Derivations of the Expected Energy Costs of Search and Replication in Wireless Sensor Networks

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ABSTRACT

We develop closed-form expressions of the expected minimum search energy cost and replication energy cost for both unstructured sensor networks (which use blind sequential search for querying) and structured sensor networks (which use efficient hash-based querying). We use both the square grid and random topology to derive each cost modeling. We find that the search cost of unstructured networks is proportional to the number of nodes N and inversely proportional to $(r + 1)$ (where r denotes the number of copies of the target event). The search cost of structured networks is proportional to $\sqrt{N}/\sqrt{r}$ while the replication cost of both structured and unstructured networks is proportional to $\sqrt{N}(r-1)$. Furthermore, the proportionality of those costs is independent of whether the topology is grid or random, which implies that the two topologies have common structural characteristics in terms of search and replication costs.

1. INTRODUCTION

There are two popular ways in which the wireless sensor network can be operated. One way is to be operated in a continuous data gathering mode, and the other way is to consider the wireless sensor network as a decentralized data storage system. We call the network of the latter mode the data-centric sensor network. While the network in the continuous data-gathering mode is popular, mainly because it is easier to analyze and simpler to implement, continuous data gathering from all sensors is generally very inefficient if most of the sensed information is not essential, or if there are multiple sinks that may need different subsets of the sensed information at different times. In this case, however, the data-centric network might be more suitable for the energy efficiency. In such a data-centric approach, the sensed data can be stored either locally or at one or more remote locations within the network. Event information is obtained by sinks through queries that are issued on an on-demand basis.

When a sink knows where the nearest copy of the target event information is stored (e.g. using hash-based data centric storage techniques such as GHT [5], DIM [6], etc.), the search cost is the energy expenditure to send a query for the event to the target node and bring the information back through the shortest path. On the other hand, when a sink has no clue where the target resides, it resorts to search for it blindly, which often is led to some sort of flooding. We call the network which adopts the former scheme the structured network and the latter scheme the unstructured network. While the structured network has smaller search energy cost at the cost of maintaining the location information of all events, the unstructured network doesn't need the overhead of maintenance.

In this work, we focus on deriving closed-form expressions for the expected minimum search energy cost and replication cost considering structured and unstructured sensor networks with grid and random deployments. Since the energy is one of the most precious resources and searching is one of fundamental operations in the data-centric sensor network, it can be very useful to derive the closed-form expressions of such costs in the sense that they can be basis of other analytical modelings with tractable optimization. There are several related works as follows. For the structured network, the minimum search cost is related with the shortest path between a sink and a source. For the unstructured network, we use the expanding ring search mechanism in which the sink sends a series of controlled flooding until it finds the event information. Chang and Liu [4] have found the way how to construct the series of controlled flooding in order to minimize the expected search cost given the distribution of the event's location. And Krishnamachari and Ahn [2] have found the approximate closed-form expression for the minimum search cost under the uniform random node deployment and the uniform distribution of the location of target event.

2. ASSUMPTIONS

The following are the key assumptions in our work:

- N nodes are deployed with constant density in a two-dimensional square area. The constant density implies that if the network size is increased, the deployment area grows proportionally.
- The radio radius of a node is R for all nodes.
- There are m atomic events that are sensed in the environment. The distribution of events is assumed to be uniform in the deployment area.
- A total of r_i copies of each event are maintained with a uniform distribution in the network by creating $r_i - 1$ additional replicas when the event is first sensed.
- For each event i, there are a total of q_i queries that are generated uniformly by the nodes in the network. Each query is a one-shot query (i.e. requires a single response, not a continuous stream), and is satisfied by locating a single copy of the corresponding event.
- We assume that the links over which transmissions take place are lossless (e.g., using blacklisting) and present no interference due to concurrent transmissions (e.g., due to low traffic conditions or due to the use of a scheduled MAC protocol).

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- The total energy cost for storage and querying is assumed to be proportional to the total number of transmissions. Particularly, the unit successful transmission cost is assumed one since it turns out to play a role only on scaling. And this proportionality assumption is reasonable particularly for sleep-cycled sensor networks where radio idle times are kept to a minimum.

