An ATM Queueing System with Long-Range Dependent Traffic: Providing QoS Guarantees

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Abstract

Many types of network traffic have been shown to exhibit long-range dependence. In this work, we investigate several performance issues regarding the dynamics of an ATM queueing system with long-range dependent arrivals. We develop a new framework, based on a probabilistic envelope process characterization of the input traffic, capable of accurately predicting queueing statistics. We compute a delay bound based on this framework and show that it is a tighter bound than Cruz's bound. We believe that this framework can be effectively used to provide Quality of Service Guarantees in ATM networks.

1 Introduction

Recently, Bellcore researchers [1] claimed that local area network traffic presents long-range dependence (LRD) [2] and can be accurately modeled by a self-similar process. In fact, although there is undeniable evidence that other types of traffic, *e.g.* variable bit rate (VBR) video [3, 4] and wide-area network traffic [5], also exhibits LRD, there is still not a complete and clear understanding on the meaning of LRD and its impact on network performance and network management protocols.

For example, a series of extensive simulation [6, 7] and analytical studies [8, 9, 10] pointed out that LRD might have pervasive effect on queueing performance, *i.e.* there is clear evidence that it can potentially cause massive cell losses in ATM networks. In fact, Duffield [11] showed that the buffer overflow probability for an ATM queueing system with fractional Brownian arrivals follows a Weibull distribution. Nevertheless, another set of studies inspected ATM systems driven by VBR video traffic [12, 13, 14] and concluded that the cell losses were caused by the short-range dependence (SRD), even though this traffic source exhibits LRD.

In this work we investigate the dynamics of an ATM queueing system driven by a LRD source. We revisit Mandelbrot's work [2] on the meaning of LRD dependence in order to explain its impact on queueing performance. We use time-scale analysis to show that a LRD source can sustain *high* rates for very long time intervals, possibly generating very long busy periods. We also show that depending on the buffer size and link utilization cell losses are more likely to occur at small or large time-scales. Based on this result, we can predict if the losses are related to the long-range or short-range dependence.

We also introduce a fractional Brownian motion (fBm) envelope process and show that it provides an accurate bound for real LAN traffic and can be used to accurately predict delay bounds and other queueing statistics. In fact, we develop a new delay bound calculus based on this envelope process and show that i) it agrees with the results obtained by large deviation theory [11], ii) it is a tighter bound than Cruz's calculus [15, 16], and iii) it agrees with the delay experienced by real network traffic. In summary, we propose a new framework based on the envelope process characterization of the input traffic that is capable of providing Quality of Service in ATM networks.

Therefore, our goals are twofold:

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- To explain the LRD phenomenon and its implication on network performance.
- To propose a new and more general traffic characterization based on envelope processes that is capable of handling long-range dependent sources.

2 What is Long-Range Dependence ?

In this section we attempt to explain the behavior of an ATM queueing system driven by LRD traffic. An ATM node can be modeled as a single-server queueing system, with deterministic service rate given by c. The arrival traffic is defined by the process $A_H(t)$ with mean $\bar{a}$ and variance σ^2. Mandelbrot [2] studied long-range dependence in Economic time series. He explains this phenomenon as a tendency for large values to be followed by large values, in such a way that those time series seem to go through a succession of *cycles* whose wavelength is of the order of the magnitude of the total sample size. It implies that i) traffic sources exhibiting LRD can sustain *high* transmission rates for very long intervals and ii) it is not possible to define a maximum burst size for those sources.

2.1 Quantifying LRD

We can quantify the LRD phenomenon by investigating for how long the source is likely to transmit at *high* rates. Following Norros' work [8], assume that the arrival process $A_H(t)$ is a fBm process given by

$$A_H(t) = \bar{a}t + \sqrt{\bar{a}v}Z(t)$$

where $\overline{a} > 0$ is the mean input rate, $v > 0$ is the variance coefficient, $H \in [\frac{1}{2}, 1)$ is the self-similar (Hurst) parameter and $Z(t)$ is a normalized fractional Brownian motion. We investigate the probability that the *instantaneous* average arrival rate, defined as $\frac{A_H(t)}{t}$, exceeds k times its mean rate.

$$P(\frac{A_H(t)}{t} > k\bar{a}) = P(\bar{a}t + \sqrt{\bar{a}v}Z(t) > k\bar{a}t) = P(Z(t) > \frac{t(k\bar{a} - \bar{a})}{\sqrt{\bar{a}v}})$$

By the self-similarity property $Z(t) = t^H Z(1)$, we have

$$P(Z(1) > \frac{t\bar{a}(k-1)}{\sqrt{\bar{a}v}t^H}) = \overline{\Phi}(\frac{\bar{a}(k-1)t^{1-H}}{\sqrt{\bar{a}v}})$$

where $\overline{\Phi}(y) = P(Z(1) > y)$ is the residual distribution function of the standard Gaussian distribution. In fact, using the approximation [8]

$$\overline{\Phi}(y) \approx (2\Pi)^{-1/2}(1+y)^{-1}exp(-y^2/2) \approx exp(-y^2/2) \tag{1}$$

we obtain

$$P(\frac{A_H(t)}{t} > k\bar{a}) \approx exp(-\frac{\bar{a}^2(k-1)^2 t^{2-2H}}{2\bar{a}v}) \tag{2}$$

