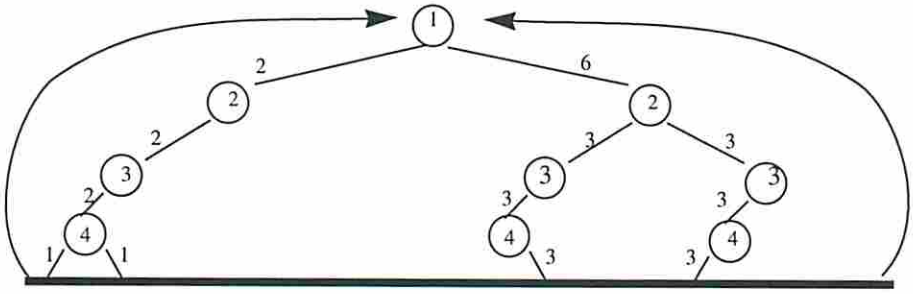

Fig.2(b)

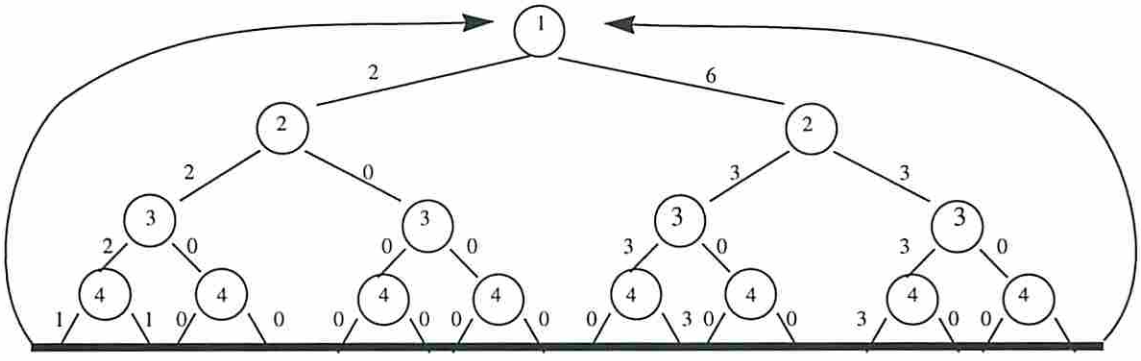

Fig.3

Definition 1. We define the *information source*, to be the pair <*S*,*P*>, where *P* is a function from *S* into [0,1] satisfying the condition:

$$P(v) = \sum_{x \in S} P(vx) \qquad (1)$$

for all v in S, where vx represents the event corresponding to the joint occurence of the strings v and x.

The above condition, simply states that the sum of the counts attached to the immediate successors of node v equals its own value $P(v)$. As we can easily see in Fig.2, condition (1) is satisfied at every node in this representation[1]. In addition, based on the counts of the terminal edges, we may easily compute the probability of occurence for a particular vector in the sequence. For instance, the probability of occurence for string '1001' is 3/8 (because the count on the terminal edge that corresponds to '1001' is 3 and the length of the sequence is 8) while the probability of string '1111' is zero, '1111' being a 'forbidden' vector for this particular sequence.

Traditionally, data compression algorithms (e.g. Huffmann, Ziv-Lempel, Cleary-Witten) [16] have been targeted to work at byte- or word-level. In contrast, DMC is normally used to process one bit of the input at a time rather than one symbol at a time. In principle, there is no reason why a symbol-oriented version of the method can not be used. However, for practical reasons (less storage required and more efficiency in computations), the bit version of DMC is preferred.

1. This is actually similar to Kirchoff's law for currents.

3. Power-oriented data compaction

In this section, first we review the main concerns about preserving the input statistics to obtain accurate power estimates. Second, we present a new approach based on DMC by generalizing the model introduced in Section 2.

3.1 Problem formulation

Input pattern dependence has a dramatic impact on power dissipation estimates. If one ignores the input statistics (which give the actual correlations among the primary inputs), power estimation results can be seriously impaired. Capturing only signal probabilities at the primary inputs of the circuit is not enough for accurate estimates therefore, for power estimation purposes, it is critical to distinguish between sequences which exhibit the same signal probabilities on different bit lines, yet showing very different spatial and temporal correlations.

Assuming that a gate level implementation is available, to estimate the total power dissipation, one can sum over all the gates in the circuit the average power dissipation due to the capacitive switching currents, that is: $P_{avg} = \frac{f_{clk}}{2} \cdot V_{DD}^2 \cdot \sum_n (C_n \cdot sw_n)$ where f_{clk} is the clock frequency, V_{DD} is the supply voltage, C_n and sw_n are the capacitance and the average switching activity of gate n, respectively. From here, the average switching activity per node (gate) is the key parameter that needs to be correctly determined, mostly if we are interested in a node-by-node basis power estimation. However, this parameter is highly sensitive to the input statistics, namely it depends significantly on transition and conditional probabilities among different signal lines.

Having these issues in mind, the vector compaction problem can be formulated as follows: for a k-bit sequence of length n (consisting of vectors $v_1,v_2,...,v_n$), find another sequence of length $m < n$ (consisting of the subset $u_1,u_2,...,u_m$ of the initial sequence), such that the average transition probability on the primary inputs is preserved *wordwise*. More formally, for any generic input v and u (seen as a collection of bits) in the original and in the compacted sequence, respectively, the following holds:

$$\left| P(v^- = X \wedge v^+ = Y) - P(u^- = X \wedge u^+ = Y) \right| < \varepsilon \qquad (2)$$

In relation (2), v^-, v^+ (u^-, u^+) denote the current and the next vector, respectively, in the original (compacted) sequence and X, Y are any two patterns that appear in the initial sequence. This condition simply requires that the joint transition probability for any group of bits is preserved within a given level of error.