3. MODELING SEARCH COSTS

3.1 Under the Unstructured Network of Grid Topology

We derive this search cost expression using the trajectory-based query. We consider a two dimensional square grid topology of N nodes. The number of node in a row or a column is $\sqrt{N}$. Each node can communicate with the direct neighbors: the nodes located on the up, down, left, or right direction. We assume that a querier issues a query which follows a path covering the whole network. Note that by Lemma 1 we can find a path or linear topology corresponding to the given grid topology which covers all nodes without visiting any node more than once if $\sqrt{N}$ is even. If it is odd, then we need only one node which is visited twice by Lemma 2. We therefore ignore the latter situation. Consider that linear topology. And let X denote the hop count to the nearest copy of the desired event. The probability that the nearest one among r copies is located no more than k hops away from the querier is then given by the expression:

$$P\{X \leq k\} = 1 - \left(1 - \frac{k}{N}\right)^r$$

Its probability mass function is simply as follows;

$$P\{X = k\} = \left(1 - \frac{k-1}{N}\right)^r - \left(1 - \frac{k}{N}\right)^r$$

Since each successful transmission is assume to incur a unit cost one, the expected search cost of unstructured system with the grid topology, denoted by $C_{s,ug}$, is given by,

$$\begin{aligned} C_{s,ug} &= \sum_{k=0}^{N-1} k\, P\{X = k\} \\ &= \sum_{k=1}^{N-1} k \left\{ \left(1 - \frac{k-1}{N}\right)^r - \left(1 - \frac{k}{N}\right)^r \right\} \\ &= \sum_{i=2}^{N} \left(\frac{i}{N}\right)^r - (N-1)\left(\frac{1}{N}\right)^r \\ & \qquad (\because \text{ using the telescoping method}) \\ &= \frac{1}{N^r}\left\{\sum_{i=2}^{N} i^r - (N-1)\right\} \end{aligned}$$

Since the following inequalities holds,

$$\sum_{i=2}^{N} i^r \geq \int_1^N i^r di = \frac{N^{r+1} - 1}{r+1}$$

$$\sum_{i=2}^{N} i^r \leq \int_2^{N+1} i^r di = \frac{(N+1)^{r+1} - 2^{r+1}}{r+1}$$

the bounds of $C_{s,ug}$ are given by,

$$C_{s,ug} \leq \frac{1}{N^r}\left(\frac{(N+1)^{r+1} - 2^{r+1}}{r+1} - N + 1\right)$$

$$C_{s,ug} \geq \frac{1}{N^r}\left(\frac{N^{r+1} + r}{r+1} - N\right)$$

This lower bound provides a good approximation (especially when r is not large) and a lower bound is more meaningful in this work, we can approximate the search cost as follows:

$$\begin{aligned} C_{s,ug} &= \frac{1}{N^r}\left(\frac{N^{r+1} + r}{r+1} - N\right) \\ &\approx \frac{N}{r+1} \end{aligned}$$

Note that we ignore the cost of way back here since it can be shown to be proportional to $\sqrt{N/r}$ from section 3.3, which is ignorably small compared to the cost to locate a copy.

LEMMA 1. *Given a square grid graph G where the number of node in a row or column is n which is even, there is always a path starting from any node s in G such that the path visits all node in G exactly once.*

Proof: Note that the path is a Hamiltonian path starting from $s \in G$ to any other node $t \in G, t \neq s$. Let $B = (V^0 \cup V^1, E)$ be a bipartite graph with $|V^0| \geq |V^1|$. We will think of the vertices of B as colored by two colors, black and white. All the vertices of V^0 will be colored by one color, the *majority color*, and the vertices V^1 by the *minority color*.

The Hamiltonian path problem (B, s, t) is *color compatible* if

1. B is *even* ($|V^0| = |V^1|$) and s and t have different color *or*
2. B is *odd* ($|V^0| = |V^1| + 1$) and s and t are colored by the majority color (i.e., $s, t \in V^0$).

