Abstract—In this paper, we characterize the achievable rate region for any 802.11-scheduled static multi-hop network. To do so, we first characterize the achievable edge-rate region, that is, the set of edge rates that are achievable on the given topology. This requires a careful consideration of the inter-dependence among edges, since neighboring edges collide with and affect the idle time perceived by the edge under study. We approach this problem in two steps. First, we consider two-edge topologies and study the fundamental ways by which they interact. Then, we consider arbitrary multi-hop topologies, compute the effect that each neighboring edge has on the edge under study in isolation, and combine to get the aggregate effect. We then use the characterization of the achievable edge-rate region to characterize the achievable rate region. We verify the accuracy of our analysis by comparing the achievable rate region derived from simulations with the one derived analytically. We make a couple of interesting and somewhat surprising observations while deriving the rate regions. First, the achievable rate region with 802.11 scheduling is not necessarily convex. Second, the performance of 802.11 is surprisingly good. For example, in all the topologies used for model verification, the max-min allocation under 802.11 is at least 64% of the max-min allocation under a perfect scheduler.

I. INTRODUCTION

A central question in the study of multi-hop networks is the following: Given an arbitrary multi-hop topology and a collection of source-destination pairs, what is the achievable rate region of this arbitrary multi-hop network? Researchers have formulated a multi-commodity flow problem to answer this question [1, 2]. These papers assume optimal scheduling with different interference models at the MAC layer in their formulations. However, the MAC protocol used in all the multi-hop networks being deployed is IEEE 802.11, see, for example, [3–6]. Characterizing the achievable rate region of an arbitrary multi-hop network with 802.11 scheduling is still an open problem and is the focus of this work. This characterization will have several applications. For example, it will allow researchers who propose new rate control or routing protocols for multi-hop networks with 802.11 scheduling to compare the performance of their scheme with the optimal value.

Setting up a multi-commodity flow formulation for 802.11-scheduled multi-hop networks runs into the following problem: What is the achievable edge-rate region of the given multi-hop topology? The achievable edge-rate region is the region characterizing the set of edge rates achievable on the given multi-hop topology. For example, for a wireline network, this region is simply characterized by the constraint that the sum of flow rates at each edge is less than the data rate of the edge. For a multi-hop network with optimal scheduling, this region is characterized using independent sets [1]. Characterizing this region is the main missing step in the characterization of the achievable rate region for 802.11-scheduled multi-hop networks.

Related Work: There is a large body of interesting work on modeling the behavior of IEEE 802.11 in a multi-hop network. This work can be subdivided into five broad categories. (i) [7, 8] present a detailed analysis for specific topologies under study (like the flow in the middle topology or the chain topology), but their methodology cannot be applied to any arbitrary topology. (ii) [9–11] propose a methodology independent of the topology at hand, but in order to keep the analysis tractable, they simplify the operation of the 802.11 protocol. In particular, they ignore lack of coordination problems due to topology asymmetries, and/or certain aspects of the protocol like the binary exponential backoff mechanism. (iii) [12–14] focus on modeling and analyzing interference at the physical layer. To eliminate MAC issues which complicate the analysis without affecting the physical layer model, they assume that all transmitters are within range of each other, and ignore certain aspects of the 802.11 protocol like the binary exponential backoff mechanism and ACK packets. Our work is complementary to papers of this category. We use a simplified physical layer model but a complete model for 802.11 MAC layer with no assumption on the topology at hand. (Section VII discusses how the more sophisticated physical layer model proposed by papers of this category can be incorporated with the MAC layer analysis presented in this paper.) (iv) [15–17] are perhaps the closest to our work. They present a general methodology without making any simplifications to the 802.11 protocol. But their methodology cannot be applied to nodes that have multiple receivers and hence, cannot be used to study any arbitrary multi-hop topology. Further, these papers do not incorporate all the possible dependencies which can exist between both neighboring and non-neighboring edges. (v) [18] proposed a complete model to derive the one-hop throughput for 802.11 in multi-hop topologies. This model is more accurate than the previous ones because it uses a Markov chain to capture the complete network state in each of its states. However, the Markov chain has an exponential number of states which precludes the model’s use for any decent sized network. (For example, a typical 20 node network will require constructing and solving a Markov chain with more than 500000 states.) To summarize, an accurate, general and scalable method to characterize the achievable edge-rate region for an 802.11-scheduled multi-hop network is still missing.