3.2 A DMC-based approach

An attempt to solve the vector compaction problem for power estimation was recently presented in [14]. In that paper, the authors use elements from probabilistic automata theory to synthesize stochastic machines which can be used in a standalone mode for sequence compaction. From a practical point of view, however, this approach has two inherent limitations:

- The values in the initial transition matrix themselves are important in the decomposition process: some distributions of transition probabilities tend to favor a small number of degenerate matrices, as opposed to others which result in much longer decompositions. In these cases, the decomposition becomes the critical step as far as running time is

concerned and one should therefore allow limited precision in the calculations to simplify the decomposition process.
• The compaction technique on stochastic machines is a multiple-step compaction technique. An initial pass through the sequence is performed to extract the statistics of interest; after that, the stochastic machine is synthesized and then the new sequence is generated. This is especially disavantageous for large sequences when the on-line computer memory and time requirements become prohibitive.

The disavantages mentioned above can be eliminated by using DMC modeling. To this end, in what follows we introduce an original framework for power-oriented data compaction.

From Section 3.1, it follows that the spatiotemporal correlations that characterize a particular sequence are the key factor in power estimation. Differently stated, not only a particular vector v_i in a given sequence is important, but also its relative position in that sequence matters. More precisely, different interleavings among the vectors belonging to the same initial set (v_1, v_2,..., v_n) (e.g.(v_1,...,v_i, v_j, v_k,...,v_n), (v_1, ...,v_i, v_k, v_j, ...,v_n) or (v_1, ...,v_k, v_i, v_j, ...,v_n)) define completely different input sequences. Coming back to the model presented in Section 2, we observe that DMT_0 alone cannot capture this property; we say that DMT_0 has *no memory* and therefore the relative order of vectors in the initial sequence is irrelevant for DMT_0's construction. Obviously, DMT_0 is a poor structure for S because it does not preserve properly the order of events. In Fig.2b for instance, the value of 3/8 is the probability to see the particular string (state) '1001' in the original sequence but this gives us no indication at all about the sequencing of this vector relative to another one, say '0001'.

To solve properly the compaction problem, we refine now the above structure by incorporating in it *first-order memory effects*. Specifically, we consider a more intricate structure, namely a tree called DMT_1 (*Dynamic Markov Tree of order 1*), where from the node representing any vector v there is an emergent arc to each value x connecting v to the successor node, associated with the string vx.

Example 2: For the same sequence in Example 1, suppose we want to construct its corresponding tree DMT_1. We begin as in DMT_0 and for each leaf that represents a valid combination in the original sequence, we construct a new tree (having the same depth as DMT_0) which is meant to preserve the *context* in which the next combination occurs. For instance, the vector v_2 = 0001 follows immediately after v_1 = 0000; consequently when we reach the node that corresponds to v_1 (the leftmost path in Fig.4a), instead of going back to the root (and therefore 'forgetting' the context), we start to build a new tree (rooted at the current leave of DMT_0) as indicated in Fig.4a. Basically, we added a new path which corresponds to '0001'. The newly constructed tree will preserve the context in which v_2 = 0001 occurred that is, immediately after v_1 = 0000 (denoted by $v_1 \rightarrow v_2$). After processing the pair (v_1,v_2), we come back to the root and continue with (v_2,v_3) as shown in Fig.4b; v_2 alone leads us to the second leftmost edge of DMT_0 from where, to construct DMT_1, we have to add the path '1001' which corresponds to v_3. In this way, we indicate the sequenting between v_2 and v_3 that is, $v_2 \rightarrow v_3$.

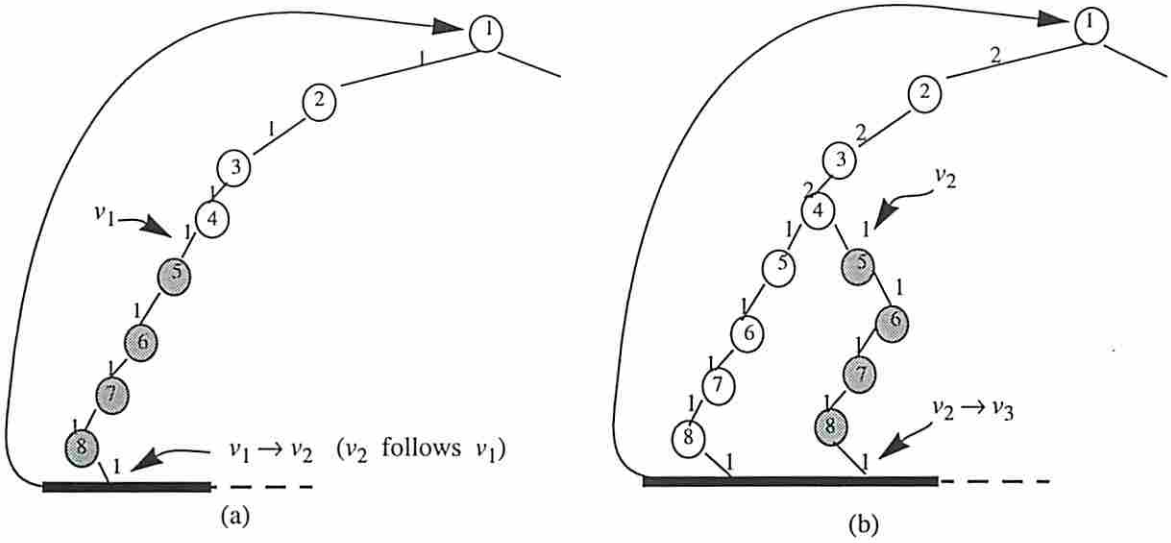

Fig.4

What is important to note here, is that *all* vectors in the original sequence are processed that is, *none of them is skipped* during the construction of DMT_1. This is the theoretical basis for accurate modeling of the input sequences as first-order Markov sources of information. Similarly, continuing this process for all leaves in DMT_0 in Fig.2b, we end up by building the whole tree DMT_1 as shown in Fig.5.

