Analysis of Slotted Multi-Access Techniques for Wireless Sensor Networks

Kiran Yedavalli and Bhaskar Krishnamachari
Department of Electrical Engineering - Systems
University of Southern California, Los Angeles, California, 90089
Email: {kyedaval, bkrishna}@usc.edu

***Abstract*— In this paper, we study the *single-packet medium access* problem, which occurs in many wireless sensor network applications. In this problem, each contending node in the one-hop network has a single packet to transmit and the node is in contention of the channel only until its packet is transmitted. We analyze the performance of slotted multi-access techniques for this problem using Markov chains and flow equations. We verify the accuracy of these analyses through simulations. We also present a thorough performance evaluation of these techniques in terms of delay and energy consumption for various design parameters.**

I. Introduction

Wireless sensor networks (WSN) are increasingly being used in a variety of critical applications. The diversity of these applications introduce equally diverse set of conditions for wireless medium access. In this paper, we consider an important set of WSN applications, in which the medium access has a common structure and differs significantly from the conditions in which traditional medium access problems are solved. Three such applications are:

- *Location Support for Mobile Users*: In this application, the mobile unknown node (sink) determines its location using measured physical quantities and relevant data from packets transmitted by several reference nodes (the sensor nodes) in its radio range. Each reference node has a single packet, that contains its coordinates, to transmit to the unknown node.
- *Node Discovery*: Most applications of sensor networks use sleep scheduling in which sensor nodes are mostly in "sleep mode" and occasionally "wake up" to sense the environment or for other duties. As a result, every time a node wakes up, it could potentially have a different set of neighbors. In applications that require a node to be aware of its neighbors' identities every time it wakes up, the node could broadcast a discovery message and all nodes in its radio range respond with their identities. Here also, each contending node has a single packet, that contains its ID, to send to the discoverer (sink).
- *Data Gathering*: Another important application in which the medium access problem is very similar to that of the above two applications is that of one-hop data-gathering. For example, (i) large number ground sensors spread over wide expanses of land, such as forests and farms, need to transmit their data to a data-gathering aircraft that flies over them as quickly as possible. (ii) A mobile user who gathers inventory data from sensor nodes deployed on shelves of ware-houses, requires that the sensor nodes transmit their single packet of data as quickly as possible.

The underlying medium access concerns that are common in the above set of applications are:

- All contending nodes have a single packet to transmit and that they are out of contention once it is transmitted.
- All communication is one-hop.
- The delay for all nodes can transmit their packets is of critical importance to the application.
- The energy consumed in this medium access operation affects the life-time of the WSN.

We call the medium access problem with the above set of concerns as the *single-packet medium access* problem. Clearly, this problem differs significantly from that of the traditional medium access problems. The analyses of wireless medium access protocols (see [1], [2], [3]) are usually characterized by the assumptions that each node always has many backlogged packets to send and that all nodes in the network reach steady state. These assumptions no longer hold for the problem we address in this paper.

Traditionally, solutions for medium access problems have been categorized into centralized TDMA schemes, Slotted Multi-access schemes, Splitting Algorithms and Carrier Sensing techniques (see [4]). In a previous work [5], we have investigated the performance of centralized TDMA schemes, for the single-packet medium access problem in the context of location support for mobile users. In this paper, we present the performance analysis for slotted multi-access schemes. Performance analysis of splitting algorithms and asynchronous carrier sensing techniques will be part of our future work.

We consider the following slotted multi-access schemes: p-persistent slotted Aloha, slotted Aloha with constant back-off window, slotted Aloha with binary exponential back-off, and slotted CSMA with constant back-off window. To the best of our knowledge ours is the first attempt at analyzing these protocols for the single-packet medium access problem. Tay *et.al.* in [6] present a non-persistent CSMA protocol in which the transmission probabilities are optimized to minimize collisions. However, the problem considered by the authors is considerably different from that of ours as they assume that the contending nodes have many backlogged packets to send and consequently the network is in steady state. Also, the authors concentrate on minimizing the delay for this first packet successfully transmitted.

The rest of the paper is organized as shown in Table I.

Section II	Problem Description
Section III	p-persistent Slotted Aloha
Section IV	Slotted Aloha with Constant Back-off
Section V	Slotted Aloha with Binary Exponential Back-off
Section VI	Slotted CSMA with Constant Back-off
Section VII	Performance Evaluation
Section VIII	Conclusion & Future Work

TABLE I
PAPER ORGANIZATION

II. PROBLEM DESCRIPTION

In the section we list the assumptions we make and define the metrics that are used to measure the performance of the protocols.

Assumptions:

- The number of nodes in the radio range of the sink is N.
- Each node has a *single* packet to send to the sink.
- Time is split into time slots of equal length.
- All nodes in the one-hop network are synchronized.
- Nodes contend for the channel at the beginning of a time slot.
- When more than one node transmits in the same time slot, collision occurs. Nodes involved in a collision that leads to packet delivery failure, detect the failure through acknowledgements from the sink within the same time slot. We assume that the effect of acknowledgements on protocol performance is negligible.
- The transmission power of the nodes is such that all packets successfully transmitted by them reach the sink without any errors.

Performance Metrics:

- *Expected delay for the delivery of the first k packets* ($D(k)$): The number of time slots required for the first $k,\ \ 1 \leq k \leq N$ packets are successfully delivered to the sink.
- *Expected number of transmissions per node for the deliver of the first k packets* ($E(k)$): The energy consumed by each node for it to successfully transmit its packet to the sink is measured in terms of the number of packet transmissions it has to make for the same.

III. P-PERSISTENT SLOTTED ALOHA

In p-persistent Slotted Aloha, each contending node (the sink is not a contending node) attempts to transmit its packet with probability p in each time slot, until it is successfully transmitted. In this case we assume that the packet length is equal to the length of the time slot.

