Fundamental properties of mobility for realistic performance analysis of mobility-assisted networks

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Abstract— **Traditional mobile ad hoc routing protocols fail to deliver any data in Intermittently Connected Mobile Ad Hoc Networks (ICMN's) because of the absence of complete end-to-end paths in these networks. To overcome this issue, researchers have proposed to use node mobility to carry data around the network. These schemes are referred to as mobility-assisted routing schemes.**

A mobility-assisted routing scheme forwards data only when appropriate relays meet each other, and the time it takes for them to first meet each other is referred to as the meeting time. The time duration they remain in contact with each other is called the contact time. If they fail to exchange the packet during the contact time, then they have to wait till they meet each other again. This time duration is referred to as the inter meeting time. A realistic performance analysis of any mobility-assisted routing scheme requires a knowledge of the statistics of these three quantities. These quantities vary largely depending on the mobility model at hand. This paper studies these three quantities for the three most popularly used mobility models: random direction, random waypoint and random walk models. Hence, this work allows to do a realistic performance analysis of any routing scheme under any of these three mobility models.

I. Introduction

Intermittently connected mobile networks (ICMN's) are networks where most of the time, there does not exist a complete end-to-end path from the source to the destination. Even if such a path exists, it may be highly unstable because of the topology changes due to mobility and may change or break soon after it has been discovered. This situation arises when the network is quite sparse, in which case it can be viewed as a set of disconnected, time varying cluster of nodes. Examples of such networks include sensor networks for wildlife tracking and habitat monitoring [1], [2], military networks [3], deep-space inter-planetary networks [4], nomadic communities networks [5], networks of mobile robots [6] vehicular ad hoc networks [7] etc.

Traditional mobile ad hoc routing protocols will fail for these networks because they require the existence of complete end-to-end paths to be able to deliver any data. To overcome this issue, researchers have proposed two approaches. Either messages get carried by specialized mobile nodes which follow predefined paths between disconnected parts of the network [8], [9], or all mobile network nodes may act as relays and as they move, carry messages from one relay node to another, until the message reaches the destination [10], [11], [12], [13], [14]. What the above two approaches share in common is that node mobility is exploited to carry messages around the network as part of the routing algorithm. We refer to these schemes collectively as *mobility-assisted routing schemes*.

Since message transmission occurs only when nodes meet each other, the time elapsed between such meetings is the basic delay component. Therefore in order to evaluate the performance of any mobility-assisted routing scheme, it is necessary to know the statistics of encounter times between nodes, called *meeting times*. These are the times until a node, which, say, just received a message, first encounters a given other node that can act as a relay. Once two nodes meet, the time these two nodes remain in contact with one another will determine the time duration they have to exchange packets. This time duration is referred to as the *contact time*. Contention in the network can cause the transmission between these two nodes to fail. Then the nodes will have to wait till they meet again to get another transmission opportunity. The time till the two nodes, which start from within range of each other and then move out of each other's range, meet again is called the *inter meeting time*. Performance analysis of any mobility assisted routing scheme with contention in the network will require a knowledge of the statistics of the inter meeting times. These three quantities constitute the basic components in the realistic performance analysis (any analysis without considering finite bandwidth and contention in the network will be unrealistic) of any scheme, and they largely vary depending on the specific mobility model in hand. This paper studies these three fundamental quantities for the three most popular mobility models: the random direction, random waypoint and random walk mobility models.

Although, there has been a lot of effort to theoretically characterize the performance of mobility assisted routing schemes for intemittently connected mobile networks [11], [15], [16], [17], [18], [19], [20], [21], [22], the statistics of these fundamental properties have remained largely unstudied for most mobility models. Most of these papers assume infinite bandwidth and no contention in the network. The performance of a routing scheme in such a network depends only on the meeting times. So, the meeting times for different mobility models have been studied in more detail than the inter meeting and the contact times. In particular, [11] finds the expected meeting time value for random walk mobility model, and [21]

N	Size of the torus
K	The transmission range
$E[M_{mm}]$	Expected meeting time for mobility process 'MM'
$E[M_{mm}^{+}]$	Expected inter meeting time for mobility process 'MM'
$E[\tau_{mm}]$	Expected contact time for mobility process 'MM'

TABLE I

NOTATION USED

finds the expected meeting time for the random waypoint and random direction mobility models. In addition to studying its expected value, researchers have also studied the tail of the distribution of the meeting times. In particular, [23] proves that the tail of the distribution of the meeting times under random walk is exponential and [15] observes this via simulations. Finally, [22] is the only prior work which attempts to analyze the performance of mobility assisted routing with contention in network and to do so, it derives the expected inter meeting time for random walk mobility.

In this paper, wer compute the expected inter meeting times of the random direction and random waypoint mobility models. We also formally prove that the tail of the distribution of the meeting and inter meeting times under random direction and random waypoint mobility is memoryless. We show through simulations that the distribution of the inter meeting times under random walk mobility is heavy tailed. Finally, we find the expected contact time for all the three mobility models. Hence, we determine all the necessary quantities to do a realistic performance analysis of mobility-assisted routing under the three most popular mobility models.

The outline of the paper is as follows: Section II presents our notation, assumptions and then formally defines the meeting time, the inter meeting time and the contact time. Section III finds the expected delay for Direct Transmission routing scheme in terms of these statistics to demonstrate their utility in the performance analysis of mobility assisted routing schemes. Sections IV, V, VI finds these statistics for the random direction, random waypoint and random walk mobility models respectively. Finally, Section VII concludes the paper.

II. Notation and Definitions

We first introduce our notation and state the assumptions we will be making throughout the remainder of the paper.

(a) All nodes exist in a two dimensional torus U of area N and have a transmission range equal to K. The position of node i at time t is denoted as $X_i(t)$.

(b) Time is slotted. Two nodes exchange packets only if they are within each other's transmission range at the start of the time slot.

Now we formally define the meeting time, the inter meeting time and the contact time of a mobility model.

Definition 2.1 (Meeting Time): Let nodes i and j move according to a mobility process 'MM' and start from their stationary distribution at time 0. The meeting time (M_{mm}) between the two nodes is defined as the time it takes them to first come within range of each other, that is $M_{mm} = min_t\{t : \|X_i(t) - X_j(t)\| \leq K\}$.

Definition 2.2 (Inter Meeting Time): Let nodes i and j move according to a mobility process 'MM'. Let the nodes start from within range of each other at time 0 and then move out of the range of each other at time t_1, that is $t_1 = min_t\{t : \|X_i(t) - X_j(t)\| > K\}$. The inter meeting time (M_{mm}^{+}) of the two nodes is defined as the time it takes them to first come within range of each other again, that is $M_{mm}^{+} = min_t\{t - t_1 : \|X_i(t) - X_j(t)\| \leq K\}$.

Definition 2.3 (Contact Time): Let nodes i and j move according to a mobility process 'MM' and assume they come within range of each other at time 0. The contact time τ_{mm} is defined as the time they remain in contact with each other before moving out of the range of each other, that is $\tau_{mm} = min_t\{t - 1 : \|X_i(t) - X_j(t)\| > K\}$. (Note that t_1 defined in Definition 2.2 is the same $\tau_{mm} + 1$.)