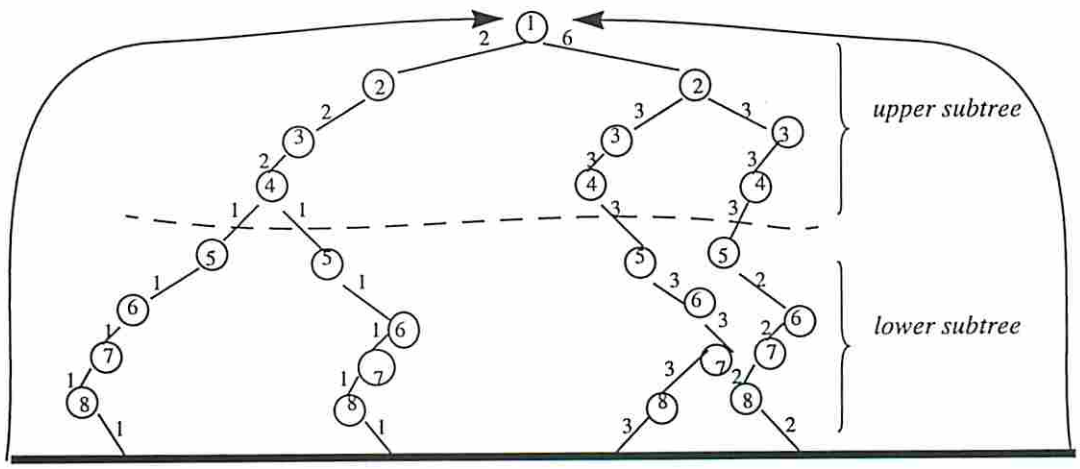

Fig.5

In Fig.5, we separated by a dashed line the two subtrees that constitute DMT_1. The upper subtree (levels 1 to 4) represents DMT_0, that is, it sets up the state probabilities for the sequence; the lower subtrees (levels 5 to 8), give the actual sequencing between any two successive vectors. To keep the counts in these subtrees consistent, while we traverse the lower subtrees and update the counts on their edges, we also accordingly increment the counts on the paths in the upper subtree (in fact, all vectors except the first and the last are processed exactly twice, once in the upper DMT_0 and next in the lower DMT_0). In practice the counts of these two subtrees may differ by one, due to the finite length of the sequences. This aspect can be seen in Fig.5 at level 5 on the rightmost path at the border between

the upper and lower subtree. A practical solution to this issue is to consider the input sequence as being cyclic that is, to link the last combination in the sequence with the very first one or simply add two dummy states, one before and one after the initial sequence.

Obviously, DMT_1 provides more information than DMT_0. To give an example, string '1001' can follow only after '0001' or '1100', information that cannot be gathered by analyzing DMT_0 alone.

Proposition 1 [19]. We write the probability of a vector string $v = v_1 v_2 ... v_n$ as follows:

$$P(v) = P(v_1) \cdot P(v_2 | v_1) \cdot \ldots \cdot P(v_n | v_1 v_2 \cdots v_{n-1}) \tag{4}$$

where the conditional probabilities are uniquely defined by: $P(x|v) = P(vx) / P(v)$.

This property, used in connection with the counts on the edges, allows a quick calculation of the transitions probabilities that characterize a particular sequence. For example, if we want to calculate the transition probability '1001' → '1100' we have from Proposition 1 $P(v) = P(v_1 v_2) = P(v_1) \cdot P(v_2 | v_1) = 3/8$ which is exactly the count on the path '10011100' in the tree DMT_1 divided by the sequence length.

Theorem 2. Any sequence in S can be modeled as a first-order Markov source using the structure DMT_1 and parameters P. We call this process Dynamic Markov Chain (DMC) modeling.

Sketch of proof: If $v = v_1 v_2$ is a string in the structure DMT_1 such that v_1 is in the upper tree and v_2 is in the lower tree, then $P(v_2 \mid v_1) = P(v) / P(v_1)$. Thus, the parameters stored on the edges of DMT_1 structure provide the conditional probabilities that characterize the lag-one Markov chain for the sequence in S. ■

Theorem 3. The structure DMT_1 and parameters P are equivalent to a stochastic sequential machine.

Sketch of proof: DMT_1 defines a Markov source (based on Theorem 2). Any Markov source is characterized by a stochastic matrix A. According to the decomposition Theorem 1 given in [14], this matrix is uniquely associated to a stochastic machine (a finite-state machine with randomly generated inputs). Thus, DMT_1 is equivalent to a SSM. ■

Generally speaking, the theory of stochastic sequential machines is far more developed than the theory of DMC modeling. However, the DMC modeling technique based on DMT_1 seems to be more effective as it offers a much more compact structure and generally outperforms the compaction techniques based on stochastic machines. Specifically:

- Sequence compaction based on DMT_1 avoids the time consuming decomposition process necessary in stochastic machines' synthesis.
- Using DMT_1 one can avoid the need for partitioning the input vectors into groups of bits. Therefore, one can expect to improve the accuracy, especially in those cases when the input patterns are highly correlated.
- DMT_1 is constructed dynamically (new nodes are added only 'on demand') therefore it offers a much more compact data structure than matrix A does.

The DMC modeling technique can be successfully used to model such complex spatiotemporal correlations. The structure DMT_1 just introduced is general enough to capture completely the correlations among all bits of the same

input vector and also between successive input patterns. Indeed, the recursive construction of DMT_1 by considering successive bits in the upper and lower subtrees completely captures the word-level (spatial) correlations for each individual input vector in the original sequence. Furthermore, cascading lower subtrees for each path in the upper subtree, gives the actual sequencing (temporal correlation) between successive input patterns. This model captures completely spatial correlations and first-order temporal correlations. However, it has conceptually no inherent limitation to be further extended to capture temporal dependencies of higher orders. For instance, if we continue to define recursively DMT_2 (as a function of DMT_1), we can basically capture second-order temporal correlations (Fig.6).