The Hamiltonian path problem (K, s, t) is *forbidden* if it satisfies one of the conditions as follows:

1. K is a 1-rectangle, and either s or t is not a corner.
2. K is a 2-rectangle, and (s, t) is a nonboundary edge (i.e., (s, t) is an edge, and it is not on the outermost face).
3. (K, s, t) is isomorphic to (K', s', t') which satisfies:
   (a) K' is a n-by-m grid graph with $n = 3$ and m even.
   (b) s' is colored differently from t' and the left corners of K'.
   (c) $s'_x < t'_x - 1$ *or* $S'_y = 2$ and $s'_x < t'_x$.

We say that a Hamiltonian path problem (K, s, t) is *acceptable* if it is color compatible and not forbidden. Then, it has been proven by Itai et al. ([7]) that there exists a Hamiltonian path from s to t in K if and only if (K, s, t) is acceptable. Since G is a square grid graph with even number of rows, it is not forbidden. Moreover G is even and two of four corner nodes are white while the other two corners are black. Hence, if s is black (or white), we set $t \neq s$ as one of the corners that are white (or black). Then, (G, s, t) is color compatible. Therefore, there exists such a Hamiltonian path.

□

LEMMA 2. *Given a square grid graph G where the number of node in a row or column is n which is odd, there is always a path starting from any node s in G such that the path visits one node either once or twice but the other nodes exactly once.*

Proof: Since G is a square grid graph with odd number of rows, it is not forbidden and is odd (i.e. $|V^0| = |V^1| + 1$). Now, we set $t \neq s$ as one of the four corners each of which has the majority color. If s has the majority color, then (G, s, t) is color compatible and so, there exists a Hamiltonian path for (G, s, t) by [7].

If s has the minority color, its neighbors have the majority color. Denoting one of it neighbors as $s' \neq t$, there exists a Hamiltonian path for (G, s', t) because it is color compatible, and so the path $\{s, H(G, s', t)\}$ visits all nodes $x \in G, x \neq s$ exactly once and s twice.

□

3.2 Under the Unstructured Network of Random Topology

We derive the search cost expression using the optimal expanding ring-based flooding query in our previous work [2]. We consider a square area with nodes deployed with a uniform random distribution with the node density ρ. Each node can communicate with any other node that is placed within a radio range R, and it is assumed that the network is sufficiently dense so that all nodes within distance kR of the querier can be reached in k hops. The width of the area is W. When modeling this search cost, we assume that the querier is located in the center of the region. From our previous work [2], the expected search cost is given by,

$$C_{s,ua} = c_1 \cdot \alpha \frac{aL^2}{n+2} \tag{1}$$

where $c_1 \approx 2.15$, α is the cost of unit successful transmission, $a = \pi R^2 \rho$ (which is the number of one-hop neighbors), n is the number of replicas excluding the original copy, and L is the smallest hop distance from the querier to cover all nodes.

Note that the above equation (1) works well for the square area topology as our previous work [2] suggests in its simulation section even though it assumes the circular area topology. Because $\alpha = 1$, $n + 1 = r$ and $aL^2 = N$ in context of this work, we can get

$$C_{s,ua} = c_1 \frac{N}{r+1} \tag{2}$$

Note that the previous work [2] also suggests in its simulation section that this model also works well even in the case that any node can be a querier equally likely.

3.3 Under the Structured Network of Grid Topology

In the structured network, the search cost is related to a path of the lowest cost from a querier to the nearest node which has one of the copies. We assume the shortest path routing scheme, so that the path would be their shortest path. We assume that all r copies of an event are evenly deployed in the network so that there are $\sqrt{r}$ rows, each of which has $\sqrt{r}$ copies of the event; and between the two consecutive columns or rows, there are $(\sqrt{N}/\sqrt{r} - 2)$ nodes which do not have a copy of the event. Furthermore, the querier can be any node with same probability in this network. Other assumptions are same as those of the unstructured network.