III. Motivating Example: Delay Analysis of Direct Transmission

In Direct Transmission, the source holds on to the message until it comes within range of the destination itself. It is one of the simplest imaginable mobility assisted routing schemes, but along with flooding (epidemic routing), it forms the basic buidling block of all the other mobility assisted routing schemes. We now analyze the expected delay of Direct Transmission to demonstrate the role played by the three fundamental quantities in the performance analysis of mobility assisted routing.

Contention leads to a loss of transmission opportunities when two nodes are within range of each other. We model the loss of transmission opportunity in a time slot due to contention as a Bernoulli Random variable with parameter p. (For more information about the meaning and the way to derive this quantity p, the interested reader is referred to [22], which has derived the expected delay of epidemic routing under contention.)

Theorem 3.1: The expected delivery delay for Direct Transmission routing scheme is

$$ED_{mm}^{dt} = E[M_{mm}] + \frac{p^{E[\tau_{mm}]}E[M_{mm}^{+}]}{1 - p^{E[\tau_{mm}]}} \tag{1}$$

where p = Pr[loss of a transmission opportunity due to contention in one time slot given the two nodes were within range of each other]

Proof: The expected time it takes for the source to meet the destination for the first time is $E[M_{mm}]$ (the expected meeting time). With probability p, they are unable to exhange the packet in one time slot. They are within range of each other for $E[\tau_{mm}]$ number of time slots. (We are making an approximation here by replacing τ_{mm} by its expected value.) Then $p^{E[\tau_{mm}]}$ is the probability that the source fails to deliver the packet to the destination when they came within range of each other. Then they will have to wait for one inter meeting time to come within range of each other. If they fail again, they will have to wait yet another intermeeting time to come within range. Thus, $ED_{dt} = E[M_{mm}] + (1-p) \left(p^{E[\tau_{mm}]}E[M^+_{mm}] + 2p^{2E[\tau_{mm}]}E[M^+_{mm}] + \ldots\right) = E[M_{mm}] + \frac{p^{E[\tau_{mm}]}E[M^+_{mm}]}{1-p^{E[\tau_{mm}]}}$. □

The expected delivery for Direct Transmission under any mobility model can be calculated by substituting the values of $E[M_{mm}]$, $E[M^+_{mm}]$ and $E[\tau_{mm}]$ for that mobility model into Equation (1).

IV. Random Direction

Definition 4.1 (Random Direction): In the Random Direction model, each node moves as follows [24]:

1. Choose a direction θ uniformly in $[0, 2\pi)$.
2. Choose a speed v uniformly in $[v_{min}, v_{max}]$ with $v_{min} > 0$ and $v_{max} < \infty$. Let $\overline{v}$ denote the average speed of a node.
3. Choose a duration T of movement from a geometric distribution with mean $\overline{T}$. The average distance travelled in a duration $\overline{L}$ is equal to $\overline{T}\overline{v}$. We assume that $\overline{L} = O\left(\sqrt{N}\right)$ to ensure fast mixing [1].
4. Move towards θ with speed v for T time slots.
5. After T time slots, pause for T_{stop} time slots where T_{stop} is chosen from a geometric distribution with mean $\overline{T}_{stop}$.
6. Goto Step 1.

The expected meeting time of the Random Direction model was evaluated in [21]. We first recap their result in Theorem 4.1 and then derive the expected inter meeting time in Theorem 4.2, the expected contact time in Theorem 4.3 and finally the distribution of the meeting and the inter meeting times in Theorem 4.4.

Theorem 4.1: The expected meeting time $E[M_{rd}]$ for the Random Direction model is given by

$$E[M_{rd}] = \frac{\left(\frac{N}{2K\overline{L}}\right)\left(\overline{T} + \overline{T}_{stop}\right)}{p_m \hat{v}_{rd} + 2(1-p_m)},$$

where $\hat{v}_{rd} \approx 1.27$ is the normalized relative speed ($\hat{v}_{rd} = \frac{E[||\vec{v}_i - \vec{v}_j||])}{\overline{v}}$ for the Random Direction model, and $p_m = \frac{\overline{T}}{\overline{T}+\overline{T}_{stop}}$ is the probability that a node is moving at any time.

Proof: See [21]. □

Theorem 4.2: The expected inter meeting time $E[M^+_{rd}]$ for the Random Direction model is approximately equal to $E[M_{rd}]$.

Proof: When the nodes move out of the range of each other, they keep moving for a duration which is geometrically distrbuted. Since we assumed that $\overline{L} = O(\sqrt{N})$, the nodes mix (reach their stationary distribution) after their respective movement duration ends. After the two nodes get mixed, the additional time it will take for them to meet again is equal to the meeting time. In general, since one movement duration is much less than the expected meeting time, $E[M^+_{rd}] = E[M_{rd}]$. □

[1] The mixing time of a mobility model is the time it takes for a node to come back to its stationary distribution after starting from any arbitrary initial distribution.

Now we find the expected contact time for the Random Direction model. To similify the exposition, we will make a couple of approximations.

(a) We approximate the geometric distribution with an exponential distribution. Exponential distribution is the equivalent continuous version of geometric distribution. In other words, we assume that both movement and pause durations are exponentially distributed. Assuming a continuous distribution simplifies the analysis because we dont have to worry about the corner cases where two time durations expire at the same time.

(b) Let $T = \frac{L}{v}$. In general, $E[T] \neq \frac{E[L]}{\overline{v}}$, but for the ease of analysis, we will assume that they are equal.

When two nodes come within range of each other, one of the following is true: (a) Both the nodes are moving or (b) Only one of the nodes is moving and the other is paused. Let $E[\tau^1_{rd}]$ denote the expected contact time given both nodes were moving when they came within range of each other and let $E[\tau^2_{rd}]$ denote the expected contact time given only one of the nodes was moving when they came within range. We derive their values in Appendix A.

Theorem 4.3: The expected contact time $E[\tau_{rd}]$ for the Random Direction model is given by

$$E[\tau_{rd}] = \frac{p_m^2}{p_m^2 + 2p_m(1-p_m)}E[\tau^1_{rd}] + \frac{2p_m(1-p_m)}{p_m^2 + 2p_m(1-p_m)}E[\tau^2_{rd}]$$

where $p_m = \frac{\overline{T}}{\overline{T}+\overline{T}_{stop}}$ is the the probability that a node is moving at any time.

Proof: The probability that both nodes are moving is equal to p_m^2. The probability that only one of the nodes is moving is equal to $2p_m(1-p_m)$. For two nodes to come within range from out of range, at least one of the nodes has to be moving. Hence, to find $E[\tau_{rd}]$, we have to condition over the fact that at least one of the two nodes is moving. Applying the law of total probability gives the result. □

We made a few approximations during the course of the analysis to keep it tractable. Since all the approximations were easily justifiable, we do not expect that they would drastically effect the accuracy of the analysis, which we verify in Figures 1(a)-1(d) where we compare the analytical and simulation results for the expected contact time of the Random Direction model.

Theorem 4.4: The tail of the distribution of the meeting time and the inter meeting time of the Random Direction model is geometric.