The network can be modeled by a Markov chain as shown in Figure 1. Each state in the chain represents the number of nodes in the network with the passage of time. The network changes from state i to state $i-1$ when there is a successful packet transmission from state i; 0 is the absorbing state.

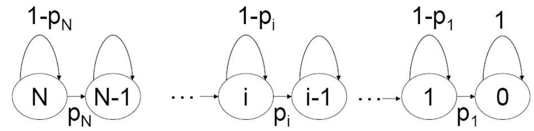

Fig. 1. Markov chain showing the states for p-persistent Slotted Aloha.

Given the transmission probability of p, the probability of making the transition from state i to state $i-1$ in the present time slot, p_i, which is same as the probability of successful transmission from state i, is equal to the probability of a single node transmitting from state i. Thus,

$$p_{suc,i} = ip(1-p)^{i-1}, \quad p_0 = 0 \tag{1}$$

The number of time slots required to transition from state i to state $i-1$ is a geometric random variable with probability of success of $p_{suc,i}$. Therefore, the expected number of time slots the network spends in state i is $\frac{1}{p_{suc,i}}$. Thus, the expected delay for the first k packets are successfully delivered to the sink is given by,

$$D(k) = \sum_{i=N}^{N-k+1} \frac{1}{p_{suc,i}} = \sum_{i=N}^{N-k+1} \frac{1}{ip(1-p)^{i-1}} \tag{2}$$

Similarly, when the network is in state i, the expected number of transmissions in a time slot is equal to ip and therefore, the expected number of transmissions for there is a successful transmission from that state is $ip \cdot \frac{1}{p_{suc,i}}$. Thus, the expected number of transmissions per node for the first k packet deliveries is given by:

$$E(k) = \frac{1}{N} \sum_{i=N}^{N-k+1} ip \cdot \frac{1}{ip(1-p)^{i-1}} = \frac{1}{N} \sum_{i=N}^{N-k+1} \frac{1}{(1-p)^{i-1}} = \frac{1-p}{Np} \left(\frac{1}{(1-p)^N} - \frac{1}{(1-p)^{N-k}} \right) \tag{3}$$

The value of k is determined by the type of application and the value of N depends on the sensor node density and the radio range of the sink. Thus both k and N are not independent variables. However, p is an independent variable that can be changed to obtain desired values of $D(k)$ and $E(k)$.

The optimal expected delay for p-persistent slotted Aloha can be obtained by minimizing $D(k)$ with respect to p. Differentiating the denominator[1] of each term in Equation 2 with respect to p and setting it equal to zero yields $p_{opt} = \frac{1}{i}$. This value of p_{opt} is clearly expected because, the delay is minimized when a single node transmits in each time slot, and this can be achieved on average if each node in network transmits with a probability which is the inverse of the number of nodes left in the network.

[1]The value of p that maximizes the denominator of each term in the sum, minimizes the sum, since all terms in the sum are positive.

Substituting p_{opt} into Equation 2,

$$D_{opt}(k) = \sum_{i=N}^{N-k+1} \left(1 + \frac{1}{i-1}\right)^{i-1} \leq k \cdot e \quad for \quad N >> 1 \tag{4}$$

Similarly, substituting $p_{opt} = \frac{1}{i}$ into Equation 3 yields $E(k) \leq e \cdot \frac{k}{N}$.

However, it is not clear how the value of i can be determined at every time slot. One possible mechanism could be that of the sink informing the sensor nodes of the number of successfully transmitted packets and the value of k through acknowledgement packets. Also, if the sink has estimates of the sensor node density and its own radio range, it can determine the initial expected value of N and using this estimate can determine the optimal value of p at which the nodes should transmit their packets after each successful transmission.

IV. Slotted Aloha with Constant Back-Off

In slotted Aloha with constant back-off, each contending node starts out by choosing a time slot uniformly at random within a constant contention window of size W. In the event that more than one nodes chooses the same time slot, a collision occurs and all nodes involved in the collision back-off, again choosing a future time slot uniformly at random from the contention window of size W. As in the previous Section, we assume that the packet length is equal to the time slot length. In addition, we assume that the back-off time counter length for each node is equal to that of the time slot.

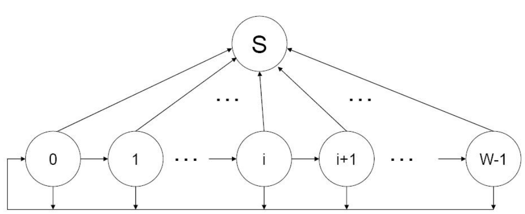

Fig. 2. Markov chain of states for a contending node using slotted Aloha with constant back-off protocol.

Figure 2 shows the Markov chain of possible states for a contending node in slotted Aloha with constant back-off protocol. A node is in state $i, \quad 0 \leq i \leq W-1$, if it had backed-off i time slots ago. The node transitions to state S, the absorbing state, if it has successfully transmitted its packet. If a node backs-off when it is in any of the W states it starts over from state 0. As stated previously, the network will not attain steady state as the number of contending nodes and the resultant probabilities of collisions change with time. In order to capture the dynamic nature of the network, we determine the rates at the network transitions between different states.

Owing to the uniform random selection of a time slot from the window of size W, the probability with which a node attempts to transmit in a given time slot, given that it hasn't transmitted until that time slot since backing-off, depends on the state i it is in the above Markov chain at the beginning of that time slot. If a node is in state i in the present time slot, the probability with which it will attempt to transmit is given by

$$p_i = \frac{1}{W-i} \tag{5}$$

Accordingly, a node will attempt to transmit with probability one, when it reaches state $W-1$. Let $n_i(t), \ i \in [0, W-1]$ be the number of nodes at time t in state i. All nodes in the network start from state 0 at time slot 0, therefore, $n_0(0) = N$.