Equation (2) shows that the probability that the fBm instantaneous average arrival rate exceeds its mean rate decays exponentially fast with t when $H = 1/2$. For a LRD process, *e.g.* if $H = 0.9$, this probability can decrease very slowly with t. We computed the average arrival rate $\bar{a}$ and variance σ^2 parameters for Bellcore's LAN trace (pAug.TL) [1] and substituted them in equation (2); figure 1 shows the result. Throughout this paper, whenever we refer to LAN traffic, we are considering this specific traffic sample. The upper and lower dotted curves correspond to the probability that the Brownian motion's instantaneous average rate is $2.0\bar{a}$ and $3.0\bar{a}$ at time t, respectively. The upper and lower solid curves correspond to the probability that the rate of a fBm with $H = 0.9$ is $2.0\bar{a}$ and $3.0\bar{a}$ at time t, respectively.

2.2 Defining Utilization

By the Strong law of Large Numbers we know that $\frac{A_H(t)}{t}$ converges to its mean $\bar{a}$. Moreover, the law of the Iterated Logarithm gives the rate at which $\frac{A_H(t)}{t}$ approaches the mean [17]. For LRD processes, this convergence can be *very slow* [3, 18]. For example, figure 2 shows the rate of convergence of a Poisson process. The three dotted curves correspond to three non-overlapping sample-paths of the normalized instantaneous average rate, *i.e.* $\frac{A(t)}{\bar{a}t}$. The solid curve corresponds to the worst-case sample path, defined by $max(\frac{A(t)}{\bar{a}t})$, for a 1,000,000 points sample. The worst-case sample path converges to the mean in a *short* period of time.

[1] This trace is available with anonymous FTP from ftp.bellcore.com and is shown to exhibit LRD.

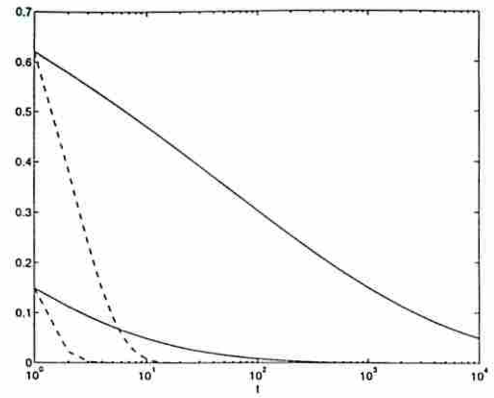

Figure 1: $P(\frac{A_H(t)}{t} > \bar{k})$ for H=0.5 (dotted curve) and H=0.9 (solid curve).

Figure 3 shows the rate of convergence for the LAN traffic. The worst-case sample path converges very slowly to the mean. This phenomenon limits the maximum possible link utilization, defined by $\frac{\bar{a}}{c}$. In fact, real network traffic is not stationary, therefore it is not adequate to define a long-term utilization. By using a self-similar model, we attempt to account for the large variability of the traffic without giving up stationarity [2]. Therefore, even though the long-term link utilization is low, the instantaneous utilization can be relatively high for very long periods of time. For example, assume that the link capacity c is given by $2\bar{a}$. We computed the instantaneous utilization, defined as $\frac{A(\tau)}{\tau c}$, for the LAN traffic over non-overlapping, consecutive, periods $\tau = 10,000$ time-slots. Figure 4 shows that in some intervals the link utilization achieves almost 80% even though the long-term utilization is 50%.

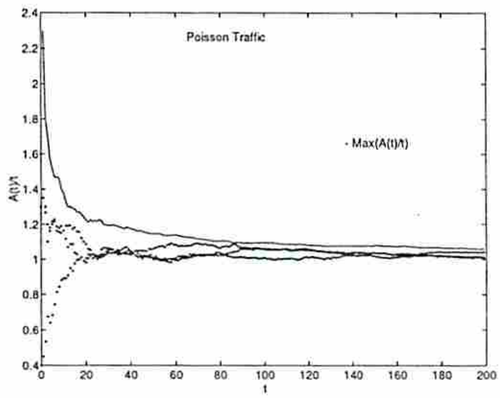

Figure 2: Sample paths of the normalized instantaneous average rate for a Poisson process.

2.3 Busy Period

A direct consequence of LRD is the presence of very long busy periods, possibly causing cell losses. In fact, we showed [19] that in an ATM queueing system with LRD traffic, at low utilization the cell losses are concentrated on the tail of the busy period distribution. In this section we compute a probabilistic bound for the maximum busy period of an ATM queueing system driven by a fBm process. We compare it to the busy period of a system with Brownian motion arrivals.