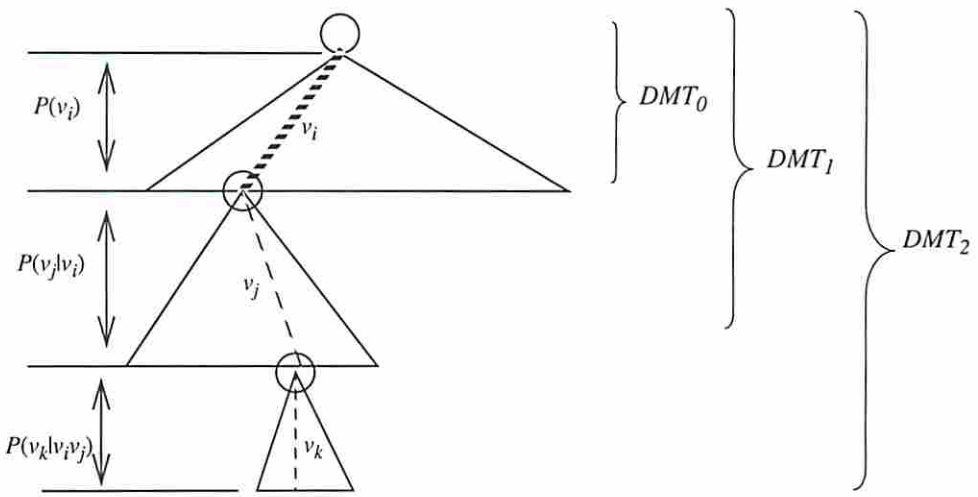

Fig.6

Theorem 4. The general structure DMT_n and parameters P can model spatiotemporal correlations of order n.

Sketch of proof: Let $v = v_1\ v_2\ ...\ v_n$ be a string in DMT_n (the substring v_i belongs to the i-level tree). Using Proposition 1, we have $P\ (v_n \mid v_1\ v_2\ ...\ v_{n-1}) = P\ (v_1\ v_2\ ...\ v_n)\ /\ P\ (v_1\ v_2\ ...\ v_{n-1})$ and thus the lag-n Markov chain characterizing the input can be fully modeled by the DMT_n structure. ■

4. A DMC-based vector compaction procedure

A practical procedure to construct DMT_1 and generate the compacted sequence is given in Fig.7a. During a one-pass traversal of the original sequence (when we extract the bit-level statistics of each individual vector $v_1,v_2...,v_n$ and also those statistics that correspond to pairs of consecutive vectors (v_1v_2), $(v_2v_3),...,(v_{n-2}v_{n-1}),(v_{n-1}v_n)$) we grow simultaneously the tree DMT_1. We continue to grow DMT_1 as long as the Markov model is smaller than a user-specified threshold (*model_size*), otherwise we just generate the new sequence up to that point and discard (flush) the model. A new Markov model is started again and the process is continued up to the end of the original sequence. The *generate_seq* procedure called by the DMC program is detailed in Fig.7b. Each generation phase is driven by the user-specified compaction parameter *ratio* that is, in order to generate a total of $m = n/ratio$ vectors, we have to keep

the same compaction ratio for every dynamically grown Markov model.

```
procedure DMC (input_file, ratio, model_size) {
 initial_state = new_state ();
 symbol = read_input (input_file);
 update_tree (symbol, upper_tree, initial_state);
 crt_state = last_state (symbol, upper_tree);
 while (!EOF (input_file)) {
  symbol = read_input (input_file);
  if (number_of_states < model_size) {
   update_tree (symbol, lower_trees, crt_state);
   update_tree (symbol, upper_tree, initial_state);
   crt_state = last_state (symbol, upper_tree);
  }
  else {
   generate_seq (upper_tree, lower_trees, ratio);
   flush_model (upper_tree, lower_trees);
   initial_state = new_state ();
   symbol = read_input (input_file);
   update_tree (symbol, upper_tree, initial_state);
   crt_state = last_state (symbol, upper_tree);
  }
 }
 generate_seq (upper_tree, lower_trees, ratio);
}
```

(a)

```
procedure generate_seq (upper_tree, lower_trees, ratio) {
 crt_symbol = generate_random ();
 lower_tree_node = last_node (upper_tree, crt_symbol);
 upper_tree_node = root (upper_tree);
 do {
  for each bit in the current vector {
   generate '0' or '1' to maximize the decrease in
   absolute error;
   if ('0' is generated) {
    lower_tree_node = left (lower_tree_node);
    upper_tree_node = left (upper_tree_node);
   }
   else {
    lower_tree_node = right (lower_tree_node);
    upper_tree_node = right (upper_tree_node);
   }
  }
  lower_tree_node = upper_tree_node;
  upper_tree_node = root (upper_tree);
 }
 while there are still vectors to be generated;
}
```

(b)

Fig.7

In all our experiments we used the DMC modeling technique based on the structure DMT_1. We also note that this strategy does note allow 'forbidden' vectors that is, those combinations that did not occur in the original sequence, will not appear in the final compacted sequence either. This is an essential capability needed to avoid 'hang-up' ('forbidden') states of the circuit during simulation process for power estimation.

Example 3: Assume that we are given the following 3-bit sequence consisting of 17 non-distinct vectors: (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}) = (001, 100, 001, 110, 111, 111, 101, 110, 011, 000, 101, 001, 100, 000, 110, 110, 011); our objective is to compact this sequence with a compaction ratio of 2.