Our Contributions: The main contribution of this work is to characterize the achievable edge-rate region for any given multi-hop topology in a scalable manner. We adopt the following methodology to characterize this region. We first find the expected service time at a particular edge in terms of the collision probability at the receiver and the idle time perceived by the transmitter of that edge. The hard part in the procedure is to find these collision probabilities and idle times because their value depends...
on the edge-rates at other edges in the network. To find the value of these variables, we decompose the local network topology into a number of two-edge topologies, derive the value of these variables for these two-edge topologies and then appropriately combine them. Finding the expected service time at each edge allows us to characterize the achievable edge-rate region. It is important to note that this “decompose and combine” approach that we follow provides an intuitive precise description of how neighboring nodes of a multi-hop wireless network affect each other under a random scheduler like 802.11.

We use the characterization of the achievable edge-rate region to characterize the achievable flow-rate region\(^1\) for any multi-hop network and a collection of source-destination pairs. We then verify the accuracy of our analysis by comparing the achievable flow-rate region derived by simulations to the one derived by analysis for different topologies. We make a couple of interesting observations from these achievable flow-rate regions. First, the achievable flow-rate region for an 802.11-scheduled multi-hop network is not necessarily convex. Second, for all the topologies studied in this paper, the max-min rate allocation under 802.11 is at least 64% of the max-min allocation under a perfect scheduler.

The outline of the rest of this paper is as follows: First, we introduce the network model and the simulation setup in Section II. Then, Sections III and IV describe the methodology to characterize the achievable edge-rate and flow-rate region respectively for any multi-hop topology and a collection of source-destination pairs. Section V verifies the accuracy of the model by comparing achievable rate regions derived theoretically and via simulations. Section VI discusses some approximations that allow to solve the coupled system of multivariate equations derived in Section III without an iterative procedure. Section VII discusses some extensions of the analytical methodology. Finally, Section VIII concludes and discusses some future directions.

### II. Preliminaries

#### A. Network Model

We assume that the static multi-hop topology is given as an input. An edge between two nodes implies that the two nodes interfere with each other (irrespective of whether they can hear each other’s transmission successfully or not). Thus, the input topology is defined by the interference graph \( G = (V, E) \) where \( V \) is the set of all nodes and \( E \) is the set of all edges. The interference is assumed to be binary, that is, a transmission emanating from one of these interfering nodes will always cause a collision at the other node, and pairwise, that is, interference happens between these node pairs only. This interference model neglects some physical layer issues like the capture effect [13] and the effect of multiple interferers [19]. However, to understand the behavior of the 802.11 MAC and derive the achievable rate region associated with 802.11 MAC layer without making any simplifications in the protocol, we purposely neglect these physical layer issues. Their absence is not altering critical MAC properties, while their inclusion would unnecessarily complicate the analysis. Note that in Section VII, we discuss how to remove the binary and pairwise assumptions on interference.

In the absence of a collision, a transmission may get lost due to physical layer imperfections like fading, hardware noise etc. Successful reception of the RTS, CTS, DATA and ACK packets transmitted on some edge \( e \in E \) in absence of collisions are modeled as Bernoulli random variables with success probability equal to \( p_{\text{RTS}}, p_{\text{CTS}}, p_{\text{DATA}} \) and \( p_{\text{ACK}} \) respectively. (Note that if two nodes are within each other’s interference range but outside each other’s transmission range, then these probabilities are equal to 0.)