It has been shown that the length of each busy period is bounded above by a constant d [20]. Let

$$d \stackrel{\text{def}}{=} inf\{t \geq 1 : A_H(t) - ct \leq 0\} \tag{3}$$

By using large deviation theory, we can compute a probabilistic bound $\hat{d}$ for the busy period.

$$\hat{d} \stackrel{\text{def}}{=} inf\{t \geq 1 : P(A_H(t) > ct) < \epsilon\}$$

where $\epsilon << 1$. Therefore, *the busy period will not exceed* $\hat{d}$ *with probability* $(1 - \epsilon)$.

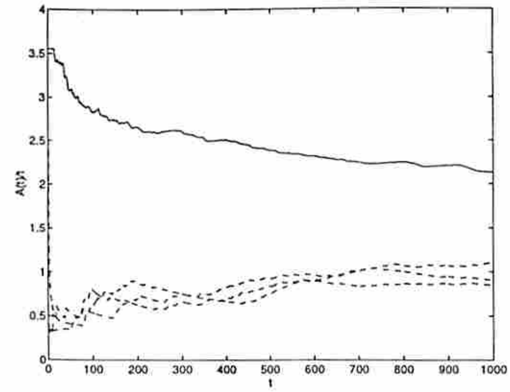

Figure 3: Sample paths of the normalized instantaneous average rate for the LAN traffic.

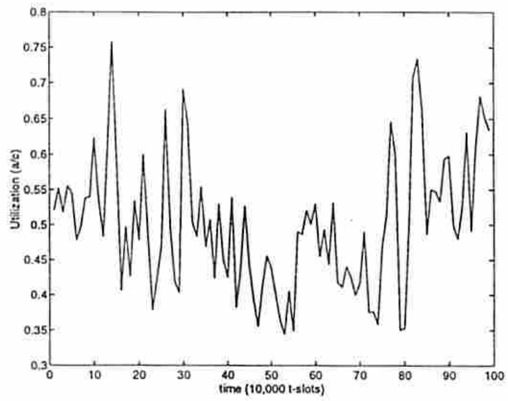

Figure 4: Instantaneous Utilization measured over 10,000 time-slots.

2.3.1 Computation of the Maximum Busy Period

By following the same approach as in the previous section, we can write

$$P(A_H(t) > ct) = \overline{\Phi}(\frac{(c-\overline{a})}{\sqrt{\overline{a}v}t^{H-1}})$$

Therefore,

$$\hat{d} = inf\{t > 0 : \overline{\Phi}(\frac{(c-\overline{a})}{\sqrt{\overline{a}v}t^{H-1}}) \leq \epsilon\}$$

where $\epsilon << 1$. Using the approximation in (1), we can write

$$\hat{d}_H = (\frac{\sqrt{(-2\log\epsilon)}\sqrt{\overline{a}v}}{(c-\overline{a})})^{\frac{1}{1-H}} = (\frac{k\sigma}{(c-\overline{a})})^{\frac{1}{1-H}} = B^{\frac{1}{1-H}} \tag{4}$$

where B is given by $(\frac{\sqrt{(-2\log\epsilon)}\sqrt{\overline{a}v}}{(c-\overline{a})})$ For the case of LAN traffic, Bellcore researchers observed H to be as large as 0.9. Therefore, the dependence on H exhibited by formula (4) shows that the busy period of the LRD system can be several orders of magnitude larger than the case of Brownian motion arrivals. For example, for $H = 1/2$ and $H = 0.90$, $\hat{d}_H$ is given by B^2 and B^{10} respectively.

2.3.2 Example

We substituted the parameters for the LAN traffic in equation (4) and compared it to the case given by $H = 1/2$. Figure 5 shows the results. The dotted curve corresponds to the case of a Brownian motion process, *i.e.* H=1/2. In this case, the maximum busy period is relatively *small* even if the link capacity is close to the average arrival rate. The solid curve shows the busy period's bound when H=0.90. In this case, since the process exhibits LRD, the maximum busy period can be extremely large ($> 100,000$ time-slots) if the link rate is close to the mean arrival rate.

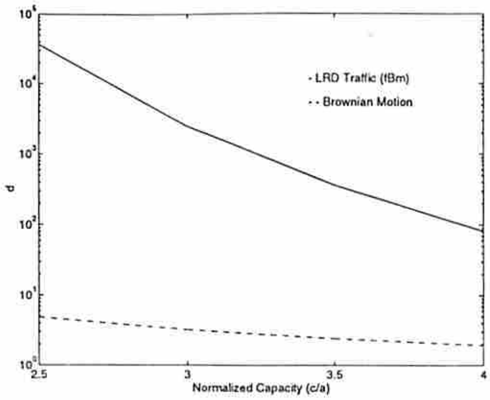

Figure 5: The busy period's bound when H=0.50 (dotted curve) and H=0.90 (solid curve).

We conclude that ATM links can be either *busy* or *idle* for very long period of times. In this case, it is necessary to allocate bandwidth dynamically in order to maximize link utilization and avoid congestion. A possible solution for this problem of *non-homogeneous link utilization*, is to change dynamically the bandwidth allocated for a given Virtual Path (VP) based on its current utilization level [21].