Proof: Let node A and node B start from their stationary distribution at time 0. Lets define one time epoch as the time duration at the end of which one of the two nodes change their state (either from moving to paused or from paused to moving). Let N_{meet} denote the number of epochs until node A meets node B, and $Pr[N_{meet} > n]$ denote the probability that node A and node B do not meet after n epochs.

Although consecutive epochs are not independent (the end of one epoch is the beginning of the next one), the random process describing the lengths and end points of the sequence

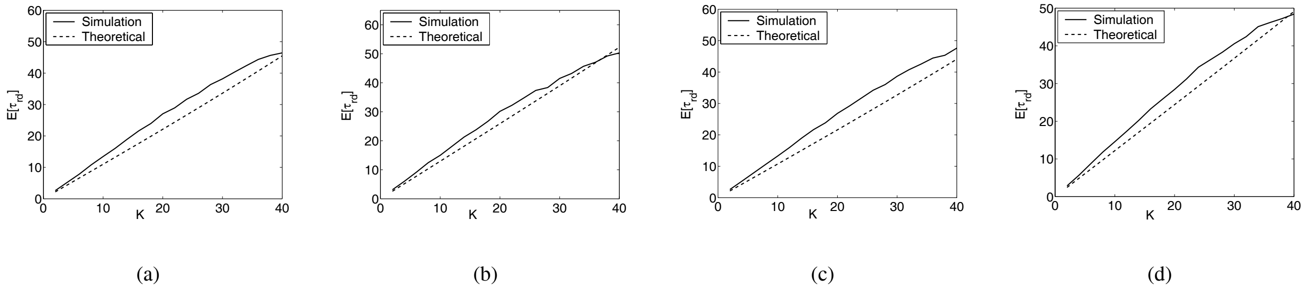


Fig. 1. Comparison of the theoretical and simulation results for the expected contact time for the Random Direction Mobility model with parameters (a) $N = 100 \times 100, \overline{T} = 300, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 100 \times 100, \overline{T} = 300, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 100 \times 100, \overline{T} = 400, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 100 \times 100, \overline{T} = 400, \overline{v} = 1, \overline{T}_{stop} = 150$

of epochs drawn is ergodic [25]. Thus, we can use the statistics of a single epoch to describe the whole process, as if the epochs were drawn independently (the argument is similar to the one made in [21] and [25]). Thus, $Pr[N_{meet} > n] = Pr[$A and B do not meet in the first n time epochs$] = (Pr[$A and B do not meet in a time epoch$])^n$. Consequently, the number of epochs needed till A meets B is geometrically distributed. Thus the distribution for the meeting time when the meeting time is much larger than one epoch time, is also geometric. Thus the tail of the distribution of the meeting time is geometric.

A similar argument holds for the inter meeting time also. □

We plot the distribution of the meeting time and inter meeting time of the Random Direction model for some sample values in Figures 2(a)-2(d) and Figures 3(a)-3(d) respectively. The tails of all the distributions are geometric.

V. Random Waypoint

Definition 5.1 (Random Waypoint): In the Random Waypoint model, each node moves as follows [26]:

1. Choose a point X in the network uniformly at random.
2. Choose a speed v uniformly in $[v_{min}, v_{max}]$ with $v_{min} > 0$ and $v_{max} < \infty$. Let $\overline{v}$ denote the average speed of a node.
3. Move towards X with speed v along the shortest path to X.
4. When at X, pause for T_{stop} time slots where T_{stop} is chosen from a geometric distribution with mean $\overline{T}_{stop}$.
5. Go to Step 1.

One iteration of these steps is referred to as an epoch.

Lemma 5.1: (a) The stationary distribution of nodes moving according to Random Waypoint model on a torus is uniform.

(b) Let L be the length of an epoch, measured as the distance between the starting and the finishing points of the epoch. Then $E[L] = 0.3826\sqrt{N}$.

Proof:

(a) See [27]. □

(b) The current position as well as the destination picked is uniformly distributed on the torus. The pdf of L can be easily evaluated using geometrical arguments to be
$$f_L(l) = \begin{cases} \frac{2\pi l}{N} & l \leq \frac{\sqrt{N}}{2} \\ \frac{4l}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2l}\right)\right) & \frac{\sqrt{N}}{2} \leq l \leq \frac{\sqrt{N}}{\sqrt{2}} \end{cases}.$$
Then $E[L] = \int_0^{\frac{\sqrt{N}}{\sqrt{2}}} f_L(l)dl = 0.3826\sqrt{N}$. □

The expected meeting time of the Random Waypoint model was evaluated in [21]. We first recap their result in Theorem 5.1 and then derive the expected inter meeting time in Theorem 5.2, the expected contact time in Theorem 5.3 and finally the distribution of the meeting and the inter meeting times in Theorem 5.4.

Theorem 5.1: The expected meeting time $E[M_{rwp}]$ for the Random Waypoint model is given by

$$E[M_{rwp}] = \frac{\left(\frac{N}{2KE[L]}\right)\left(\frac{E[L]}{\overline{v}} + \overline{T}_{stop}\right)}{p_m \hat{v}_{rwp} + 2(1 - p_m)}$$

where $\hat{v}_{rwp} \approx 1.27$ is the normalized relative speed ($\hat{v}_{rwp} = \frac{E[\|\vec{v}_i - \vec{v}_j\|]}{\overline{v}}$) for the Random Waypoint model, and $p_m = \frac{\frac{E[L]}{\overline{v}}}{\frac{E[L]}{\overline{v}} + \overline{T}_{stop}}$ is the probability that a node is moving at any time.

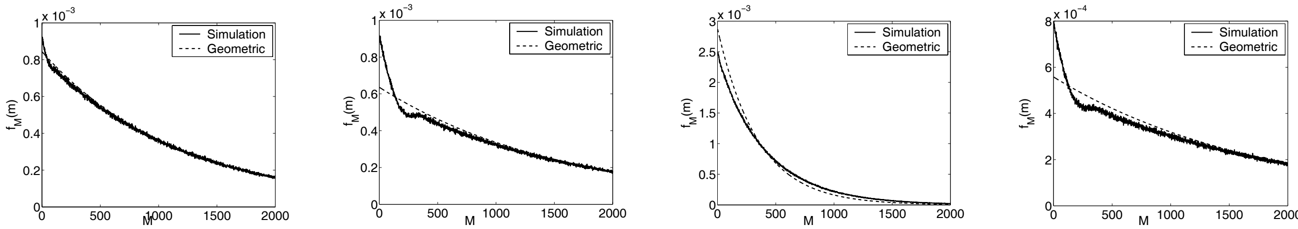
Proof: See [21]. □

Theorem 5.2: The expected inter meeting time $E[M^+_{rwp}]$ for the Random Waypoint model is approximately equal to $E[M_{rwp}]$.