The average number of nodes in state $i(>0)$ at time $t+1$ is equal to the sum of the average number of nodes in state i at time t and the average number of nodes in state $i-1$ that do not attempt to transmit at time t, minus, the sum of the average number of nodes in state i that transmit successfully and the average number of nodes in state i that back-off at time t. This arithmetic reduces to,

$$n_i(t+1) = n_{i-1}(t)(1-p_{i-1}), \quad i > 0 \tag{6}$$

The average number of nodes in state 0 at time $t+1$ is equal to the sum of the average number of nodes in state 0 at time t and the average number of nodes in all states other than S at time t that attempt to transmit and back-off, minus, the sum of the average number of nodes in state i that transmit successfully and the average number of nodes in state i that back-off at time t. This results in,

$$n_0(t+1) = \sum_{q=0}^{W-1} n_q(t) p_q [1 - \pi_q(t)] \tag{7}$$

where,

$$\pi_q(t) = (1-p_q)^{(n_q(t)-1)} \prod_{l=0 (l \neq q)}^{W-1} (1-p_q)^{n_q(t)} \tag{8}$$

The average number of nodes in state S at time $t+1$ is equal to the sum of the average number of nodes at time t and the average number of nodes that successfully transmit from all states $i, \quad 0 \leq i \leq W-1$, at time t. Therefore,

$$n_s(t+1) = n_s(t) + \cdot \sum_{q=0}^{W-1} n_q(t) p_q \pi_q(t) \tag{9}$$

The expected delay $D(k)$ is the number of time slots required such that $n_s(t) = k$, *i.e.*, $n_s(D(k)) = k$. The total expected number of transmissions in each time slot is equal to the sum of the expected number of transmissions from each state i in that time slot. Therefore, the expected number of transmissions per node ($E(k)$) is given by

$$E(k) = \frac{1}{N} \sum_{t=0}^{D(k)} \sum_{i=0}^{W-1} n_i(t) p_i \tag{10}$$

Unlike in the p-persistent slotted Aloha case, the above equations are not tractable enough to be expressed in closed form. However, the performance of the protocol can be studied by numerically evaluating the above expressions for different values of W, k and N.

V. Slotted Aloha with Binary Exponential Back-Off

In this protocol, each node starts out with a minimum congestion window of size W_0 and each time it backs-off, it doubles the size of its congestion window up to a maximum value of W_{MAX}, leading to a binary exponential increase in its window size. At each stage of the increase, the node chooses a time slot to transmit uniformly at random within the current window size. The congestion window size of the node remains constant once the maximum window size is reached. If the number of stages of increase before the maximum window size is reached is M, then the window size at stage i, ($0 \leq i \leq M-1$) is $W_i = 2^i W_0$, and $W_{MAX} = W_{M-1}$.

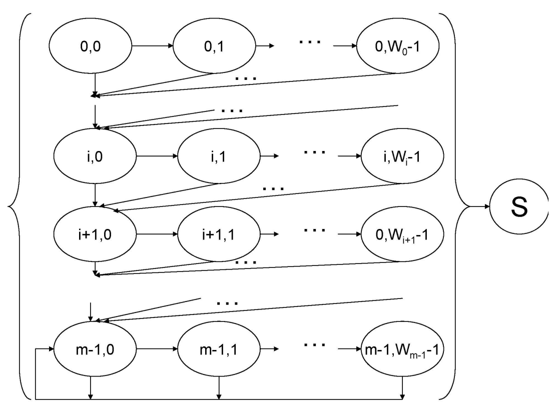

Fig. 3. Markov chain of states for a contending node using slotted Aloha with binary exponential back-off protocol.

Figure 3 shows the Markov chain[2] of states a node using this protocol goes through before it successfully delivers its packet to the sink through state S. The state (i,j) implies that the node has entered the stage i after backing-off from stage $(i-1)$, j back-off time counter slots ago. Similar to the protocol in the previous Section, we assume that the back-off time counter length for each node in the network is equal to the time slot length, where both are in turn equal to the packet length.

Let $n_{i,j}(t),\ (i \in [0, M-1],\ j \in [0, W_i - 1])$ be the number of nodes at stage i at time t that have entered this stage j time slots ago. The probability that a node in state (i,j) attempts to transmit its packet, given that it is in this state is given by,

$$p_{i,j} = \frac{1}{W_i - j} \quad (11)$$

[2]This Markov chain is equivalent to the Markov chain used in [1] for the performance analysis of 802.11. In the latter, it is assumed that the back-off counter of nodes decrements with each time slot and the node transmits when the counter reaches zero. Where as, in our chain, we assume that the back-off counter increments with each time slot and the node transmits its packet when the randomly chosen time slot is reached.

All nodes in the network start at time slot 0 at state $(0,0)$, thus, $n_{0,0}(0) = N$. However, $\forall\ t > 0$, $n_{0,0}(t) = 0$, as all nodes move to a different state in the next time slot and do not return to state $(0,0)$.