2.4 Time Scale of Cell Losses

S.Q.Li simulated an ATM system with VBR traffic [22] and claimed that the cell losses were caused by the low-frequency (long-range dependence) component of the traffic. On the other hand, recent simulation studies based on an ATM queueing system with VBR video traffic indicated that the high-frequency component of this input traffic was responsible for the majority of the cell losses in the system, [12] when small buffers (bounded delay) is considered. Therefore it is not clear in which scenario LRD is responsible for cell losses. We can answer this question by investigating in which time-scale cell losses are more likely to occur. If they do occur at large time-scales, LRD is to blame for the losses. Otherwise, the cell losses are caused by SRD and Markov models are able to predict queueing statistics.

Montgomery [23] proposes a framework to investigate the time-scale in which cell losses are more likely to occur. We apply his framework to our queueing system with an fBm source. By using the same derivation of section 3.2, the probability that over a time interval of length t the $A_H(t)$ source can overcome the potential service ct and further exceed a buffer level b is given by

$$P(A_H(t) > ct + b) = \overline{\Phi}(\frac{t(c-\overline{a})+b}{\sqrt{\overline{a}v}t^H}) \approx exp(-\frac{1}{2}g(t)^2) = exp(-\frac{1}{2}(\frac{t(c-\overline{a})+b}{\sqrt{\overline{a}v}t^H}^2))$$

Therefore, a minimizer $t^* \in arginf_{t>0}g(t)$ so that the overflow probability is maximized, would correspond to a likely time-scale on which overflow occurs in this system. Therefore, t^* is given by

$$t_H{}^* = \frac{bH}{(c-\overline{a})(1-H)}$$

The time-scale of overflow events increases linearly with buffer size. For small buffers, cells losses are more likely to occur at a small time-scale, *i.e* they are caused by the high-frequency component. Montgomery found that for a Brownian motion process, $t^* = b/(c-\overline{a})$. It is exactly the result developed here when $H = 1/2$. Therefore $t_H{}^* = \frac{H}{(1-H)}t^*$, *i.e.* for a given buffer size, cell losses in LRD systems usually occur at larger time-scales than in traditional Markovian systems. Since the low-frequency component is associated to large time-scales, we can say that for large buffer's systems, the low-frequency component (LRD) dominates bandwidth requirements. This result was predicted by S.Q.Li in [24, 25].

3 A Probabilistic Envelope Process

In this section we introduce a fBm envelope process and show that it can be used to predict queueing statistics. Cruz developed a calculus for network delay based on a deterministic characterization of the input traffic [15, 16]. Cruz's delay bound has been shown to be too conservative therefore limiting its practical use. Chang [20] extended

Cruz's work by developing a delay bound based on a stochastic characterization of the input traffic. In this work, we use a probabilistic envelope process to characterize the input traffic in order to compute delay bounds and other queueing statistics. Our approach has the advantage of combining the simplicity of Cruz's delay calculus with the multiplexing effects resulting from the use of statistical characterization.

It is well known that for a Brownian motion (Bm) process $A(t)$ with mean $\overline{a}$ and variance σ^2, the envelope process $\hat{A}(t)$ can be defined by [26]

$$\hat{A}(t) \stackrel{\text{def}}{=} \overline{a}t + k\sqrt{\sigma^2 t} = \overline{a}t + k\sigma t^{\frac{1}{2}}$$

The parameter k determines the probability that $A(t)$ will exceed $\hat{A}(t)$ at time t. In fact, using the approximation given by equation (1), we get $k = \sqrt{-2\log\epsilon}$, where $P(A(t) > \hat{A}(t)) \leq \epsilon$, see Appendix A.

This approach can be extended to LRD traffic. Let $A_H(t)$ be a fractional Brownian motion process with mean $\overline{a}$. Hurst's law states that the variance of the increment of this process is given by $Var[A_H(t+s) - A_H(t)] = \sigma^2 s^{2H}$ where $H \in [\frac{1}{2}, 1)$ is the Hurst parameter. Therefore, we can also define a fBm envelope process by

$$\hat{A}_H(t) \stackrel{\text{def}}{=} \overline{a}t + k\sqrt{\sigma^2 t^{2H}} = \overline{a}t + k\sigma t^H$$

The Brownian motion envelope process is just the special case of $H = 1/2$. Similarly, k determines the probability that $A_H(t)$ will exceed $\hat{A}_H(t)$. However, since the process exhibits LRD, if $A(t)$ exceeds $\hat{A}(t)$ at time t, it is possible that it will stay *above* it for a long period of time. This representation of the input traffic has two major advantages: i) it is quite general, *i.e* it can represent *any* traffic source, ii) the input parameters $\overline{a}, \sigma$, and H can be provided by the source or estimated in real-time from the incoming traffic sample. We will show in the next sections that we can compute a very tight delay bound based on this characterization.