Proof: When the nodes move out of the range of each other, they pick up a destination uniformly at random in the torus. After reaching their destination, they are fully mixed (back in their stationary distribution) and the additional time it takes for them to meet again is equal to the meeting time. In general, since an epoch time is much less than the expected meeting time, $E[M^+_{rwp}] = E[M_{rwp}]$. □

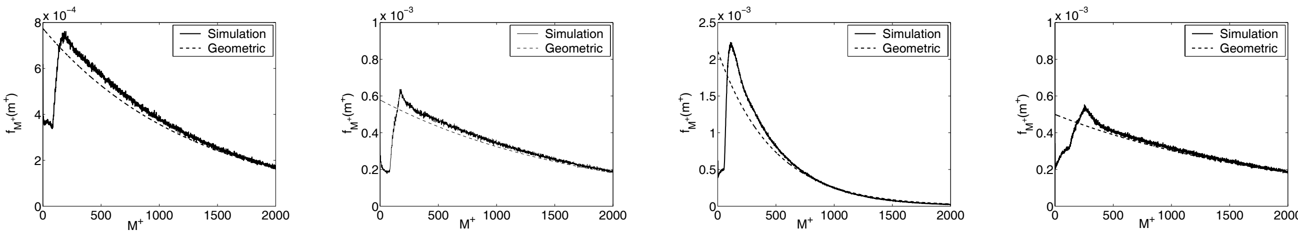
Now we find the expected contact time for the Random Waypoint mobility model. The approach is exactly the same as for the Random Direction model. Also, we will make the same two approximations as we made to find the expected contact time for the Random Direction model.

Theorem 5.3: The expected contact time $E[\tau_{rwp}]$ for the



(a) (b) (c) (d)

Fig. 2. Meeting distribution for Random Direction Mobility model with parameters (a) $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 300 \times 300, K = 70, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 500 \times 500, K = 70, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$



(a) (b) (c) (d)

Fig. 3. Inter Meeting distribution for Random Direction Mobility model with parameters (a) $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 300 \times 300, K = 70, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 500 \times 500, K = 70, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$

Random Waypoint model is given by

$$E[\tau_{rwp}] = \frac{p_m^2}{p_m^2 + 2p_m(1-p_m)} E[\tau_{rwp}^1] + \frac{2p_m(1-p_m)}{p_m^2 + 2p_m(1-p_m)} E[\tau_{rwp}^2].$$

where $p_m = \frac{\frac{E[L]}{\overline{v}}}{\frac{E[L]}{\overline{v}} + \overline{T}_{stop}}$ is the the probability that a node is moving at any time, $E[\tau_{rwp}^1]$ is the expected contact time given both nodes were moving when they came within range of each other and $E[\tau_{rwp}^2]$ is the expected contact time given only one of the nodes was moving when they came within range. We find their values in Appendix B.

Proof: The proof runs along similar lines as the proof of Theorem 4.3. □

We made a few approximations during the course of the analysis to keep it tractable. Since all the approximations were easily justifiable, we do not expect that they would drastically effect the accuracy of the analysis. To validate our claim we compare the analytical and simulation results for the expected contact time of the Random Waypoint model in Figures 4(a)-4(d), and find that the two curves match.

Theorem 5.4: The tail of the distribution of the meeting time and the inter meeting time of the Random Waypoint model is geometric.

The proof of Theorem 5.4 runs along the same line as the proof of Theorem 4.4. We plot the distribution of the meeting time and inter meeting time of the Random Direction model in Figures 5(a)-5(d) and Figures 6(a)-6(d) for some sample values. The tails of all the distributions are geometric.

VI. Random Walk

We now assume that nodes are moving on a $\sqrt{N} \times \sqrt{N}$ grid in a 2-D torus. Each node moves one grid unit in one time unit.

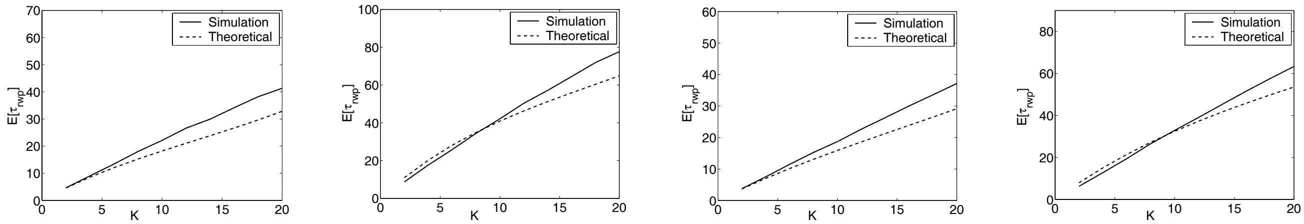
Definition 6.1 (Random Walk): In the Random Walk mobility model, each node moves as follows:

1. Choose one of the four neighboring grid points uniformly at random.
2. Move towards the chosen grid point during that time slot.
3. Goto Step 1.

The expected meeting time and the expected inter meeting time of the Random Walk model was evaluated in [21] and [22] respectively. We first recap these results in Theorems 6.1 and 6.2 and then find the expected contact time in Theorem 6.3. Then we cite the result about the tail of the distribution of the meeting time from [23]. Finally, we study the distribution of the inter meeting time using simulations.

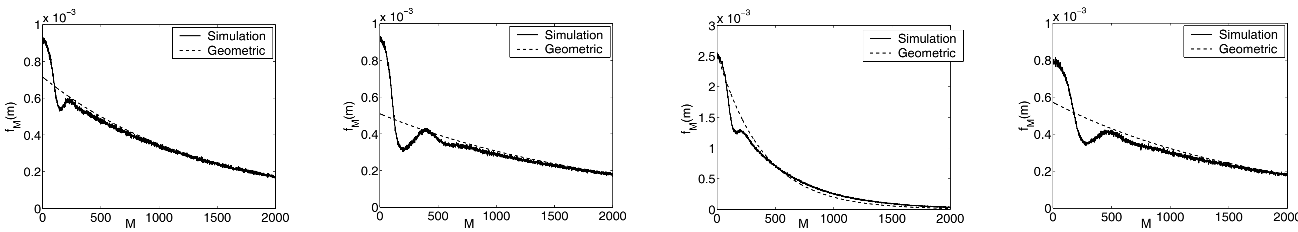
Theorem 6.1: The expected meeting time $E[M_{rw}]$ for the Random Walk model is given by

$$E[M_{rw}] = \frac{N}{2}\left(clogN - \frac{2^{K+1} - K - 2}{2^K - 1}\right),$$



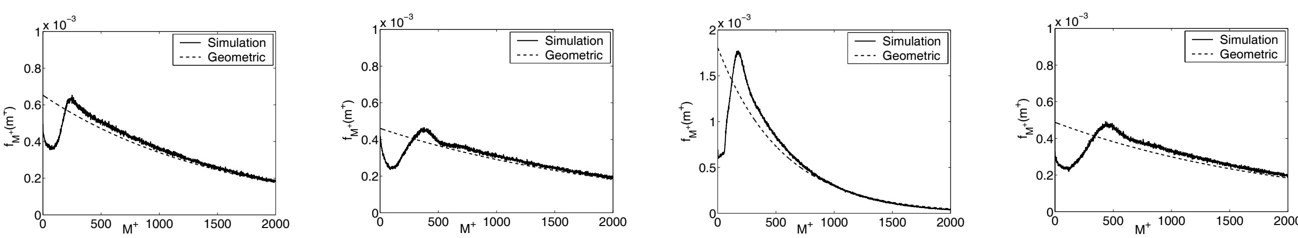
(a) (b) (c) (d)