We assume that the set of flows \( F \) is also given as an input. Each flow \( f \in F \) is represented by a source-destination pair. Let \( s(f) \) denote the source and \( d(f) \) denote the destination for flow \( f \). We assume that the arrival process for each flow \( f \) has i.i.d. (independent and identically distributed) inter-arrival times, and a long term rate equal to \( T_f \). We also assume independence between the arrival process for different flows.\(^2\) and denote the edge rate (sum of the flow rates at the edge) induced by these flows on edge \( e \) by \( \lambda_e \). A given set of edge rates \( \Lambda_E = \{ \lambda_e : e \in E \} \) is said to be achievable if the input rate at each queue in the network is less than the service rate at that queue. Then, a given set of end-to-end flow rates is said to be achievable if there exists a routing (multiple paths per flow are possible) such that the induced set of edge-rates is achievable. The achievable edge-rate and flow-rate regions are then defined as the closures of the corresponding achievable sets of rates.

We assume that each node is running IEEE 802.11 with RTS/CTS at the MAC layer. (We assume RTS/CTS because its use is suggested by the 802.11 standard and we do not want to ignore any part of the protocol.) Let \( W_0 \) and \( m \) denote the initial backoff window and the number of exponential backoff windows respectively. We assume that the basic time unit is equal to one backoff slot time. Let \( T_{\text{RTS}}, T_{\text{CTS}}, T_{\text{DATA}} \) and \( T_{\text{ACK}} \) denote the time taken to transmit one RTS, CTS, DATA and ACK packet respectively. (Note that the DATA packet includes the UDP, IP, MAC and PHY headers along with the payload.) We also assume that all packets are of the same size, so \( T_{\text{DATA}} \) is a constant. Let \( T_{\delta} \) denote the time wasted in an RTS collision and let \( T_\delta \) denote the time it takes to complete one packet transmission. Then, \( T_{\delta} = T_{\text{RTS}} + DIFS + \delta \) and \( T_{\delta} = T_{\text{RTS}} + SIFS + \delta \) where \( \delta \) is the propagation delay and \( DIFS \) and \( SIFS \) are IEEE 802.11 parameters.\(^3\)

We will be making the following two assumptions throughout the paper to simplify the analysis.

**Assumption 1:** First, we assume \( T_{\text{RTS}} \ll T_\delta \) and \( T_{\text{CTS}} \ll T_\delta \).

\(^2\)Since we assume independent inter-arrival times and independence between the arrival process for different flows, what we derive is a lower bound on the capacity region derived without any assumption on the arrival processes.

\(^3\)We do not provide a description of IEEE 802.11 protocol. Please refer to [20] for a detailed description of the protocol.

### TABLE I

<table>
<thead>
<tr>
<th>SYSTEM PARAMETERS USED TO OBTAIN NUMERICAL RESULTS</th>
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<tr>
<td>Packet Payload</td>
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<td>MAC Header</td>
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<td>PHY Header</td>
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\(^1\)Achievable flow-rate region is also referred to as the achievable rate region. Both these terms are used interchangeably in this paper.
The protocol description recommends the use of RTS/CTS only when the size of the DATA packet is much larger than the size of the RTS packets. This is in line with the fundamental principle that the load due to control packets should be a small fraction of the total load. Hence, this assumption is satisfied for normal protocol operation.

Assumption 2: Second, we assume that $W_0 \gg 1$. Default 802.11 parameters satisfy this assumption. In general, choosing a small value for $W_0$ will not properly regulate random access to the channel, and will cause a lot of collisions and throughput loss even for WLAN’s. Hence, this assumption is also satisfied for normal protocol operation.