We investigated the accuracy of the fBm envelope process representation by inspecting how well it can model the worst-case behavior of real LRD traffic. We substituted the parameters of the LAN traffic in the envelope process equations. In figure 7, the upper curve corresponds to the fBm envelope process with $\epsilon = 10^{-6}$. The lower curve represents the Brownian motion envelope process with $\epsilon = 10^{-1}$. The optimal envelope process (the worst-case sample path) for this trace is defined by $max_t(A(t))$ and is represented by the middle (dotted) curve. Figure 7 shows that the ordinary Brownian motion envelope process is unable to bound the behavior of the LRD source even if we choose ϵ large. On the other hand, the fBm envelope process does bound the source behavior throughout the entire *sample* space, *i.e.* $max_t(A(t))$ does not exceed $\hat{A}_H(t)$ even if $\epsilon = 10^{-6}$.

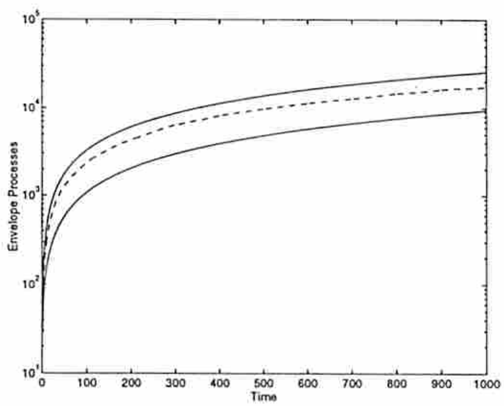

Figure 6: Envelope Processes versus LAN traffic.

3.1 Computation of the Maximum Busy Period by using Envelope Processes

We use the fBm envelope processes to compute a bound for the busy period and compare it to the result of the previous section. We compute $\hat{d}_H$ by substituting $\hat{A}_H(t)$ into equation (3). In this case, $\hat{A}_H(t) = ct$ so that we have

$$\overline{a}t + k\sigma t^H = ct$$

Therefore, $\hat{d}_H$ is given by

$$\hat{d}_H = \left(\frac{\sqrt{(-2\log\epsilon)}\sqrt{\overline{a}v}}{(c - \overline{a})}\right)^{\frac{1}{1-H}}$$

We arrive at the same formula (4) computed in section 2.3. It shows that the envelope process characterization leads to an accurate solution with little computational effort.

3.2 Leaky Bucket

The Leaky Bucket can be seen as a traffic regulator [15, 32] with output given by the process $L(t)$ so that

$$L(t) \leq Rt + S$$

In an interval of length t it can *accept* up to $(Rt + S)$ cells. If the input process $A(t)$ behaves according to this traffic descriptor, the Leaky Bucket should *accept* all incoming cells. Otherwise, if the source *misbehaves*, the Leaky Bucket should mark incoming cells. It is very hard to choose the Leaky Bucket's parameter for a *bursty* source so that it only marks cells when the process is misbehaving. By using the envelope process, we develop a framework that can be effectively used to setup the Leaky Bucket's parameter.

In order to minimize incorrectly dropping cells, we should have $L(t) \geq A_H(t), \forall t > 0$. Moreover,

$$L(t) \geq A_H(t) \leq \hat{A}_H(t), \forall t > 0$$

Therefore, we can assume that $L(t) \geq \hat{A}_H(t)$ so that we have

$$\bar{a}t + k\sigma t^H \leq Rt + S \tag{5}$$

Moreover,

$$t(\bar{a} - R) + k\sigma t^H - S \leq 0 \tag{6}$$

In order to choose the Leaky Bucket parameters, we find t^* that maximizes equation (6).

$$t^* = [\frac{k\sigma H}{R - \bar{a}}]^{\frac{1}{1-H}}$$

Substituting t^* back in equation (5) we get

$$(\bar{a} - R)[\frac{k\sigma H}{R - \bar{a}}]^{\frac{1}{1-H}} + k\sigma[\frac{k\sigma H}{R - \bar{a}}]^{\frac{H}{1-H}} - S \leq 0 \tag{7}$$

Therefore, by using (7) we can compute R given S, or vice versa. In the case of a Brownian motion the equation (7) degenerates in a simple quadratic equation. For the general case, we can solve equation (7) numerically.

3.2.1 Example

We substituted the LAN traffic parameters in equation (7); see figure 8. The dotted curve corresponds to the special case H=1/2. We can see that even if R is chosen close to the average rate, S is still relatively *small.* The solid curve corresponds to the case when H=0.90. In this case, if R is close to the mean rate, S is prohibitively large. In fact, the inability of the Leaky Bucket to police the average rate whenever the source is *bursty* has already been reported previously [29, 30].