We start building the Markov model that characterizes the initial sequence. For clarity, the construction of the tree DMT_1 is shown in Figs.8-9 for two different scenarios. First, in Fig.8, we assume that the parameter *model_size* is set by the user to the value 35; this means that the model can be grown dynamically (without any need for flushing) until this limit is reached.

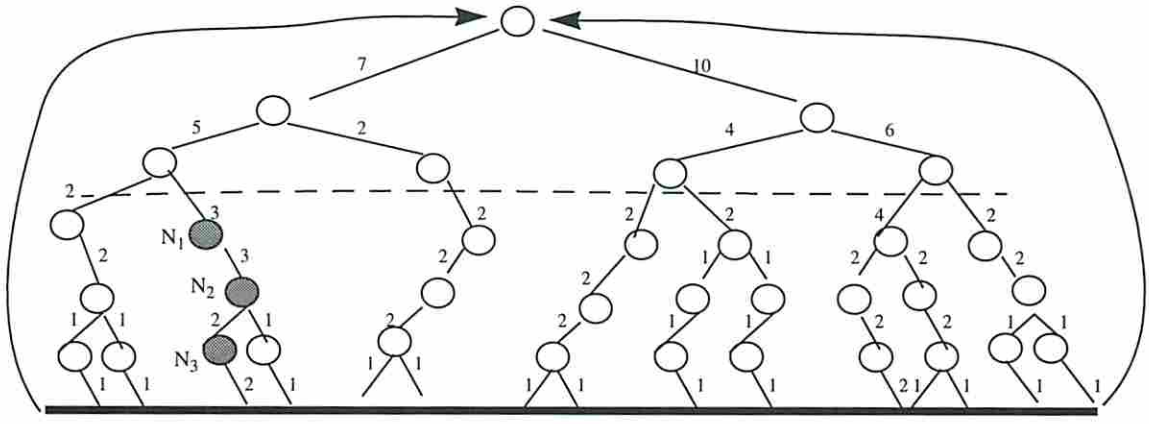

Fig.8

Once we built the Markov tree in Fig.8, we start the procedure *generate_seq* with parameter *ratio* = 2 and generate a subset of 8 vectors which best approximate the original sequence. To this effect, we use a modified version of the *dynamic weighted selection algorithm* [20]. In that approach, a similar structure with DMT_0 is built; more precisely, a full tree having on the leaves the symbols that need to be generated. The counts on the edges are dynamically updated and the symbols are generated according to their probability distribution. For this, a single random number generator is required in order to divide the interval [0,1] into subintervals that correspond to symbols' probabilities. At each level, the random number is compared to the left probability: if lower, a zero value is generated; if greater, a one value is generated and the number is decreased by the left probability. In our case, this strategy is used only to generate the first vector. After that, to ensure a minimal level of error, we use an *error controlling mechanism* in a greedy fashion. More precisely, at each level in the lower Markov tree, in order to decide whether a zero or one has to be generated, we compute the transition probabilities for both alternatives and choose the one that minimizes the absolute error accumulated up to that point . Simultaneously, the upper tree is parsed from the root to the leaves, according to the bits generated in the lower subtree. The procedure is then resumed until the needed number of vectors is generated.

In our example, if we assume that $x = 0.23$ is the first randomly generated number, based on the tree in Fig.8, since $0.23 < 7/17$ we take the left edge, generate a value 0 and x remains unchanged. At the second level, $x = 0.23 < 5/17$ so again we generate a '0' and leave x unchanged. Now $x = 0.23 > 2/17$ so a '1' is generated and x becomes $x = 0.23 - 2/17 = 0.11$. For the lower subtree rooted at the node denoted by the vector '001' (that is , we parse the upper subtree according to the already generated bits 0, 0, 1), to produce the second vector, we use the error controlling mechanism. Specifically, at node N_1 in Fig.8, the only choice is to take the right edge, generating a '1'. Next, at node N_2, the absolute error made for the transition probabilities becomes $|2/17-1/8| + |1/17 - 0| = 0.066$ if we take the left edge, and $|2/17 - 0| + |1/17 - 1/8| = 0.183$ if we take the right edge (8 is the length of the sequence to be obtained). The first choice is preferred and therefore a '0' is generated. At the last level, at node N_3, the decision is quite simple as we have only one descendent. Thus, after the first vector '001', we generate '101' as the second vector. The generation procedure continues for the lower subtree rooted at the node denoted by the vector '101' until the desired length $m =$

n/ratio is achieved. Despite its locality, this decision strategy performs very well in practice; as we'll see in the experimental part, the overall level of error is very small in all practicas cases.

In the second scenario, illustrated in Fig.9, the *model_size* parameter is set by the user as being 30 therefore the tree in Scenario 1 cannot be grown as such because the limit of 30 nodes is reached before the whole sequence is scanned. As a consequence, once we reach this limit (this actually happens immediately after processing the subsequence v_1, v_2, ..., v_9), we stop growing the tree and call *generate_seq* procedure with parameter *ratio* = 2 (Fig.9a). This will produce a subsequence of 4 vectors which best approximate the first 'segment' (v_1, v_2, ..., v_9) of the original sequence. After that we flush the model (keeping only the very last processed vector v_9) and start a new Markov tree as shown in Fig.9b. When the whole sequence is exhausted, based on this new Markov tree, we generate a new subset of 4 vectors which best approximate the second 'segment' (v_{10}, v_{11}, ..., v_{17}) of the original sequence.