B. Simulation Setup

We use simulations to verify the accuracy of our analysis. We use Qualnet 4.0 as the simulation platform in this paper, since it has been shown to provide an accurate and realistic simulation environment [21]. All our simulations are conducted using an unmodified 802.11(b) MAC (DCF) with RTS/CTS. We use default parameters of 802.11(b) (summarized in Table I) in Qualnet unless otherwise stated. Auto-rate adaptation at the MAC layer is turned off and the rate is fixed at 1Mbps. We set the buffer size and maximum retry limit in 802.11 (the number of retransmission attempts after which the packet is dropped) to a very large value to avoid packet losses. This allows us to generate theoretically derived capacity region, we simulate all possible combinations of flow rates with each flow rate varying from 0 to 1 Mbps in steps of 10 Kbps and plot the achieved output rate at the destination.

III. CHARACTERIZING THE ACHIEVABLE EDGE-RATE REGION

This section characterizes the achievable edge-rate region $\Lambda_E$ for any multi-hop topology.

A. Expected Service Time of an Edge

This section finds the expected service time of a particular edge (denoted by $e$) in a particular topology (denoted by $T$) by constructing and solving a Markov chain (MC) for this edge. The states of this MC describe the current backoff window, backoff counter, and time since the last successful/unsuccessful RTS/CTS exchange (see next paragraph for details). The transition probabilities of this MC for $e$ depend on the collision probabilities at the receiver of $e$, which, in turn, depend on the exact state at the other edges in the network. In order to decouple the MCs and reduce the state space, we find the average value of the collision probabilities by averaging over all possible events which can cause a collision at the receiver. (Note that these events are not independent.) The dependence between the different edges, and, consequently, MCs, is captured via these average probabilities.

Prior work on the analysis of 802.11 has also attempted to reduce the state space of a MC describing the backoff window and counter values. For single-hop networks, the author in [22] assumed node homogeneity and independence, an approach which has been justified rigorously recently [23]. In the context of multi-hop networks [15, 17] a somewhat similar approach to ours has been used, but not all events leading to collisions have been considered, and these events have been assumed to be independent. Later sections describe how to find the value of these average collision probabilities, here we focus on finding the expected service time assuming these probabilities are given.

![Fig. 1. The Markov chain representing the evolution of a transmitter's state.](image)

The evolution of the 802.11 MAC layer state at the transmitter of edge $e$ after receiving a packet from the network layer is represented by the absorbing MC shown in Figure 1. The MC starts from the state START (which represents a packet entering the MAC layer to be scheduled for transmission) and ends in the state DONE (which represents the end of a successful packet transmission). The expected service time at $e$ is equal to the expected time it takes for the MC to reach DONE from START. The state $(j, W_i), 0 \leq j \leq W_i, 0 \leq i \leq m$, represents the transmitter state where the backoff window is equal to $W_i$ and the backoff counter is equal to $j$. The backoff counter keeps decrementing till it expires (reaches state $(0, W_i)$) which is then followed by a transmission attempt. The transmitter first attempts an RTS-CTS exchange, which fails with probability $p_{i, T}$. (Thus, $p_{i, T}$ denotes the probability that the RTS-CTS exchange at edge $e$ in topology $T$ is unsuccessful given that either the RTS/CTS exchange or the DATA/ACK exchange was unsuccessful in the previous $i$ transmission attempts.) The states $(C_k, W_i), 1 \leq k \leq T_c$ represent an unsuccessful RTS/CTS exchange $k$ time-units before, while the states $(T_k, W_i), 1 \leq k \leq T_c$ represent a successful RTS-CTS exchange $k$ time-units before, followed by the DATA-ACK exchange which fails with probability $p_{i, T}$. (Thus, $p_{i, T}$ denotes the probability that the DATA-ACK exchange is unsuccessful given that the RTS-CTS exchange was successful, and either the RTS/CTS exchange or the DATA/ACK exchange was unsuccessful in the previous $i$ transmission attempts.) If the DATA-ACK exchange is successful, the MC moves to the state DONE. If either the RTS/CTS or the DATA/ACK exchange is unsuccessful, the backoff window is set to $W_{i+1}$ if $i < m$, and to $W_m$ if $i = m$, and the backoff counter is chosen uniformly at random in between 0 and the new backoff window value and the
MC jumps to the corresponding state.