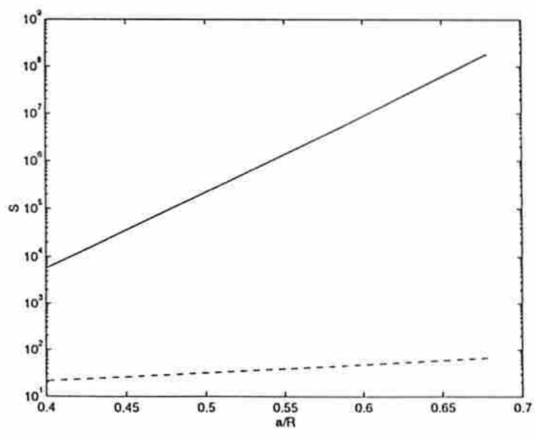

Figure 7: Leaky Bucket Parameters

3.3 Maximum Queue Size

Previously, Duffield [11] showed that the tail of the distribution of a fractional Brownian storage system follows a Weibull distribution. In this section, we derive a probabilistic bound for the maximum number of cells in the system, by taking advantage of the simplicity of the envelope process defined in the previous section. We show that our result is the same result obtained by Duffield. Consider a continuous-time queueing system, with deterministic service given by c. Without loss of generality, assume that i) the arrival process is given by $A(t)$, continuous and differentiable and ii) a busy period starts at time *0* and ends at time t_1. Assume also that $A(0) = 0$. For a given busy period, the number of cells in the system (unfinished work)[2] at time $t \leq t_1$ is given by $q(t) = A(t) - ct \geq 0$. Therefore, the maximum number of cells in the system for this busy period occurs at time t^* such that $\frac{dq(t^*)}{dt} = 0$. In this case,

$$\frac{dA(t^*)}{dt} = c \tag{8}$$

The envelope process $\hat{A}(t)$ defines a probabilistic upper-bound for $A(t)$. Therefore, we can compute an upper-bound for the maximum number of cells in the system, by substituting $\hat{A}(t)$ in equation (8). In this case, the maximum number of cells, given by q_{max}, occurs at time t^* such that

$$\frac{d\hat{A}(t^*)}{dt} = c$$

We can say that $q(t)$ *will not exceed* q_{max} *with probability* $(1-\epsilon)$. In other words, whenever the arrival process does not exceed the envelope process, the maximum number of cells size will not exceed its estimated value. For the case of the fBm process, we have that

$$q(t) = \hat{A}_H(t) - ct = \bar{a}t + k\sigma t^H - ct$$

Therefore, t^* is given by

$$t^* = [\frac{k\sigma H}{(c-\bar{a})}]^{\frac{1}{1-H}}$$

Moreover q_{max} is given by

$$q_{max} = \hat{A}_H(t^*) - ct^* \tag{9}$$

$$q_{max} = (c-\bar{a})^{\frac{H}{H-1}}(k\sigma)^{\frac{1}{1-H}}H^{\frac{H}{1-H}}(1-H)$$

Since the fBm process does not exceed $\hat{A}(t)$ with probability $(1-\epsilon)$, the maximum number of cells will be bounded by q_{max} with the same probability. We find $c^{'}$ so that q_{max} is equal to K. In other words, a buffer of size K will overflow with probability ϵ if the link capacity is $c^{'}$. Therefore, $c^{'}$ is given by

$$c^{'} = a + K^{\frac{H-1}{H}}(k\sigma)^{\frac{1}{H}}H(1-H)^{\frac{H-1}{H}}$$

This result was also obtained by Norros [31] using Duffield's equation ! For the special case *H=1/2*, equation (9) degenerates into a simple quadratic equation. Therefore, q_{max} is given by

$$q_{max} = \frac{k^2\sigma^2}{4(c-\bar{a})} = \frac{\sigma^2 \log\epsilon}{2(\bar{a}-c)} \tag{10}$$

This result is exactly the same result given by the diffusion equation for a deterministic service system, see Appendix B. On the other hand, if $H = 0.90$, q_{max} is given by

$$q_{max} = (c-\bar{a})^{-9}(k\sigma)^{10}0.04$$

We define β to be the ratio between q_{max} for H equal to 0.90 and 0.50 respectively. Therefore,

$$\beta = \frac{q_{max}{}^{0.90}}{q_{max}{}^{0.50}} = 0.16[\frac{k\sigma}{(c-\bar{a})}]^{8}$$

[2]The unfinished work is given by the formula: number of cells * cell transmission time.

If the traffic source has *large* variance, β can also be very large. In fact, network traffic has also been shown to suffer from the infinite variance syndrome (IFV) [2]. In this case, q_{max} can be much larger than the bounds computed by traditional Markov queueing models.

In summary, we derived a simple framework for computing accurate queueing bounds that is capable of handling LRD sources. Furthermore, this framework can be easily extended to handle statistical multiplexing of heterogeneous sources.

3.4 Example

We substituted the LAN traffic parameters in (9). Figure 9 shows that if the link utilization is greater than 40%, *i.e.* $c < 2.5\overline{a}$, the fBm queueing system can exhibit a queue size *100* times greater than the Brownian motion's maximum queue size. The over-optimistic queueing results of traditional models have been reported earlier in [6, 7, 11].