Whenever a node attempts to transmit in a time slot, either it backs-off due to collisions or packet errors, or successfully delivers its packet to the sink. Therefore, the number of nodes that enter state $(i,j),\ j \neq 0$, at $t+1$ is equal to the average number of nodes in state $(i, j-1)$ that do not attempt to transmit at t. And all nodes in state $(i,j),\ j \neq 0$, at time t leave that state either through successful transmission or back-off or by just moving to state $(i, j+1)$ at time $t+1$. Thus,

$$n_{i,j}(t+1) = n_{i,j-1}(t)(1 - p_{i,j-1}) \quad (12)$$

The average number of nodes that enter state $(i,0),\ 0 < i < M-1$, at time $t+1$ is equal to the sum of the average number of nodes that back-off from all states $(i-1, j)$ in the previous back-off stage $i-1$, at time t. And all nodes in state $(i,0)$ at time t would leave to other states by time $t+1$. Therefore, for $i \neq 0, M-1$,

$$n_{i,0}(t+1) = \sum_{q=0}^{W_{i-1}-1} n_{i-1,q}(t) p_{i-1,q} [1 - \pi_{i-1,q}(t)] \quad (13)$$

where,

$$\pi_{i,j}(t) = (1 - p_{i,j})^{n_{i,j}(t)-1} \prod_{k=0}^{M-1} \prod_{\substack{l=0 \\ (k \neq i, l \neq j)}}^{W_k - 1} (1 - p_{k,l})^{n_{k,l}(t)} \quad (14)$$

The number of nodes that enter state $(M-1, 0)$ at time $t+1$ is equal to the average number of nodes that back from all states of stages $M-2$ and $M-1$ at time t. Therefore,

$$n_{M-1,0}(t+1) = \sum_{q=0}^{W_{M-1}-1} n_{M-1,q}(t) p_{M-1,q} [1 - \pi_{M-1,q}(t)] + \sum_{q=0}^{W_{M-2}-1} n_{M-2,q}(t) p_{M-2,q} [1 - \pi_{M-2,q}(t)] \quad (15)$$

Consequently, the average number of nodes that enter state S at time slot $t+1$ is equal to the sum of the average number of successful deliveries from all states (i,j) in the Markov chain of Figure 3 to the sink.

$$n_s(t+1) = n_s(t) + \cdot \sum_{i=0}^{M-1} \sum_{j=0}^{W_i - 1} n_{i,j}(t) p_{i,j} \pi_{i,j}(t) \quad (16)$$

The expected delay $D(k)$ is such that $n_s(D(k)) = k$. And the expected energy consumption per node for the successful delivery of the first k packets to the sink is:

$$E(k) = \frac{1}{N} \sum_{t=0}^{D(k)} \sum_{i=0}^{M-1} \sum_{j=0}^{W_i - 1} n_{i,j}(t) p_{i,j} \quad (17)$$

One of the most important aspects of counting the number of nodes at each state by the above, average expressions is the handling of fractional values. If $n_{i,j}(t)$ is a real number less than one, then that number can be approximately assumed to be the probability of existence of a node at state (i, j). All real numbers greater than one are used without change. It should be noted the number of states with fractional values of $n_{i,j}(t)$'s, increases with increase in their number.

VI. Slotted CSMA with Constant Back-off

In this protocol, each node starts out by choosing a time slot uniformly at random from a constant window of size W, to transmit its packet. Unlike in the previous protocols, the packet length here is assumed to be R (> 1) time slots long, while the back-off time counters of the nodes are equal to a time slot in length. When a node reaches its randomly chosen time slot it senses the channel and transmits if the channel is free of any transmissions. If the channel is busy, the node backs-off again choosing a future time slot uniformly at random from the constant window of size W. If more than one nodes choose the same time slot to transmit and the channel is free, we assume that the nodes will detect the collision and back-off within the time slot[3].

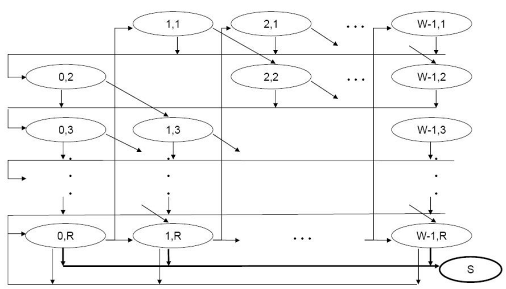

Fig. 4. Markov Chain for slotted CSMA with constant back-off.

Figure 4 shows the Markov chain for slotted CSMA with constant back-off window. This is an extension of the Markov chain for slotted Aloha with constant back-off window of Figure 2 with $R - 1$ additional states for each state j. The state (j, r), $0 \leq j \leq W-1$, $1 \leq r \leq R$,denotes that the nodes in this state have backed-off j time slots ago and there was successful transmission r slots ago. The medium is free of all transmissions when $r > R - 1$. Thus, all states (j, r), $r > R$ can be collapsed into a single state of (j, R), indicating a free channel. It should be noted that the state $(0, 1)$ does not exist. This is because, this state means that there was simultaneous collision and success transmission in the same time slot, which is not possible. As a result of this, all diagonal states in which $r = j+1$, do not exist, except for the diagonal element in the channel free states (j, R).

Equations similar to that of the previous two sections can be written for the above Markov Chain also. However, numerical evaluations of such equations and comparison with simulations results reveal that the averages do not work in this case, as they have worked for the previous two protocols. One possible reason is that with increased number of states, the number of states with fractional numbers is much higher and in such cases the averages are not good approximations of the network dynamics. Thus, the true probability mass functions (PMFs) of existence of nodes instead of their averages for each state in the Markov chain need to be determined. But, for large values of N, R and W, determining the PMFs for each state is intractable. However, we present an example for small values of N, R and W, through which we show that the Markov chain is accurate.

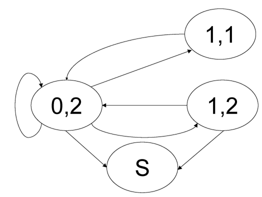

Fig. 5. Markov chain for the example with $R = 2, W = 2$.