Note that $p_{e_i}^{c,T}$ and $p_{t,r}^{c,T}$ depend on $i$ which denotes the number of successive transmission failures. Since the probability that there are more than $m + 1$ successive transmission failures is small for the default values of 802.11, we approximate $p_{c,i}^{e,T}$ and $p_{t,r}^{e,T}$ for $i > m$ by $p_{c,m}^{e,T}$ and $p_{t,r}^{e,T}$. In case one decides to not use the default parameters of 802.11 and set $m$ to a smaller value, then one can introduce additional states in the MC till some value $m' > m$ such that the probability of $m' + 1$ successive transmission failures is small.

This MC does not capture the duration of time the backoff counter may get frozen due to another transmission within the transmitter’s neighborhood (due to the physical/virtual carrier sensing mechanism of the 802.11 protocol). To capture this, let $p_{idle}^{e,T}$ denote the proportion of time the channel around the transmitter of edge $e$ is idle conditioned on the event there is no successful transmission ongoing at $e$. We now use the MC to derive the expected service time at edge $e$ (denoted by $E[S_{e_i}]$) in Equation (1) in terms of the collision and idle probabilities. For ease of presentation, we define the following two additional variables: Let $E[T^c_{W_i}]$ and $E[T^s_{W_i}]$ for $1 \leq i \leq m$ denote the additional time required to reach the start of a successful packet transmission given that the backoff window just got incremented to $W_i$ due to an unsuccessful RTS-CTS and DATA-ACK exchange respectively.

$$E[T^c_{W_i}] = T_e + \frac{W_i+1}{2^{\pi_i}} + p_{c,i}^{e,T} E[T^c_{W_{n_i}}] + \left(1 - p_{c,i}^{e,T}\right) p_{t,i}^{e,T} E[T^c_{W_{n_i}}]$$

$$E[T^s_{W_i}] = T_s + \frac{W_i+1}{2^{\pi_i}} + p_{s,i}^{e,T} E[T^s_{W_{n_i}}] + \left(1 - p_{s,i}^{e,T}\right) p_{t,i}^{e,T} E[T^s_{W_{n_i}}]$$

$$E[S_{e_i}] = T_s + \frac{W_i+1}{2^{\pi_i}} + p_{c,i}^{e,T} E[T^c_{W_i}] + \left(1 - p_{c,i}^{e,T}\right) p_{t,i}^{e,T} E[T^s_{W_i}]$$

where $n_i = \begin{cases} 1 & \text{if } 1 \leq i \leq m-1,  \\ m & \text{if } i = m. \end{cases}$

To derive the value of the expected service time at a particular edge $e$ using Equation (1), one has to first find the value of $p_{c,i}^{e,T}$, $p_{s,i}^{e,T}$ and $p_{idle}^{e,T}$ for that edge. The next two sections describe how to find the value of these variables for any edge in a given multi-hop topology.

Note that we have neglected the effect of post-backoff in this MC. (Post-backoff refers to backing off right after the transmission of the last packet in the queue, in anticipation of a future packet for which there will be no backoff if post-backoff has completed in the meantime.) Since we are interested in determining the boundary of the capacity region, this will have a negligible impact on the accuracy. This is because the boundary of the capacity region depends on the service rate of the backlogged edges, such edges are almost always busy and don’t post-backoff, and their dependence on non-backlogged edges is nearly unaffected by the post-backoff taking place in these non-backlogged edges.