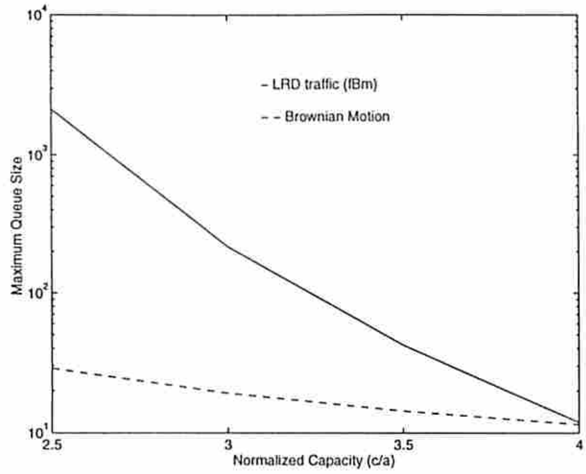

Figure 8: Maximum Queue Size for $\epsilon = 10^{-6}$.

3.5 Delay Bounds

The maximum number of cells in the system corresponds to the maximum delay that a cell can suffer in a *FIFO* queueing system. Therefore, the maximum delay in the system, d_{max}, is given by

$$d_{max} = \frac{q_{max}}{c}$$

We compare d_{max} with Cruz's delay bound [15]. Cruz's calculus assumes that the traffic behaves according to the envelope process $L(t) = Rt + S$. Based on this characterization Cruz's delay bound, d_b, is given by the maximum busy period in the system [15, 32]:

$$d_b = \frac{S}{c - R} \tag{11}$$

where c is the link capacity. We expect d_{max} to be a tighter delay bound than d_b because i) it assumes a more accurate characterization of the input traffic, ii) it computes the maximum queue size, not the maximum busy period. Moreover, it is a more flexible bound since we can choose a conservative or a optimistic approach based on ϵ.

We compare d_{max} to d_b for different values of link utilization, given by $\rho = \frac{\bar{a}}{c}$. First, we have to find R and S in order to use Cruz's calculus. For this example, we keep $\rho \leq 0.70$. In order to satisfy equations (5) and (11), we have $\bar{a} < R < c$ so that $R \in (0.70c, c)$. Therefore, we fix $R = 0.90c$ and compute S based on the framework derived in section 3.2. We substituted R, S and c in the equation (11) in order to compute d_b and compare it to d_{max}. We also used a simulation driven by the LAN trace in order to compute the maximum delay and compare it to the delay bound estimates. Figure 10 shows the result when $H = 0.85$ and $\epsilon = 10^{-3}$. The upper dotted curve corresponds to d_b, the middle solid curve corresponds to d_{max} and the lower curve is the maximum delay computed by the simulation. We can see that d_{max} is indeed a much tighter bound than d_b. In fact, since the busy period can be extremely large, d_{max} estimate can be 100 times smaller than d_b. Moreover, d_{max} is very close to the simulation results.

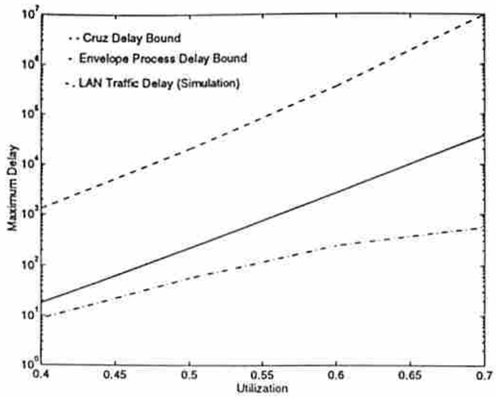

Figure 9: Delay bound estimate versus Cruz's delay estimate.

3.6 Statistical Multiplexing

Assume that we have N independent sources ${A_H}^i(t)$ defined by the following parameters: mean $\bar{a}_i$,standard deviation $\bar{\sigma}_i$ and Hurst parameter H_i, for $i \in [1, N]$. The aggregate traffic is given by $A_H(t) = \sum_{i=1}^{N} {A_H}^i(t)$. The envelope process of each source and of the aggregate traffic is given by $\hat{A}_H^i(t)$ and $\hat{A}_H(t)$, respectively. We can compute q_{max} by using two different approaches:

- Case 1: Define N independent envelope processes, compute t^* numerically, and substitute it into equation (9) in order to find q_{max}.
- Case 2: Define a unique envelope process from the aggregate traffic and compute q_{max}.

Case 1:

If we define N envelope processes and add them together, equation (8) is given by

$$\sum_{i=1}^{N} \bar{\sigma}_i H_i t^{H_i - 1} = c - \sum_{i=1}^{N} a_i \tag{12}$$

where c is the link capacity allocated for the aggregate traffic. Similar equation was proposed in [33]. We can solve it numerically in order to find t^* and substitute it in equation (9) in order to find q_{max}. The advantage of this approach is that we do not need to compute the Hurst parameter of the aggregate traffic.

Case 2:

Since the sources are independent, the aggregate traffic can be defined by the following parameters:

$$\bar{a} = \sum_{i=1}^{N} \bar{a}_i$$

$$\bar{\sigma} = \sqrt{\sum_{i=1}^{N} \bar{\sigma}_i^2}$$

$$H = max(H_i)$$

A conservative approach for choosing the Hurst parameter is to use the largest H, since it dominates the behavior of the aggregate traffic[7]. Nevertheless, this approach might be too conservative.