Figure 5 shows the Markov chain for $R = 2$ and $W = 2$. We perform Monte Carlo evaluation of the Markov chain and compare it with simulation results. In Monte Carlo evaluation we start with N nodes in state $(0, 2)$ and at each time slot a node chooses to transmit from state $(0, 2)$ with probability 0.5, and from states $(1, 2)$ and $(1, 1)$ with probability 1. The three possible events in a time slot are *no-transmission*, *successful transmission* and *collision*. In the event of no-transmission, all nodes from state $(0, 2)$ move to state $(1, 2)$ and all nodes from state $(1, 2)$ move to state $(0, 2)$. In the event of a successful transmission, the successfully transmitted node moves to state S from state $(0, 2)$ or $(1, 2)$, the remaining nodes in $(0, 2)$ move to $(1, 1)$ and those remaining in $(1, 2)$ move to $(0, 2)$. In the event of a collision, all colliding nodes from $(0, 2)$ remain in $(0, 2)$ and remaining nodes move to $(1, 2)$, however, all colliding and non-colliding nodes in $(1, 2)$ move to $(0, 2)$. All nodes in $(1, 1)$ move to $(0, 2)$ in every time slot.

The delay in a trial is the number of time slots required for all the nodes to reach state S. The expected delay is calculated as the average of many (1000) trials. Figure 6 compares the Monte Carlo evaluation results to that of simulations and shows that they match very well. Finding a tractable analytical model for slotted CSMA remains a part of our ongoing work.

VII. Performance Evaluation

In this section, we present evaluation results of the four protocols, discussed in the previous sections, for the single-packet medium access problem. We first verify the accuracy of the analyses by comparing them with simulation results. Figures 9 - 10 plot the analyses and simulation results for p-persistent slotted Aloha, slotted Aloha with constant back-off and slotted Aloha with binary exponential back-off. Figure 11 plots the simulation results for slotted CSMA with constant back-off

[3]This can be achieved by reserving a tiny portion of time at the end of the time slot for an acknowledgement from the sink indicating a collision.

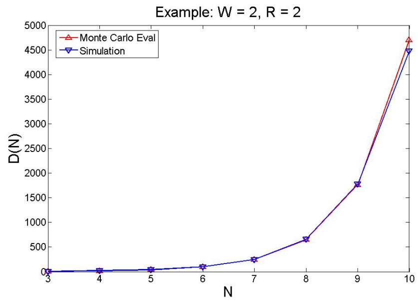

Fig. 6. Comparison of Monte Carlo Evaluation of slotted CSMA using the Markov Chain of Figure 4 and that of simulations for the example in which $R = 2, W = 2$.

window and Figure 12 compares all the four protocols. The main observations from the Figures are as follows:

- The analysis matches very well with simulations, in almost all cases one curve is exactly above the other. Any differences between the curves can be attributed to the approximation of fractional node values described in Section V.
- In p-persistent Slotted Aloha, in the absence of optimal probability of transmission, the probability that gives the best delay depends on the value of N (Figure 7(a)), as expected. A trade-off between delay and energy can be observed in Figures 7(a) and (b) – while energy consumption is the least for $p = 0.01$, the delay due to it is maximum for lower values of N. This is expected, as lower probability results in nodes attempting to transmit fewer times with larger time intervals between tries, thus increasing the delay and reducing the number of transmissions, as consequently reducing the energy consumption.
- In p-persistent slotted Aloha, optimal delay does not lead to optimal energy consumption. However, the energy consumption associated with optimal delay is not too high, in fact, it is close to the lowest values in Figure 7(b). This phenomenon can be observed for values of $k < N$ also, in Figures 7(c) and (d).
- From Equation 3 in Section III it can be deduced that $E(N)$ is a super-linear function of N and a sub-linear[4] function of k. The exact phenomenon can be observed in Figures 7(b) and (d).
- For slotted Aloha with constant back-off window, the delay for large number of nodes can be reduced by increasing the window size, however for small numbers of nodes, the delay is reduced by reducing the window size (Figure 8(a)). This is expected, because, when the value of N is low, the delay is limited by N, thus increasing the window size would introduce unnecessary delay in transmissions of packets. For high values of N, the delay is limited by the transmission probability of nodes, which depends on the window size, thus increasing the window size would help in reducing the delay by reducing the transmission probabilities. The energy consumption, however, reduces with increasing window size for all numbers of nodes (Figure 8(b)), which can be explained by the same reasoning as above.
- The same behavior as above can be observed for $k < N$ in slotted Aloha with constant back-off (Figures 8(c) and (d)). For a fixed value of N, the delay for successful transmission of the first k packets reduces with increasing window size initially, but the delay for higher values of k is lower for $W = 64$ and $W = 128$ than for $W = 256$. The energy consumption per node reduces with increasing window size.
- The super-linear and sub-linear natures of $E(N)$ and $E(k)$, $(k < N)$, respectively, can be observed in the case of slotted Aloha with constant back-off also.
- The slotted Aloha protocol with binary exponential back-off has an additional parameter of M, the number of stages, compared to slotted Aloha with constant back-off. In order to understand the effect of this parameter on the delay and energy consumption it is required to determine the dynamics of each stage as a function of time. The number of nodes $n_i(t)$ in stage i Figure 3 is calculated as follows using equations from Section V:

$$n_i(t) = \sum_{j=0}^{W_i-1} n_{i,j}(t), \quad 0 \le i \le M-1 \qquad (18)$$

Figure 9 plots the number of nodes in each of the five stages in slotted Aloha with binary exponential back-off as a function of time for $M = 5$ and $W_0 = 32$.

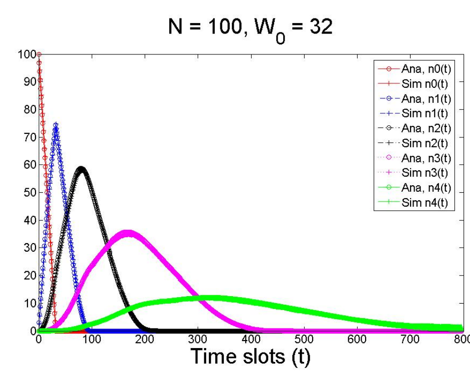

Fig. 9. Comparison of Analysis and Simulations for slotted Aloha with Binary Exponential Back-off.