Fig. 4. Comparison of the theoretical and simulation results for the expected contact time for the Random Waypoint Mobility model with parameters (a) $N = 100 \times 100, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 100 \times 100, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 150 \times 150, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 150 \times 150, \overline{v} = 1, \overline{T}_{stop} = 150$



(a) (b) (c) (d)

Fig. 5. Meeting distribution for Random Waypoint Mobility model with parameters (a) $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 300 \times 300, K = 70, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 500 \times 500, K = 70, \overline{v} = 1, \overline{T}_{stop} = 150$



(a) (b) (c) (d)

Fig. 6. Inter Meeting distribution for Random Waypoint Mobility model with parameters (a) $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 50$ (b) $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) $N = 300 \times 300, K = 70, \overline{v} = 1, \overline{T}_{stop} = 50$ (d) $N = 500 \times 500, K = 70, \overline{v} = 1, \overline{T}_{stop} = 150$

Proof: See [11]. □

Theorem 6.2: The expected inter meeting time $E[M_{rw}^+]$ for the Random Walk model is given by

$$E[M_{rw}^+] = \frac{N}{2}\left(\frac{2^{K+1} - K - 2}{2^K - 1}\right)\left(\frac{1}{2K+1}\right.$$

$$\left.\left(4K - (2K+1) - \left(\frac{1 + l_{K-1}}{1 + l_K}(2K-1)\right)\right)\right),$$

where $l_K = 1 + \sum_{t=1}^{K-1} \prod_{r=1}^{t} \frac{q_r}{p_r}$ and $p_r = \frac{2r+1}{4r}$, $q_r = \frac{2r-1}{4r}$.

Proof: See [22]. □

Now we find the expected contact time for the Random Walk mobility model.

Theorem 6.3: Let $E[CM]_k$ denote the expected additional time two nodes will remain in contact with each other when the distance between them is equal to $k \leq K$. By definition, the expected contact time $E[\tau_{rw}]$ for the Random Walk mobility model is equal to $E[CM]_K$. The $E[CM]_k$'s can be found by solving the following set of linear equations:

$$\begin{array}{ll} E[CM]_k = 1 + \frac{16K-20}{64K}E[CM]_{k-2} + \\ \frac{16K+12}{64K}E[CM]_{k+2} + \frac{32K+8}{64K}E[CM]_k & 3 < k < K \\ E[CM]_k = 1 + \frac{7}{48}E[CM]_{k-2} + \\ \frac{15}{48}E[CM]_{k+2} + \frac{26}{48}E[CM]_k & k = 3 \\ E[CM]_k = 1 + \frac{3}{32}E[CM]_{k-2} + \\ \frac{11}{32}E[CM]_{k+2} + \frac{18}{32}E[CM]_k & k = 2 \end{array}$$

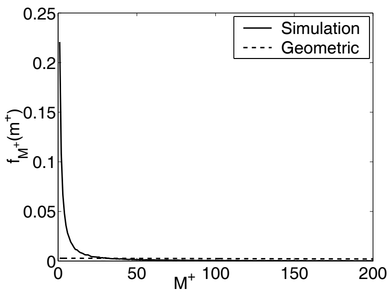


Fig. 7. Inter Meeting distribution for Random Walk Mobility model (with parameters $N = 80 \times 80$, $K = 4$) and an exponential distribution having the same mean.

$$E[CM]_k = 1 + \tfrac{9}{16}E[CM]_k + \tfrac{7}{16}E[CM]_{k+2} \quad k = 1$$

$$E[CM]_k = 1 + \tfrac{4}{16}E[CM]_k + \tfrac{12}{16}E[CM]_{k+2} \quad k = 0$$

$$\begin{array}{ll} E[CM]_k = 1 + \frac{16K-20}{64K}E[CM]_{k-2} & \\ \quad + \frac{32K+8}{64K}E[CM]_k & k = K \end{array}$$

Proof: Let one node be static and the other node move two grid units in one time unit. Both the steps are picked uniformly at random. This model is equivalent to one where both nodes move one grid unit in one time unit. The distance between the two nodes can either increase by two, decrease by two or stay the same depending on the node movement. The probability of these events can found by simple combinatrics. The law of total probability allows us to write the set of linear equations using these probabilities.

When the nodes come within range, the distance between them is equal to K. Hence, by definition, the expected contact time $E[\tau_{rw}]$ is equal to $E[CM]_K$. $\square$

Theorem 6.4: The tail of the distribution of the meeting time of the Random Walk model is memoryless.

Proof: See [23]. $\square$

Now we look at the distribution of the inter meeting time of the Random Walk model. We plot the distribution and the distribution of a geometric random variable having the same mean, in Figure 7. Its easy to see that they don't match.

The probability that the two nodes meet again in the first few time slots after moving out of range, is very high. Still, the expected value of the inter meeting time is very large ($O(N)$ where N is the area of the grid). This suggests that inter meeting times are heavy tailed.

We plot the inter meeting distribution for some sample network paramaters and the discretized Bounded Pareto in Figures 8(a)-8(d). Its easy to see that discretized Bounded Pareto fits the inter meeting time distribution pretty well.

Now we discuss how to find the parameters of the Bounded Pareto distribution in terms of the network parameters. The pdf of the Bounded Pareto distribution is,

$$f_X(x) = \frac{\alpha s^\alpha}{1-\left(\frac{s}{S}\right)^\alpha} x^{-\alpha-1} \quad s \le x \le S \tag{2}$$

The Bounded Pareto pdf has three unknowns, s, S and α. For the inter meeting time distribution, $s = 1$ as its going to take at least one time unit for the nodes to come back within range. We need two equations to find the other two unknowns. We will use $Pr[M^+_{rw} = 1] = \frac{\alpha}{1-\left(\frac{1}{S}\right)^\alpha}$ and $E[M^+_{rw}] = Pr[M^+_{rw} = 1]\left(1 - S^{1-\alpha}\right)$ to find S and α. Now, since $S > E[M^+_{rw}] >> 1$, $Pr[M^+_{rw} = 1] = \alpha$ and $S = \left(\frac{E[M^+_{rw}](1-\alpha)}{Pr[M^+_{rw}=1]} + 1\right)^{\frac{1}{1-\alpha}}$.

The value of $E[M^+_{rw}]$ was stated in Theorem 6.2. Now we derive $Pr[M^+_{rw} = 1]$.

The probability that the inter meeting time is equal to 1 can be easily evaluated using simple combinatrics. Nodes can either move at a distance of two away from each other in one time unit or still remain within range. So, when the nodes move out of range, they are at a distance of $K + 2$ from each other after the first time slot. The inter meeting time will be equal to one only if they come back within range of each other in the next time slot. We found the probability that the distance between two nodes decreases by two in one time unit given their current distance in Theorem 6.3. Using these results, we find $Pr[M^+_{rw} = 1]$ for the Random Walk mobility model is given by,

$$Pr[M^+_{rw} = 1] = \begin{cases} \frac{16(K+2)-20}{64(K+2)} & K > 1 \\ \frac{7}{48} & K = 1 \\ \frac{3}{32} & K = 0 \end{cases}. \tag{3}$$

The parameters of the Bounded Pareto distributions plotted in Figures 8(a)-8(d) were derived using the preceeding method.