3.6.1 Homogeneous Sources: Multiplexing Gain

Assume that there are N homogeneous sources with parameters H, $\bar{a}$ and $\bar{\sigma}$. First, we consider each queueing system separately. The link capacity for each system is given by c. The envelope process is given by

$$\hat{A}_H^i(t) = \bar{a}t + k\bar{\sigma}t^H$$

Therefore $q^i{}_{max}$ is defined by the following equations:

$$t_i{}^* = [\frac{k\bar{\sigma}H}{(c-\bar{a})}]^{\frac{1}{1-H}}$$

$$q^i{}_{max} = \hat{A}^i_H(t_i{}^*) - ct_i{}^*$$

Next, we multiplex the sources together and compute q_{max}. Since the sources are independent, the aggregate traffic $A_H(t)$ is defined by $N\bar{a}$, $\sqrt{N}\bar{\sigma}$ and H. The Hurst parameter is preserved under multiplexing of identical sources [7]. We investigate q_{max} when the link capacity is Nc. In this case, we can write

$$\hat{A}_H(t) = N\bar{a}t + \sqrt{N}k\bar{\sigma}t^H$$

Therefore,

$$t^* = [\frac{\sqrt{N}k\bar{\sigma}H}{N(c-\bar{a})}]^{\frac{1}{1-H}} = N^{\frac{1}{2(H-1)}}t_i{}^*$$

Moreover,

$$q_{max} = N(\bar{a}-c)N^{\frac{1}{2(H-1)}}t_i{}^* + N^{\frac{H}{2(H-1)}}N^{\frac{1}{2}}k\bar{\sigma}(t_i{}^*)^H$$

$$q_{max} = N^{\frac{(H-1/2)}{(H-1)}}q_{max}{}^i$$

Therefore, there is a significant gain when we multiplex homogeneous sources. For example, if $H = 1/2$, $q_{max} = q_{max}{}^i$. This result is predicted by the diffusion equation and is the basis of the effective bandwidth approximation [11]. It is interesting to note that for LRD sources the multiplexing gain is even greater. For example, if $H = 0.90$, $q_{max} = \frac{q_{max}{}^i}{N^4}$. We are currently investigating the multiplexing effects when we aggregate heterogeneous sources. The preliminary results indicate that it is also possible to achieve significant multiplexing gains.

4 Conclusion

We investigated the long-range phenomenon and its impact on network management and queueing performance. We showed that the busy period of an ATM queueing system driven by LRD arrivals can be very long. We also derived a simple framework that can identify if cell losses are caused by SRD or LRD. Moreover, we proposed a more general and accurate characterization of the arrival process based on a fBm envelope process. Based on this characterization:

- We proposed a general calculus for choosing the Leaky Bucket's parameters. Furthermore, we explained why it is difficult to choose its parameters in order to police the source's average rate.
- We derived a new delay bound that is tighter than Cruz's bound. In fact, it agrees closely with simulation results driven by real LAN data.
- We showed that there is a significant multiplexing gain when we aggregate LRD sources.

In summary, our major contribution was i) to propose a more general and accurate characterization of the arrival traffic, and ii) to derive an accurate delay calculus that supports multiplexing, based on this representation. Moreover, this traffic characterization simplified the theoretical analysis of several ATM performance issues.

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Appendix A

Since A(t) is a Brownian motion process we can write

$$P(\frac{A(t)-\bar{a}t}{\sigma t^H} > k) = \overline{\Phi}(k)$$

where $\overline{\Phi}(y)$ is the residual distribution function of the standard Gaussian distribution. Using the approximation

$$\overline{\Phi}(y) \approx (2\Pi)^{-1/2}(1+y)^{-1}exp(-y^2/2) \approx exp(-y^2/2) \tag{13}$$

we find k such that

$$\overline{\Phi}(k) \leq \epsilon$$

Therefore, k is given by

$$k = \sqrt{-2\log\epsilon}$$

Appendix B

In order to validate our work, we compare q_{max} to the upper-bound of the unfinished work computed by an ordinary diffusion equation. Following the same approach described in [34], let W be the unfinished work in the system, m be the average and δ be the variance of the rate of work accumulating in the system, respectively. The input flow is given by process $A(t)$ with mean $\bar{a}$ and variance σ^2. The output flow is given by the deterministic process ct. Therefore, we can write

$$m = \bar{a} - c \tag{14}$$

$$\delta = \sigma^2 \tag{15}$$

The diffusion approach describes the unfinished work in a queueing system by a Fokker-Planck equation. At steady state, the solution of this equation is given by

$$P(W \geq w) = e^{\frac{2mw}{\delta}} \tag{16}$$

We define an upper bound for the unfinished work in the system at steady state, w^*, by

$$w^* = inf\{w > 0 : P(W > w) \leq \epsilon\} \tag{17}$$

Therefore, we can write

$$w^* = \frac{\delta\log\epsilon}{2m} \tag{18}$$

Substituting (14) and (15) into (18) we get

$$w^* = \frac{\sigma^2\log\epsilon}{2(\bar{a}-c)} \tag{19}$$

Therefore, when the output process is deterministic, the diffusion equation gives the same result of equation (10) !

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