It can be seen from this Figure that, the final stage contributes the maximum to the delay. For example, the first four stages contribute close to half of the delay and the final stage (stage 5) contributes the other half of the delay in the above Figure. This is expected, as nodes in the lower stages, with higher probability of transmissions and resultant higher probabilities of collisions, have to move through $M - 1$ stages before they reach the final stage with lower probability of transmissions. Thus, the

[4] A function is super-linear with respect to a variable if the second derivative is positive with respect to that variable and it is sub-linear if the second derivative is negative.

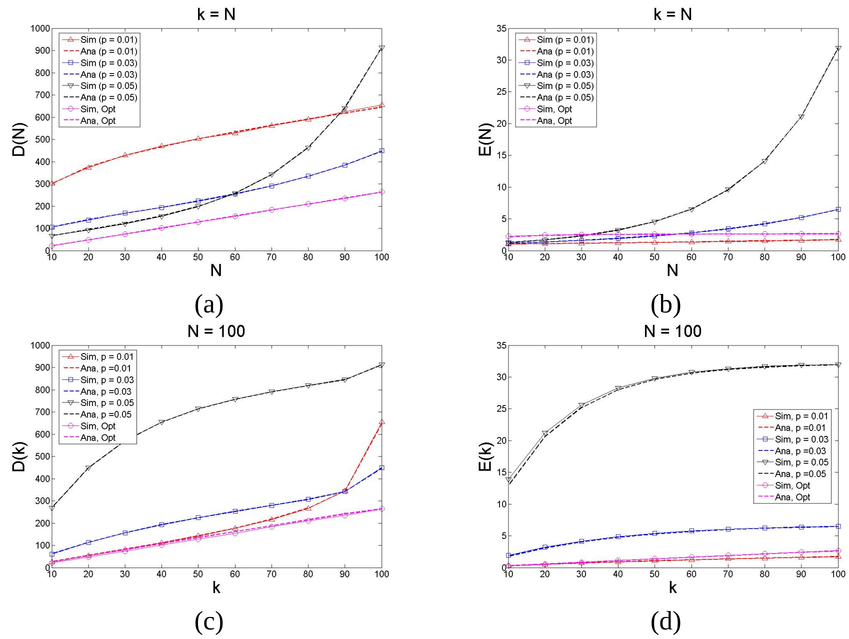

Fig. 7. Comparison of Analysis and Simulations for p-persistent Slotted Aloha.

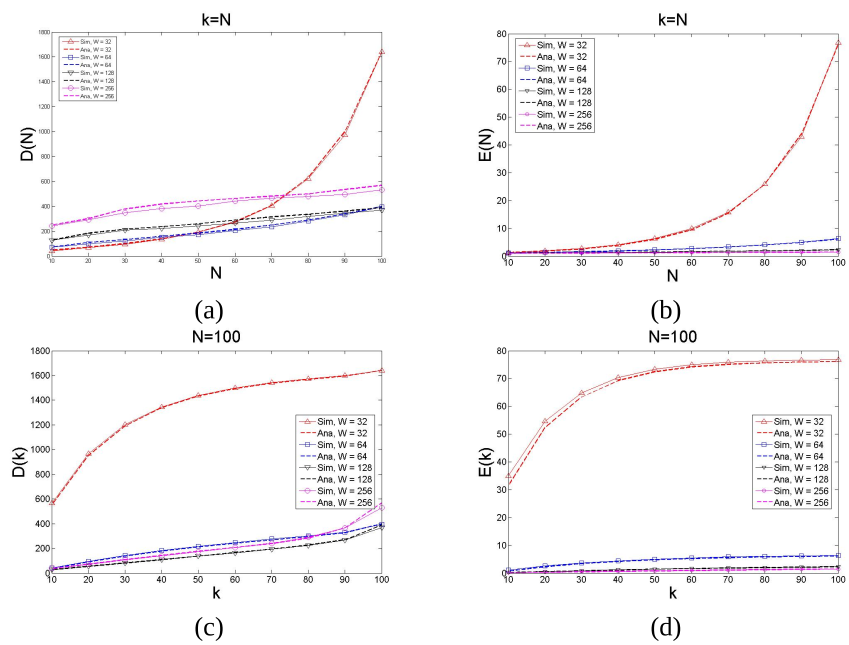

Fig. 8. Comparison of Analysis and Simulations for Slotted Aloha with constant back-off window.

delay can be expected to reduce if the number of stages is reduced to such an extent that the probabilities of transmissions in the final stages do not lead to excessive collisions. Figure 10(a) shows that the delay reduction can be achieved by reducing M from 5 to 3. Similar reasoning as above would suggest that increasing the value of M would increase the delay, which can also be observed in this Figure. However, the energy consumption per node reduces increasing number of stages (Figure 10(b)), even though the increase is negligible for lower number of nodes.

- Alternatively, if the number of stages is held constant, and if the initial window size is increased, the delay is expected to increase because the transmission probabilities of nodes increases for all stages, thus keeping the medium idle for longer periods between packet transmissions. This phenomenon can be observed in Figure 10(c). Again, for the same reason, the energy consumption per node reduces with increasing window size, and can be observed in Figure 10(d). It should be noted that the reduction in energy consumption is more sensitive to change in initial window size W_0 than to change in the number of stages.