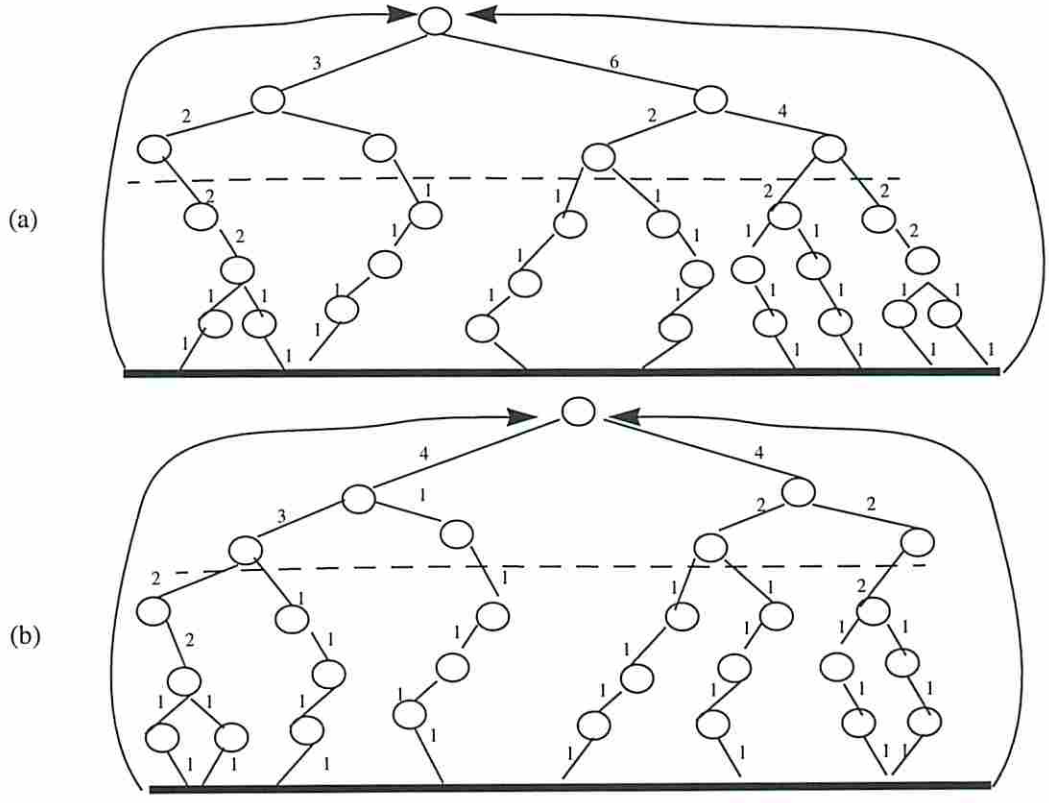

Fig.9

In general, by alternating the generation and flush phases in the DMC procedure, the complexity of the model can be effectively handled. The issue of accuracy in the context of these repeated flushes is discussed in the subsequent section.

5. Practical considerations

5.1 Complexity related issues

The DMC modeling approach offers the significant advantage of being a *one-pass adaptive technique*. As a one-pass technique, there is no requirement to save the whole sequence in the on-line computer memory. Starting with an

initial empty tree DMT_1, while the input sequence is scanned incrementally, both the set of states and the transition probabilities change dynamically making this technique highly adaptive.

Input sequences having a large number of bits k are very common in practice; the success of DMC models for sequence compaction when k is large is based on two key observations:

• The larger the value of k is, the sparser the structure of DMT_1 will be.

To motivate this, assume a finite input sequence of length n ($n \ll 2^k$). Intuitively, in a worst-case scenario when DMT_1 is completely skewed (that is, all vectors are distinct), DMT_1 will have a number of nodes proportional to $2nk$ (in all other cases, due to the sharing of paths among nondistinct vectors, the number of nodes will be smaller). On the other hand, the corresponding full tree (statically constructed) with the same depth, will have a number of nodes proportional with 2^{2k}. Therefore the sparsity of the tree DMT_1 (compared to the corresponding full tree) will increase with k as: $Sparsity \propto \frac{2nk}{2^{2k}}$. Assuming for instance an input sequence on 60 bits having a length of 100,000 vectors, then the sparsity of DMT_1 is about 10^{-29}. The DMC modeling technique exploits this observation by starting with an initially empty model and dynamically growing the Markov tree that characterizes the input sequence. By doing so, one can expect to build much smaller trees than the ones otherwise obtained by using a static model based on an initial full tree. Indeed, in practice the dynamic growing of the Markov model performs very well and the experimental results presented in the next section will support this claim.

• Biased sequences which usually occurs in practice as candidates for power estimation, contain a relatively small number of distinct patterns which arise in many different contexts in the whole sequence therefore a probabilistic model is ideally suited for modeling them.

We point out that both these observations can be efficiently exploited only by a probabilistic technique such as DMC modeling; a deterministic technique (e.g. [13]) has no such inherent capability and therefore cannot avoid all the difficulties that arise from this type of complexity.

However, a natural question still remains: when should this growing process be halted ? If it is not halted, there is no bound on the amount of memory needed. On the other side, if it is completely halted we lose the ability to adapt if some characteristics of the source message change. A practical solution is to set a limit on the number of states in the DMC [17] as we actually did in Example 3. When this limit is reached, the Markov model is flushed and a new model is started. Although this solution may appear as too drastic, in practice it performs very well. The intuition behind this property is the capability of DMC model to adapt very fast to changes that occur while the input is scanned. A less extreme solution to limit model growing is also possible; we can keep a backup buffer that retains the last p vectors emitted by the source and whenever the model should be discarded, we may reuse this information to avoid starting the new model from the scratch.