VII. Conclusions and Discussion

Realistic performance analysis of mobility-assisted routing with contention in the network requires a knowledge of the statistics of the meeting time, inter meeting time and the contact time. Delay analysis of Direct Transmission shows the importance of these quantities for performance analysis. These quantities vary largely depending on the mobility model in hand. In this paper, we compute the expected inter meeting times of the random direction and random waypoint mobility models. We also prove that the tail of the distribution of the meeting and inter meeting time for random direction and random waypoint is memoryless. We show through simulations that the inter meeting time distribution for random walks is heavy tailed. Finally, we find the expected contact time for all the three mobility models.

In future, we plan to study these quantities for other mobility models too, for example, the community based mobility model [21]. We also plan to provide realistic performance analysis of many routing schemes, in addition to Direct Transmission.

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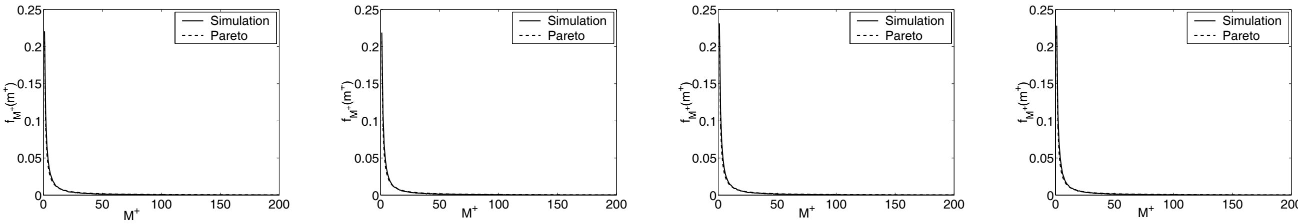

(a) (b) (c) (d)

Fig. 8. Inter Meeting distribution for Random Walk Mobility model with parameters (a) $N = 80 \times 80, K = 4$ (b) $N = 120 \times 120, K = 4$ (c) $N = 100 \times 100, K = 6$ (d) $N = 150 \times 150, K = 6$

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APPENDIX

A. Expected Contact Time for the Random Direction Mobility model

Lemma 1.1: The expected contact time given both nodes were moving when they came within range of each other, $E[\tau^1_{rd}]$, is given by,

$$E[\tau^1_{rd}] = (1 - p_1)\frac{4K}{1.27\pi\overline{v}} + p_1\left(\frac{0.6366K}{1.27\overline{v}} + E[et \mid 1p1m]\right)$$

where $p_1 \approx \int_0^{\pi} \frac{1}{\pi}\left(1 - e^{\left(\frac{4K sin(\phi)}{1.27\overline{v}T}\right)}\right) d\phi$ is the probability that one of the two nodes pause while they are within range of each other and $E[et \mid 1p1m]$ is the expected additional time the two nodes remain within range after one of the nodes paused. We will find the value of $E[et \mid 1p1m]$ in Lemma 1.3.

Proof: When both the nodes are moving when they come within range of each other, either they move out of each other's range before any of them pauses or one of them pauses before they move out of range.

(a) They move of each other's range before pausing: Let one node be static and let the other node move at a speed $\vec{v}_i - \vec{v}_j$. This model is equivalent to the model when both nodes are moving at speeds $\vec{v}_i$ and $\vec{v}_j$ respectively. We will work with the former model during this proof as well as all the subsequent proofs. The angle of $\vec{v}_i - \vec{v}_j$ is uniformly distributed between $[0, 2\pi)$.

So, when these two nodes come within range of each other, the angle ϕ in Figure 9 will be uniformly ditributed

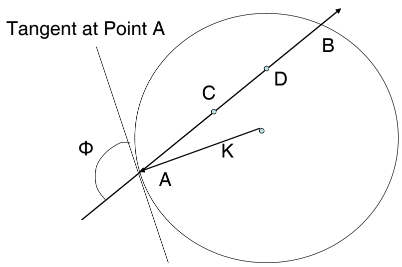

Fig. 9. The first node enters the transmission range of the second node at an angle ϕ to the tangent at A and moves along the chord AB.

within $[0, \pi)$. It cannot be greater than π as $\phi > \pi$ implies that the nodes were already in contact with each other. They will remain in contact with each other while the first node travels along the chord AB in Figure 9. The length of the chord AB is equal to $2Ksin(\phi)$. E[distance for which the nodes remain in contact with each other] = E[length of chord AB] = $\int_0^\pi \frac{1}{\pi} 2Ksin(\phi)d\phi = \frac{4K}{\pi}$. The expected speed of moving node is $E\left[\|\vec{v}_i - \vec{v}_j\|\right] = 1.27\overline{v}$. Thus the expected time they remain in contact with each other is approximately equal to $\frac{4K}{1.27\pi\overline{v}}$.

(b) One of the nodes pauses before they move out of each other's range: We again work with the model where one of the nodes is static and the other node is moving at a speed $\vec{v}_i - \vec{v}_j$. The moving node is equally likely to pause anywhere on the chord AB in Figure 9 since the distribution of movement duration is memoryless. Let the node stop at point C which is $0 \leq x \leq 2Ksin(\phi)$ distance away from A. Thus $f_{X|\Phi}(x \mid \phi) = \begin{cases} \frac{1}{2Ksin(\phi)} & 0 \leq x \leq 2Ksin(\phi) \\ 0 & \text{otherwise} \end{cases}$. Multiplying by $f_\Phi(\phi)$ and integrating over ϕ gives us $f_X(x)$. The expected distance node travels before pausing can then be evaluated to $0.6366K$. The expected time the node travels before pausing is equal to $\frac{0.6366K}{1.27\overline{v}}$. $E[et \mid 1p1m]$ is the additional time spent within range of each other.

Now we find p_1 to complete the proof. $p_1 =$ the probability that one of the two nodes pause before moving out of range. Since the movement duration of both the nodes is exponential with mean $\overline{T}$, p_1 given ϕ and $\|\vec{v}_i - \vec{v}_j\|$ is equal to $1 - e^{-\frac{2Ksin(\phi)}{\frac{\overline{T}}{2}\|\vec{v}_i - \vec{v}_j\|}}$. To simplify exposition, we replace $\|\vec{v}_i - \vec{v}_j\|$ by its expected value. Hence, $p_1 \approx \int_0^\pi \frac{1}{\pi}\left(1 - e^{\left(\frac{4Ksin(\phi)}{1.27\overline{v}\overline{T}}\right)}\right) d\phi$ which can be evaluated numerically. □

In the previous lemma, we found the expected contact time given both nodes were moving when they came within range of each other. Next lemma evaluates the expected contact time when only one node was moving when they came within range of each other.