Note that a querier is in the square $\mathfrak{D}$ in which there are four copies of the event which locate at the four corners of $\mathfrak{D}$. Note that the width of $\mathfrak{D}$ is $\sqrt{N}/\sqrt{r}$ hop distance, hence the expected hop distance from the querier to the nearest of these four copies becomes the expected search cost. Consider the square $\mathfrak{D}_1$ that is one of four equal-sized small squares after dividing the square $\mathfrak{D}$ by four; each side of $\mathfrak{D}_1$ has $\lceil \frac{\sqrt{N}}{2\sqrt{r}} \rceil$ nodes. If a node in the square $\mathfrak{D}_1$ sends a query for the event, the appropriate corner node has its nearest copy. Suppose the the corner node is located at $(1,1)$ and a querier in $\mathfrak{D}_1$ at (x,y), $1 \leq x \leq m, 1 \leq y \leq m, (x,y) \neq (1,1)$ without loss of generality, where $m = \frac{\sqrt{N}}{2\sqrt{r}}$. Since the hop distance $H_{(x,y)}$ between the two nodes is $(x-1+y-1)$ and every node except the corner node can be the querier equally likely, the expected hop distance is given as follows:

$$\begin{aligned} E[X] &= \frac{\sum_{x=1}^{m}\sum_{y=1}^{m} H_{(x,y)}}{m^2-1} = \frac{m^2(m-1)}{m^2-1} \\ &= \frac{\frac{N}{4r}\left(\frac{\sqrt{N}}{2\sqrt{r}} - 1\right)}{\frac{N}{4r} - 1} \\ &\approx \frac{\sqrt{N}}{2\sqrt{r}} \end{aligned}$$

Therefore, the search cost which consists of the locating cost and the feedback cost (note that both costs are same) is as follow;

$$C_{s,sg} = 2E[X] = \frac{\sqrt{N}}{\sqrt{r}}$$

3.4 Under the Structured Network of Random Topology

Note that the expected hop distance from a querier to the nearest copy of an event is given in the section II.D. of our previous work [2] as follows:

$$E[X] = \frac{\sqrt{N}}{2R\sqrt{\rho}} \cdot \frac{r \cdot \Gamma(r)}{\Gamma(r+3/2)}$$

Since both lower bound and upper bound of $\frac{r \cdot \Gamma(r)}{\Gamma(r+3/2)}$ is proven to be proportional to $1/\sqrt{r}$ by Lemma 3, we can approximate $E[X]$ to be proportional to $1/\sqrt{r}$ in terms of r with good accuracy as follow:

$$E[X] \approx \frac{c_2}{2R\sqrt{\rho}} \cdot \frac{\sqrt{N}}{\sqrt{r}}$$

where $0.66 < c_2 < 1.71$ (we can get the more accurate value of c_2 using the curve fitting.)

Therefore, the search cost composed of the locating cost and the feedback cost is given by,

$$\begin{aligned} C_{s,sa} &= 2E[X] \\ &= \frac{c_2}{R\sqrt{\rho}} \cdot \frac{\sqrt{N}}{\sqrt{r}} \end{aligned}$$

LEMMA 3. *For $r \geq 1$, the following double inequality holds:*

$$0.4\sqrt{e}\,\frac{1}{\sqrt{r}} < \frac{r \cdot \Gamma(r)}{\Gamma(r+3/2)} < e^{0.531}\frac{1}{\sqrt{r}}$$

Proof: From Robbins 1955 [1], Stirling's approximation can be extended to the following double inequality:

$$\sqrt{2\pi}r^{r+\frac{1}{2}}e^{-r+\frac{1}{12r+1}} < \Gamma(r+1) < \sqrt{2\pi}r^{r+\frac{1}{2}}$$

Using this inequality,

$$\begin{aligned} r \cdot \Gamma(r) &= \Gamma(r+1) \\ &> \sqrt{2\pi}r^{r+\frac{1}{2}}e^{-r+\frac{1}{12r+1}} \\ \Gamma(r+\frac{3}{2}) &< \sqrt{2\pi}(r+\frac{1}{2})^{r+1}e^{-r-\frac{1}{2}+\frac{1}{12r+6}} \end{aligned}$$