5.2 Accuracy related issues

To see how the flushing technique affects the accuracy, let's assume that an input sequence of length n is modeled by the *DMC* approach. Suppose that during the building of the Markov model, flushing occurs after the first n_1 vectors, then after the next n_2 vectors, and so on. If the number of flushes is f, then $n_1 + n_2 + ... + n_f = n$. Let $v_i\,(u_i)$ be a vector from the initial (compacted) i-th subsequence (obtained due to successive flushes) and $v\,(u)$ a vector from the initial (compacted) sequence. We note that:

$$P(v^- = X \wedge v^+ = Y) = \frac{\sum_{i=1}^{f} n_i \cdot P(v_i^- = X \wedge v_i^+ = Y)}{n} \text{ and}$$

$$P(u^- = X \wedge u^+ = Y) = \frac{\sum_{i=1}^{f} \frac{n_i}{r} \cdot P(u_i^- = X \wedge u_i^+ = Y)}{\frac{n}{r}} \text{ where } r \text{ is the compaction ratio.}$$

If the i-th subsequence is approximated with an error less than ε_i, then the accuracy for the whole sequence is

$$\varepsilon = \frac{\sum_{i=1}^{f} n_i \cdot \varepsilon_i}{n} \leq max(\varepsilon_i)\ . \tag{5}$$

Therefore, as long as the partial DMC models accurately the transition probabilities for the initial subsequences, the transition probabilities for the entire sequence are preserved up to some ε. Differently stated, we do not have to worry about the number of flushes needed to manage complexity if the individual Markov models capture accurately the characteristics of the subsequences.

An extreme case may occur when all patterns in the original sequence are distinct that is, there are no two identical vectors in the whole sequence. In such a case all terminal edges in the upper subtree of DMT_1 will have only one successor in the lower subtree, therefore the probabilistic choices during the generation phase will degenerate into a pure deterministic generation of a subsequence from the initial sequence. To avoid this, we propose to use a random pairwise generation phase that is, to select randomly the first vector in the upper subtree of DMT_1 and generate deterministically the second vector from the lower subtree of DMT_1. An alternative solution would be to partition the initial sequence into disjoint groups of bits and apply the DMC modeling technique to each individual group separately. As partitioning criteria we may use either functional considerations (e.g. separation between control bits and data-path bits) or the level of correlations among individual bits as was originally proposed in [14]. By partitioning, we increase the chances for repeated patterns in each group of bits therefore making probabilistic arguments more reasonable. The disadvantage however, is that this solution will ignore correlations across group boundaries.

6. Experimental results

The overall strategy is depicted in Fig.10.

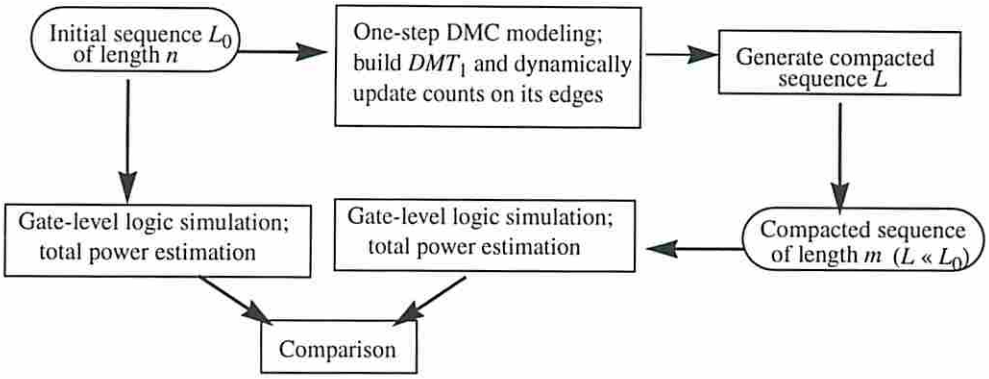

Fig.10

Basically, we verified our ability to compact large input sequences which may also be used as power benchmarks in the design process. We assume that the input data is given in the form of a sequence of binary vectors. While this is a valid assumption at the logic level, it requires some justification at the architectural level. An instruction stream at the architectural level can be converted to a binary stream using the information about opcodes and dynamic instruction traces that resolve memory references and ambiguities. The obtained binary stream can be then subjected to any compaction technique developed for bit-level specifications.

Starting with an k-bit input sequence of length n, we perform a one-pass traversal of the original sequence and simultaneously build the basic tree DMT_1; during this process, the frequency counts on DMT_1's edges are dynamically updated.

The next step in Fig.10 does the actual generation of the output sequence (of length m). As explained in Section 4, to generate the new sequence we use a modified version of the dynamic weighted selection algorithm presented in [20]. If the initial sequence has the length n and the new generated sequence has the length $m < n$ then the outcome of this process is a compacted sequence, equivalent to the initial one as far as total power consumption is concerned; we say that a *compaction ratio* of $r = n/m$ was achieved.

Finally, a validation step is included in the strategy; for short sequences we used the commercial tool PowerMill [2] whilst for long sequences we resorted to an in-house gate-level logic simulator developed under SIS. The total power consumption of some ISCAS'85 and ISCAS'89 benchmarks has been measured for the initial and the compacted sequences, making it possible to assess the effectiveness of the compaction procedure (under both zero- and real-delay models).

In Tables 1-2, we provide only the real-delay results for two types of initial sequences. Sequences of type 1 are large input streams having the same initial length n =100,000 and being then prime candidates for compaction; type 1 refers to biased sequences obtained by doing bit-level logical operations on ordinary pseudorandom sequences. The sequences of type 2 (having the length 4,000) are highly biased sequences obtained from real industry applications. As shown in Table 1, sequences of type 1 were compacted with two different compaction

ratios (namely r = 50 and 100); we give in this table the total power dissipation measured for the initial sequence (column 3) and for the compacted sequence (columns 4, 5). In the last column, we give the time is seconds (on a Sparc 20 workstation with 64 Mbytes of memory) necessary to read and compress data with DMC modeling. Since the compaction with DMC modeling is linear in the number of nodes in the structure DMT_1, the values reported in the last column are far less than the actual time needed to simulate the whole sequence. During these experiments, the number of states allowed in the Markov model was 20,000 on average.