When only one node is moving, either they will move out of each other's range before the paused node restarts again and the moving node pauses, or the moving node pauses or the paused node restarts before they move out of each other's range. The derivation has to account for all the three scenarios.

Lemma 1.2: The expected contact time given only one of the nodes was moving when they came within range of each other, $E[\tau_{rd}^2]$, is given by,

$$E[\tau_{rd}^2] = (1-p_2)\frac{4K}{\pi\overline{v}} + p_2\left(\frac{0.6366K}{\overline{v}} + p_{21}E[et \mid 2p] + p_{22}E[et \mid 2m]\right)$$

where $p_2 \approx \int_0^\pi \frac{1}{\pi}\left(1 - e^{\left(\frac{2Ksin(\phi)}{\frac{1}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}}\right)}\right) d\phi$ is the probability that the paused node restarts again or the moving node pauses before moving out of each other's range, $p_{21} = \frac{\frac{1}{\overline{T}}}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}$ is the probability that the moving node pauses before the paused node restarts and $p_{22} = \frac{\frac{1}{\overline{T}_{stop}}}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}$ is the probability that the paused node restarts before the moving node pauses. $E[et \mid 2p]$ and $E[et \mid 2m]$ are the expected additional times the two nodes remain within range after both of them are paused and after both of them start moving respectively. We will find their value in Lemma 1.3.

Proof:

(a) Both nodes move out of the range of each other without any of them changing state: The expected time they remain in contact is $\frac{4K}{\pi\overline{v}}$. The proof goes along the same lines as in proof of Lemma 1.1 (a). Except here, the expected relative speed is $\overline{v}$.

(b) The moving node pauses or the paused node starts moving before they move out of each other's range: The expected time before one of the nodes change their state is $\frac{0.6366K}{\overline{v}}$. The proof goes along the same lines as in proof of Lemma 1.1 (b). Except here, the expected relative speed is $\overline{v}$.

  (i) The moving node pauses before the paused node restarts: The movement duration is exponentially distributed with mean $\overline{T}$ while the pause duration is exponentially distributed with mean $\overline{T}_{stop}$. Hence, the probability that the paused node restarts before the moving node pauses = p_{21} can be easily derived using the properties of exponential distribution to be $\frac{\frac{1}{\overline{T}}}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}$. $E[et \mid 2p]$ is the additional time spent within range of each other.

  (ii) The paused node restarts before moving node pauses: The probability that the moving node pauses before the paused node restarts = $p_{22} = 1 - p_{21}$. $E[et \mid 2m]$ is the additional time spent within range of each other. Note that using exponential distribution instead of geometric simplifies the analysis as the probability that both the nodes change state at the same time is zero.

Now we find the value of p_2 to complete the proof. p_2 can be derived in a manner similar to the derivation of p_1 in Lemma 1.1. Except the relative speed here is $\overline{v}$ and the mean of the running exponential is $\frac{1}{\frac{1}{T}+\frac{1}{\overline{T}_{stop}}}$. $\square$

Next we find the values of $E[et \mid 1p1m]$, $E[et \mid 2p]$ and $E[et \mid 2m]$.

Lemma 1.3: $E[et \mid 1p1m]$, $E[et \mid 2p]$ and $E[et \mid 2m]$ are related to each other through the following set of linear equations:

$$E[et \mid 1p1m] = (1 - p_{1p1m})\frac{0.6366K}{\overline{v}} + p_{1p1m}\left(\frac{4K}{3\pi\overline{v}} + p_{21}E[et \mid 2p] + p_{22}E[et \mid 2m]\right), \quad (4)$$

$$E[et \mid 2p] = \overline{T}_{stop}/2 + E[et \mid 1p1m]. \quad (5)$$

$$E[et \mid 2m] = (1-p_{2m})\frac{0.6366K}{1.27\overline{v}} + p_{2m}\left(\frac{4K}{3\pi 1.27\overline{v}} + E[et \mid 1p1m]\right) \quad (6)$$

where p_{1p1m} is the probability that one of the nodes change their state (either the paused node starts moving or the moving node pauses) before they go out of the range of each other, p_{21} is the probability that the moving node pauses before the paused node starts moving and $p_{22} = 1 - p_{21}$, and p_{2m} is the probability the one of the nodes pause before they go out of the range of each other.

Proof: We will derive each of the equations one by one.

(4) $E[et \mid 1p1m]$ is the additional time two nodes remain in contact when only one node is paused and the other node is moving. Either of the following can happen in the succeeding time slots:
   (a) The two nodes move out of the range of each other without either of the nodes changing states: The expected distance the node travels before going out of range is $0.6366K$. (This is the expected length from a point anywhere on chord AB in Figure 9 to point B.) The expected relative speed is $\overline{v}$. Hence, the expected duration the two nodes remain in contact is $\frac{0.6366K}{\overline{v}}$.
   (b) The moving node pauses before moving out of range. The additional time both nodes spend within range of each other is $E[et \mid 2p]$.
   (c) The paused node starts moving before the nodes move out of range. The additional time both nodes spend within range of each other is $E[et \mid 2m]$.

   Let $p_{1p1m} = Pr[$ one of the node changes its state $]$ and let $E[s]$ be the distance travelled by the node before one of them changes state .

   The movement duration is exponentially distributed with mean $\frac{1}{T}$ and the pause duration is exponentially distributed with mean $\frac{1}{\overline{T}_{stop}}$. Hence, $p_{1p1m} \approx \int_0^{\pi}\int_0^{2K sin(\phi)} \frac{1}{2\pi K sin(\phi)}\left(1 - e^{\left(\frac{-\lambda x}{\overline{v}}\right)}\right) dx d\phi$ where $\lambda = \frac{1}{T} + \frac{1}{\overline{T}_{stop}}$. The probability that the moving node pauses first $= p_{21} = \frac{\frac{1}{T}}{\frac{1}{T}+\frac{1}{\overline{T}_{stop}}}$.

   Now we figure out $E[s]$ which is the distance travelled before one of the nodes changed states. $E[s] = E[$distance between points C and D on the chord in Figure 9 $]$ $= \int_0^{\pi}\int_0^{2K sin(\phi)}\int_0^{x} \frac{s-x}{4\pi K^2(sin(\phi))^2} ds dx d\phi = \frac{4K}{3\pi}$. Hence, the expected time spent before one of the two nodes change their states $= \frac{4K}{3\pi\overline{v}}$.

(5) $E[et \mid 2p]$ is the additional time two nodes remain in contact when both the nodes are paused. The expected time before one of the nodes starts moving is $\overline{T}_{stop}/2$ as the paused duration is exponentially distributed with mean $\frac{1}{\overline{T}_{stop}}$. The additional time both the nodes spent within range is equal to $E[et \mid 1p1m]$.

(6) $E[et \mid 2m]$ is the additional time two nodes remain in contact when both the nodes are moving. Either of the following can happen in the succeeding time slots:
   (a) The two nodes move out of the range of each other without either of the nodes pausing. The expected duration the two nodes remain in contact is $\frac{0.6366K}{1.27\overline{v}}$. (The derivation is similar to the one for 4 (a).)
   (b) One of the two nodes pause before moving out of the range of each other. The expected time spent before one of the node pauses = $\frac{E[s]}{1.27\overline{v}}$. The additional time both nodes spend within range of each other is $E[et \mid 1p1m]$.