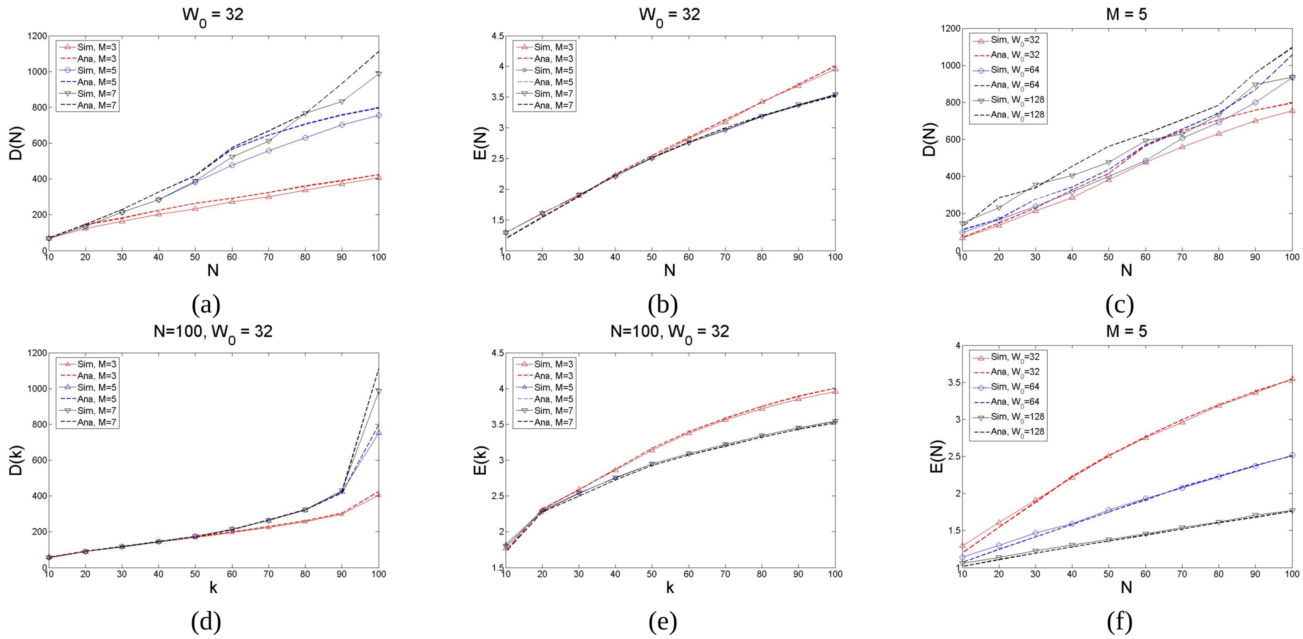

Fig. 10. Comparison of Analysis and Simulations for slotted Aloha with Binary Exponential Back-off.

- For $k < N$, for slotted Aloha protocol with binary exponential back-off, while the delay increases with increasing number of stages, the energy consumption per node reduces (Figures 10(e) and (f)).
- It is noteworthy that both $E(N)$ and $E(k)$, $(k < N)$ are sub-linear for slotted Aloha with binary exponential back-off. Further, maintaining the shift in pattern from slotted Aloha with constant back-off window, the concavity increases with increasing number of stages (Figure 10(b)).
- Figure 11 shows the simulation results for constant back-off slotted CSMA. We plot the delay normalized by the length of the packet. For packets sizes $R < W$, the normalized delay increases with decreasing packet sizes. For $R > W$, the normalized delay is independent of R (Figures 11(a) and (c)). However the energy consumption is independent of the packet size (Figures 11(b) and (d)). The above trends in normalized delay and energy consumption per node can be observed for a fixed value of N and $k < N$ (Figure 11(e) and (f)).

  It should be noted that, we have assumed, in the case of a collision, nodes involved in the collision would detect it within the same time slot. This increases the performance of slotted CSMA protocol for $R > W$. In this case, the normalized delay in time slots is equal to the number of nodes, implying that on average one node transmits per time slot, every time slot.
- Figure 12 compares the analytical results of the four slotted Aloha protocols. The comparison between slotted CSMA and other protocols is possible either through normalization of slotted CSMA results or scaling of the other protocol results by the packet length to back-off timer ratio. We use the former approach. However, it is not required to normalize the energy consumption per node. In Figures 12(a) and (b), the parameters for each protocol were chosen such that the delay is the minimum among the considered values. Figures 12(c) and (d) show the energy vs delay scatter plot for the protocols. In these plots, a point represents the corresponding protocol's delay and energy values for certain value of N and the protocol parameters. These plots suggest that a single protocol does not perform the best over all ranges of network parameters. They show the existence of delay-energy trade-offs for choosing the appropriate protocol based on its parameters.

VIII. Conclusion & Future Work

In this paper, we investigated the performance of four different slotted multi-access protocols for the single-packet medium access problem. We presented transient Markov chain analysis for p-persistent slotted Aloha, slotted Aloha with constant back-off window, slotted Aloha with binary exponential back-off window and slotted CSMA with constant back-off through the use of flow equations that captured the network dynamics as a function of time. Comparison of analytical results with that of the simulations shows that the analysis is very accurate. We also presented the performance of each of the above protocols in terms of expected delay and energy consumption per node as a function of various protocol parameters and suggested the most appropriate protocol that is dependent on these parameters.