Thus,

$$\begin{aligned} \frac{r \cdot \Gamma(r)}{\Gamma(r+\frac{3}{2})} &> r^{-\frac{1}{2}}\left(1 - \frac{1}{2r+1}\right)^{r+1} e^{\frac{1}{2}+\frac{5}{(12r+1)(12r+6)}} \\ &> 0.4\sqrt{e}\,\frac{1}{\sqrt{r}} \end{aligned} \tag{3}$$

1. The expected search energy cost

i) Grid topology

(a) The unstructured network

$$C_{s,ug} = \frac{N}{r+1}$$

(b) The structured network

$$C_{s,sg} = \frac{\sqrt{N}}{\sqrt{r}}$$

ii) Random Topology

(a) The unstructured network

$$C_{s,ua} = c_1 \frac{N}{r+1}$$

(b) The structured network

$$C_{s,sa} = \frac{c_2}{R\sqrt{\rho}} \cdot \frac{\sqrt{N}}{\sqrt{r}}$$

2. The Expected Replication Energy Cost

i) Grid Topology

$$C_{r,g} = \frac{2}{3}\sqrt{N} \cdot (r-1)$$

ii) Random Topology

$$C_{r,a} = \frac{0.521405}{R\sqrt{\rho}}\sqrt{N}(r-1)$$

Table 1: Summary of expected energy costs

The inequality of the equation (3) holds because of the following facts. Since $\left(1 - \frac{1}{2r+1}\right)^{r+1}$ is an increasing function and its value is $4/9$ when $r = 1$, this term is larger than 0.4 for all $r \geq 1$. In addition, $e^{\frac{1}{2}+\frac{5}{(12r+1)(12r+6)}}$ is greater than $e^{\frac{1}{2}}$ since r is positive.

In the other hand, using the above double inequality of Robbins in the other way,

$$\begin{aligned} r \cdot \Gamma(r) &< \sqrt{2\pi} r^{r+\frac{1}{2}} e^{-r+\frac{1}{12r}} \\ \Gamma(r+3/2) &> \sqrt{2\pi}(r+\frac{1}{2})^{r+1} e^{-r-\frac{1}{2}+\frac{1}{12r+7}} \end{aligned}$$

Hence,

$$\begin{aligned} \frac{r \cdot \Gamma(r)}{\Gamma(r+3/2)} &< r^{-\frac{1}{2}} \left(\frac{2r}{2r+1}\right)^{r+1} e^{\frac{1}{2}+\frac{7}{12r(12r+7)}} \\ &< e^{0.531} \frac{1}{\sqrt{r}} \end{aligned} \quad (4)$$

Since $r \geq 1$, $\left(\frac{2r}{2r+1}\right)^{r+1} < 1$, and $e^{\frac{1}{2}+\frac{7}{12r(12r+7)}}$ is an decreasing function with respect to $r > 0$, the inequality (4) holds.

□

4. THE REPLICATION COST

4.1 Grid Topology

The assumptions are same as those in the search cost. Note that the expected replication cost does not depend on the querying schemes, but on the topologies. Suppose the leftmost and bottommost node is at (1,1) on the XY plane, and a node which locates in the i^{th} column from left and j^{th} row from bottom is at (i, j) on the XY plane. Suppose the original event is generated at $\overline{p} = (p_x, p_y)$ where $1 \leq p_x \leq n(= \sqrt{N}), 1 \leq p_y \leq n$. The hop distance from $\overline{p}$ to an arbitrary node at (x, y) is given by $(|x - p_x| + |y - p_y|)$. Now let's divide the whole square network into four rectangles so that a internal boundary is a line parallel to y-axis and at $p_x - 1 < x < p_x$, and the other internal boundary is a line parallel to x-axis and at $p_y - 1 < y < p_y$. Let $R_{11}(\overline{p})$ denote the sum of hop distances from $\overline{p}$ to each node in the left bottom rectangle, $R_{12}(\overline{p})$ the right bottom rectangle, $R_{21}(\overline{p})$ the left top rectangle, and $R_{22}(\overline{p})$ the right top rectangle. It is easy to calculate those quantities. For example, $R_{12}(\overline{p}) = \sum_{x=p_x}^{n} \sum_{y=1}^{p_y-1} (x - p_x + p_y - y)$