   The probability that one of the nodes pause before moving out of the range of each other $= p_{2m} \approx \int_0^{\pi}\int_0^{2K sin(\phi)} \frac{1}{2\pi K sin(\phi)}\left(1 - e^{\left(\frac{-2x}{1.27T\overline{v}}\right)}\right) dx d\phi.$

$\square$

The set of linear equation in Lemma 1.3 can be solved to get $E[et \mid 1p1m]$, $E[et \mid 2p]$ and $E[et \mid 2m]$.

B. Expected Contact Time for the Random Waypoint Mobility model

Lemma 1.4: Let $p = Pr[$node A pauses within the transmission range of node $B \mid$ node A is passing through the transmission range of node $B]$. Then $p = \frac{\frac{\pi K^2}{N}}{\frac{\pi K^2}{N}+p_{r1}+p_{r2}}$ where $p_{r1} = \frac{1}{N}\int_K^{\frac{\sqrt{N}}{2}} \frac{2\pi l}{N}\int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2r sin^{-1}\left(\frac{K}{l}\right) dr dl$ and $p_{r2} = \frac{1}{N}\int_{\frac{\sqrt{N}}{2}}^{\frac{\sqrt{N}}{\sqrt{2}}} \frac{4l}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2l}\right)\right)\int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2r sin^{-1}\left(\frac{K}{l}\right) dr dl$. $p_{r1} + p_{r2}$ is the probability that node A will pass through the transmission range of node B but not pause within node B's transmission range.

Proof: Let node A start from point X shown in Figure 10. XY_1 and XY_2 are tangents from point X to the circle denoting the transmission range of node B. If the destination of node A falls within the marked region, then node A will pass within transmission radius of node B. but not pause within node B's transmission range. l denotes the distance between point X and node B. The pdf of the random variable L is given in the proof of Lemma 5.1(b). The angle θ shown in Figure 10

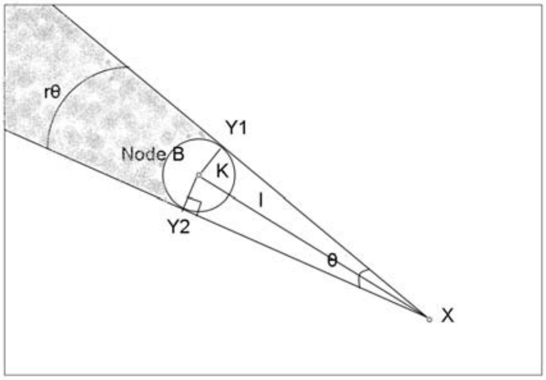

Fig. 10. Node A will pass through the transmission range of node B if and only if its destination lies in the shaded region.

is equal to $2sin^{-1}\left(\frac{K}{l}\right)$. The sector of a circle at a distance r from node X and lying within the shaded region is equal to $r\theta$. Integrating over all possible values of r and l will give the probability that the chosen destination lies in the shaded region. p_{r1} and p_{r2} represent the probabilities when $0 \leq l \leq \frac{\sqrt{N}}{2}$ and $\frac{\sqrt{N}}{2} \leq l \leq \frac{\sqrt{N}}{\sqrt{2}}$ respectively. □

Lemma 1.5: The expected contact time given both nodes were moving when they came within range of each other, $E[\tau^1_{rwp}]$, is given by,

$$E[\tau^1_{rwp}] = (1-p)^2 \frac{4K}{\pi 1.27\overline{v}} + (2p-p^2)\left(\frac{0.6366K}{1.27\overline{v}} + E[et \mid 1p1m]\right)$$

where $E[et \mid 1p1m]$ is the expected additional time the two nodes remain within range after one of the nodes paused. We will find its value in Lemma 1.7.

Proof: The proof runs along similar lines as the proof of Lemma 1.1. The only difference is that the probability that none of the nodes pause while within range of each other is equal to $(1-p)^2$. □

The next lemma evaluates the expected contact time when only one node was moving when the nodes came within range of each other.

Lemma 1.6: The expected contact time given only one of the nodes was moving when they came within range of each other, $E[\tau^2_{rwp}]$, is given by,

$$E[\tau^2_{rwp}] = (1-p)(1-p_1)\frac{4K}{\pi\overline{v}} + p_1\left(\frac{0.6366K}{\overline{v}} + E[et \mid 2m]\right) + p(1-p_1)\left(\frac{0.6366K}{\overline{v}} + E[et \mid 2p]\right),$$

where p is the probability that the moving node pauses while within range and $p_1 \approx \int_0^{\pi}\int_0^{2K sin(\phi)} \frac{1}{2\pi K sin(\phi)}\left(1 - e^{\left(\frac{-x}{\overline{T}_{stop}\overline{v}}\right)}\right) dx d\phi$ is the probability that the paused node starts moving again before both the nodes move out of each other's range. $E[et \mid 2p]$ and $E[et \mid 2m]$ are the expected additional times the two nodes remain within range after both of them are paused and after both of them start moving respectively. We will find their value in Lemma 1.7.

Proof: The proof runs along similar lines as the proof of Lemma 1.2. Only the probabilities of nodes changing states while within range of each other are different. The value of p was derived in Lemma 1.4. Now, we derive the value of p_1 to complete the proof. The moving node enters along chord AB as shown in Figure 9 and let the paused node restarts when the moving node is at C. C is equally likely to be any point on the chord AB. Given the value of x and ϕ, the probability that the paused node restarts between x and $x + dx$ is equal to $1 - e^{\left(\frac{-x}{\overline{T}_{stop}\overline{v}}\right)}$. Multiplying by the pdf's of x and ϕ and integrating gives the result. □

Next we find the values of $E[et \mid 1p1m]$, $E[et \mid 2p]$ and $E[et \mid 2m]$.

Lemma 1.7: $E[et \mid 1p1m]$, $E[et \mid 2p]$ and $E[et \mid 2m]$ are related to each other through the following set of linear equations:

$$E[et \mid 1p1m] = (1-p_1)(1-p_2)\frac{0.6366K}{\overline{v}} + p_2(1-p_1)\left(\frac{4K}{3\pi\overline{v}} + E[et \mid 2p]\right) + p_1\left(\frac{4K}{3\pi\overline{v}} + E[et \mid 2m]\right),$$

$$E[et \mid 2p] = \overline{T}_{stop}/2 + E[et \mid 1p1m].$$

$$E[et \mid 2m] = (1-p_2)^2\frac{0.6366K}{1.27\overline{v}} + (2p_2 - p_2^2)\left(\frac{4K}{3\pi 1.27\overline{v}} + E[et \mid 1p1m]\right)$$

where p_1 is the probability that the paused node starts moving again before both the nodes move out of each other's range and is the same as derived Lemma 1.6, p_2 is the probability that the moving node chooses a destination within the transmission range of the paused node and is equal to $\frac{\pi K^2}{N}$.

Proof: The proof runs along similar lines as the proof of Lemma 1.3. □