In this paper, was have assumed that the nodes are synchronized in time. This might not be true in some application scenarios. In the future we wish to analyze the performance of asynchronous CSMA for the single-packet medium access problem. In addition, as stated previously, we wish to find a tractable analytical model for slotted CSMA. In the future,

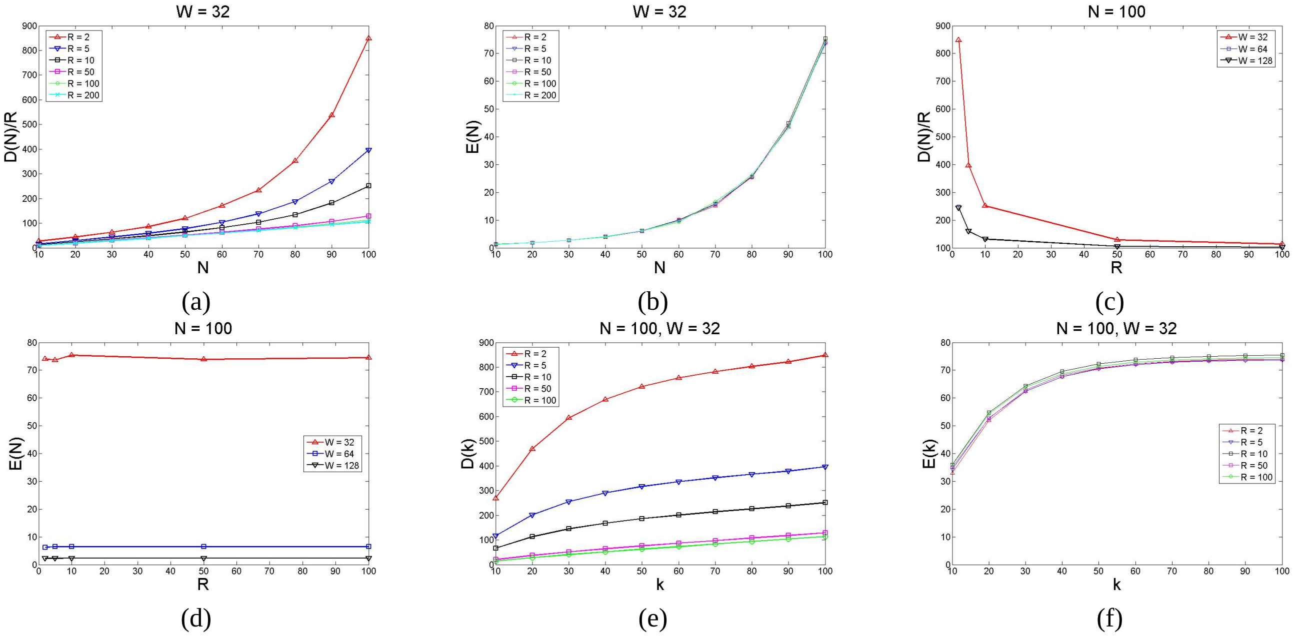

Fig. 11. Simulations results for slotted CSMA with Constant Back-off window.

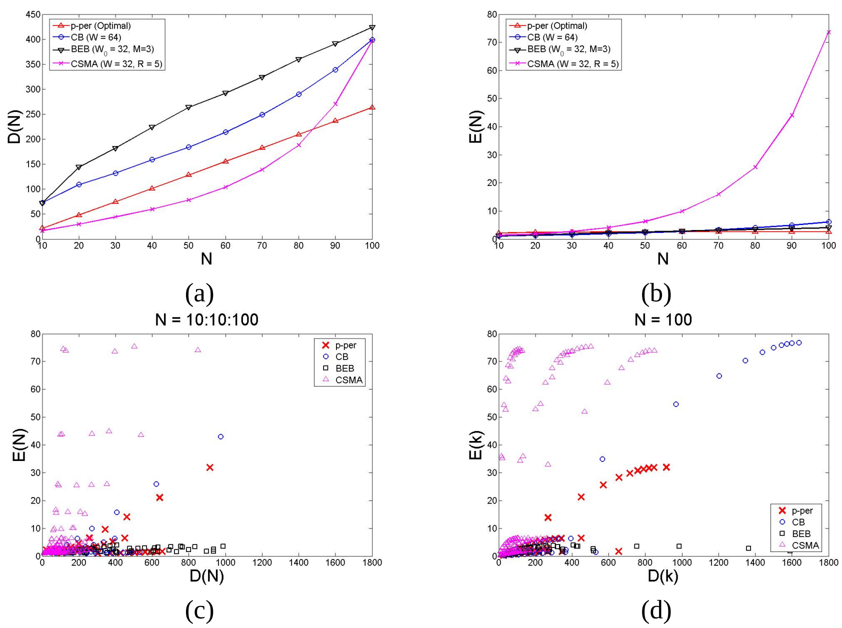

Fig. 12. Comparison of p-persistent slotted Aloha, slotted Aloha with constant back-off (CB), slotted Aloha with Binary exponential back-off (BEB) and slotted CSMA with constant window size.

we would also like to analyze the performance of splitting algorithms for the single-packet medium access problem.

REFERENCES

[1] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 535–547, March 2000.

[2] Johan Hstad , Tom Leighton , Brian Rogoff, "Analysis of Backoff Protocols for Mulitiple AccessChannels", SIAM Journal on Computing, v.25 n.4, p.740-774, Aug. 1996

[3] B.-J. Kwak, N.-O. Song, and L. E. Miller, "Performance Analysis of Exponential Backoff", IEEE/ACM Transactions on Networking, Vol. 13, pp. 343-355 (April 2005).

[4] D. Bertsekas and R. Gallager, *Data Networks (Second Edition)*. Prentice Hall, 1991.

[5] K. Yedavalli, B. Krishanamchari, and L. Venkatraman, "Fast/Fair Mobile Localization in Infrastructure Wireless Sensor Networks," *ACM Mobile Computing and Communications Review Special Issue on Localization Technologies and Algorithms*, 2006, (To Appear).

[6] Y. C. Tay, K. Jamieson, and H. Balakrishnan, "Collision Minimizing CSMA and its Applications to Wireless Sensor Networks," *IEEE Journal on Selected Areas in Communications*, August 2004.