Using those quantities, we can simply get the average distance from the event source $\overline{p}$ to a node in the network as,

$$E[H_{r,g}|\overline{p} = (p_x, p_y)] = \frac{1}{N-1} \sum_{i,j} R_{ij}(\overline{p})$$

where $H_{r,g}$ denotes the hop distance between two nodes in the network.

Therefore, the average hop distance between an event source to a node is given by,

$$\begin{aligned} E[H_{r,g}] &= \frac{1}{N} \sum_{p_x=1}^{\sqrt{N}} \sum_{p_y=1}^{\sqrt{N}} E[H_{r,g}|\overline{p} = (p_x, p_y)] \\ &= \frac{2}{3} \frac{\sqrt{N}(N+2)}{N-1} \\ &\approx \frac{2}{3}\left(\sqrt{N} + \frac{2}{\sqrt{N}}\right) \\ &\approx \frac{2}{3}\sqrt{N} \end{aligned}$$

Since there are $(r-1)$ replicas in the network, the expected replication cost is given by,

$$C_{r,g} = E[H_{r,g}] \cdot (r-1) \; = \; \frac{2}{3}\sqrt{N} \cdot (r-1)$$

4.2 Random Topology

The assumptions for the replication cost of the random topology are also same as that of the search cost of the random topology. Let $H_{r,a}$ and $D_{r,a}$ denote the number of hops needed to make one replica of an event and the corresponding Euclidean distance, respectively. Then, the expected replication cost is as follows;

$$C_{r,a} = E[H_{r,a}] \cdot (r-1) = \frac{E[D_{r,a}]}{R} \cdot (r-1) \quad (5)$$

where R is the radio radius of a node.

But, $E[D_{r,a}]$ is the average of Euclidean distances over all possible pairs of points in the square area, which is known to be as follows;

$$
\begin{aligned}
& E[D_{r,a}] \\
&= W \underbrace{\int \cdots \int_0^1}_{4} \sqrt{(x_1-y_1)^2+(x_2-y_2)^2}\, dx_1 dy_1 dx_2 dy_2 \\
&= W \frac{2+\sqrt{2}+5\ln\left(1+\sqrt{2}\right)}{15} \\
&\approx 0.521405\, W
\end{aligned}
\tag{6}
$$

Substituting the equation (6) into the equation (5), we can get the expected replication cost as,

$$
\begin{aligned}
C_{r,a} &= \frac{0.521405W}{R}\,(r-1) \\
&= \frac{0.521405\sqrt{N}}{R\sqrt{\rho}}\,(r-1) \quad \left(\because\ \rho = \frac{N}{W^2}\right) \\
&= c_3\sqrt{N}(r-1)
\end{aligned}
$$

where

$$
c_3 = \frac{0.521405}{R\sqrt{\rho}}
$$

5. CONCLUSIONS

We have derived minimum expected search costs and replication costs in the several kinds of data-centric wireless sensor networks. In Particular, we have considered structured and unstructured networks under grid and random topologies. The final expressions are summarized in Table 1. One important thing to note is that the difference of each costs between the grid topology and the random topology is only the value of coefficient. This fact implies that the grid and random topologies have common structural characteristics in terms of search and replication costs.

As an extended work of this paper, we have developed fundamental scaling laws for energy-efficient storage and querying in wireless sensor networks [3]. In the paper, we have found that the scalability of a sensor network's performance depends on whether or not the increase in energy and storage resources with more nodes is outweighed by the concomitant application-specific increase in event and query loads.

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