Table 1: Total Power (uW@20MHz) for sequences of type 1

Circuit	Number of inputs	Power for initial seq.	Power for r = 50	Power for r = 100	Time for DMC (sec)
C432	36	1816.32	1838.89	1779.60	42
C499	41	3697.84	3546.65	3622.26	48
C880	60	3314.07	3229.85	3329.31	75
C1355	41	3205.27	3044.20	3109.18	48
C3540	50	10876.22	9910.08	10687.32	61
C6288	32	110038.69	114199.50	109077.42	37
s344	9	751.58	748.54	719.53	10
s386	7	818.11	844.58	848.80	8
s838	34	1052.05	1061.73	1091.14	41
s1196	14	3687.47	3702.32	3580.63	16
s9234	36	9192.75	9157.31	9209.75	43
		Average relative error (%)	2.80	2.93	

As we can see, the quality of results is very good even when the length of the initial sequence is reduced by 2 orders of magnitude. Thus, for C432 in Table 1, instead of simulating 100,000 vectors with an exact power of 1816.32 uW, one can use only 2000 vectors with an estimate of 1838.89 uW or just 1000 vectors with a power consumption estimated as 1779.60 uW. This reduction in the sequence length has a significant impact on speeding-up the simulative approaches where the running time is proportional to the length of the sequence which must be simulated.

The sequences of type 2 were compacted for two compaction ratios (r = 5 and r =10) using PowerMill [2]; to asses the potential of efficiency of the approach, for both original and compacted sequences, we report also the actual running time required by PowerMill to provide power estimates. The number of states allowed for the Markov model construction, was 5,000 on average; the CPU time for DMC modeling was below 3 seconds in all cases.

Table 2: Total Current (mA) for sequences of type 2

		Initial sequence		Compacted sequence		
Circuit	Number of inputs	Current (mA)	Time to simulate (sec)	Current (mA) r = 5	Current (mA) r = 10	Time to simulate (sec) r = 10
C432	36	0.4135	1186	0.4352	0.4066	120
C499	41	0.8188	2675	0.8337	0.8290	235
C880	60	0.7907	2289	0.7995	0.8023	274
C1355	41	1.1375	2993	1.1549	1.1461	284
C1908	33	1.2976	4034	1.2821	1.2833	367
C3540	50	3.4490	9467	3.3582	3.5719	1082
C6288	32	14.5749	88032	14.8020	13.3095	5005
			Average relative error (%)	2.15	2.63	

As it can be seen in Table 2, the average relative error is below 3% while the speed-up in power estimation is about one order of magnitude on average. For example, using the original sequence of 4000 vectors, PowerMill took for C432 about 1186 seconds to estimate a total current of 0.4135 mA. On the other side, using the sequence generated with DMC of only 400 vectors (r = 10), PowerMill estimated a total current of 0.4066 mA in only 120 seconds. This speed-up in power estimation becomes more significant for larger modules (e.g.C6288). We note also, that the results presented both tables 1 and 2, are significantly better than those reported in [14] in terms of accuracy, running time and memory requirements.

Finally, we compare our results with simple random sampling of vector pairs from the original sequences [21]. In simple random sampling, we performed 1,000 simulation runs with 0.99 confidence level and 5% error level on each circuit[1]. We report in Table 3 the maximum and average number of vector pairs needed for total power values to converge [11]. We also indicate the percentage of error violations for total power values, using as thresholds 5%, 6% and 10%. Using different seeds for the random number generator (and therefore choosing different initial states in the sequence generation phase), we run a set of 1,000 experiments for the DMC technique. In Table 4, we give the DMC results for the same thresholds as those used in simple random sampling.

Table 3: Results obtained for Simple Random Sampling

	Number of vector pairs		Error violations (%)		
Circuit	Max.	Avg.	> 5%	> 6%	>10%
C432	3300	2176	1.1	0.7	0.2
C499	1500	862	1.4	1.3	0.4
C880	3990	2705	1.8	0.4	0.7
C1355	1380	814	1.7	1.0	0.2
C1908	1620	846	1.9	1.3	0.2
C6288	7470	5422	1.4	1.4	0.3

Table 4: Results obtained for DMC Approach

		Error violations (%)		
Circuit	Number of vectors	> 5%	> 6%	>10%
C432	2000	6.7	1.9	0.0
C499	800	0.3	0.0	0.0
C880	2000	1.4	0.1	0.0
C1355	800	0.2	0.0	0.0
C1908	800	1.9	1.2	0.0
C6288	2000	0.0	0.0	0.0

Once again, the results obtained with DMC modeling technique score very well and prove the robustness of the present approach. As we can see, using fewer vectors, the accuracy of DMC is higher than the one of simple random sampling in most of the cases. Noteworthy examples are benchmarks C499, C880, C6288 where less than 60% of the maximal number of vector pairs needed in random sampling for convergence are sufficient for DMC to achieve higher accuracy and confidence levels.

7. Conclusion

In this paper, we addressed the vector compaction problem from a probabilistic point of view. Based on dynamic Markov Chain modeling, we proposed an original approach to compact an original sequence into a much shorter equivalent one, which can be used after that with any available simulator to derive power estimates in the target circuit.

The mathematical foundation of this approach relies in Markov models; within this framework a family of

1. This means that the probability of having a relative error larger than 5% is only 1%.

dynamic Markov trees is introduced and characterized as an effective and flexible way to model complex spatiotemporal correlations which occur during power estimation. The results obtained both on combinational and sequential benchmarks show that large compaction ratios of 1-2 orders of magnitude can be obtained without much loss in accuracy in total power estimates.

The issues brought into attention on this paper represent an important step to reduce the gap between simulative and nonsimulative techniques which are currently the norm.

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