B. Derivation of Collision and Idle Probabilities for Two-Edge Topologies

This section finds the collision and idle probabilities for all possible two-edge topologies. These two-edge topologies reveal the types of inter-dependence which can exist between two edges in a multi-hop network and an analysis for these topologies will serve as the building block for the analysis of more complex topologies as will be seen in the next section. [15] identified four different categories of two-edge topologies which can exist in a given multi-hop network and analyzed them to study unfairness in 802.11 networks. Here we derive the achievable edge-rate region for these topologies. (This list is exhaustive, that is, all possible two-edge topologies belong to one of these four categories.)

We use the following notation throughout this section: $e_1$ and $e_2$ denote the two edges under consideration, and $\lambda_{e_1}$, $j = 1, 2$, denote the edge rates (in packets/time unit). Further, let $T_{e_1}$ and $R_{e_1}$, $j = 1, 2$, denote the transmitter and the receiver of the two edges. Finally, let $E_{RTS}$ and $E_{CTS}$, $t, r \in \{T_{e_1}, T_{e_2}, R_{e_1}, R_{e_2}\}$, denote the event that the RTS and the CTS packet transmitted by node $t$ is not correctly received at node $r$ due to physical layer errors respectively. For example, $E_{RTS}$ denotes the event that the CTS transmitted by $R_{e_1}$ is not correctly received at $T_{e_2}$ due to physical layer errors.

1) Coordinated Stations (CoS): A two-edge topology is a coordinated station topology if $T_{e_1}$ and $T_{e_2}$ interfere with each other. Figure 2(a) shows an example of a coordinated station topology.

We first state the value of $p_{c,i}^{e,CoS}$ in the following lemma.

**Lemma 1:** $p_{c,i}^{e,CoS} = 1 - (p_{RTS}^{e,T} \times p_{c,ACK}^{e,T})$, $0 \leq i \leq m, j = 1, 2$.

**Proof:** For this topology, the RTS-CTS exchange will successfully avoid any DATA collision and the DATA-ACK exchange will be unsuccessful only when the DATA or the ACK packet gets corrupted due to physical layer effects.

We next derive the value of $p_{c,i}^{e,CoS}$ Note that the analysis presented in [22] can be directly applied for this topology to derive the value of $p_{c,i}^{e,CoS}$ under saturation conditions (when transmitters always have a packet to send). The following lemma finds this probability for non-saturation conditions.

**Lemma 2:**
(i) $p_{c,i}^{e,CoS} = 1 - (p_{RTS}^{e,T} \times p_{c,ACK}^{e,T})(1 - \lambda_{e_2} E[S_{e_2}] p_{e_1}^{e,T})$, $0 \leq i \leq m$.
(ii) $p_{c,i}^{e,CoS} = 1 - (p_{RTS}^{e,T} \times p_{c,ACK}^{e,T})(1 - \lambda_{e_1} E[S_{e_1}] p_{e_2}^{e,T})$, $0 \leq i \leq m$.

where $\frac{2}{W_{m+1}} \leq p_{e_1}^{e,T} \leq \frac{2}{W_{m+1}}$ is the probability that the backoff counter at edge $e$ is equal to 0.

**Proof:** We first look at edge $e_1$. The RTS/CTS exchange is unsuccessful if either the RTS or the CTS is lost due to physical layer errors or an RTS collision happens at $R_{e_1}$. An RTS collision will occur only if the backoff counter at edge $e_2$ also expires in the same slot duration resulting in both $T_{e_1}$ and $T_{e_2}$ sending an RTS packet. Thus, $p_{c,i}^{e,CoS} = P(e_2$ has a packet to send) $\times p_{e_2}^{e,T}$, (a) $P(e_2$ has a packet to send) $= \lambda_{e_2} E[S_{e_2}]$ as the probability that a queuing system is non empty is equal to $AE[S]$ where $\lambda$ is the packet arrival rate into the system and $E[S]$ is the expected service time. (b) As derived in [22], $p_{e_2}^{e,T}$ is upper bounded by $\frac{2}{W_{m+1}}$ and lower bounded by $\frac{2}{W_{m+1}}$. Putting everything together.

![Fig. 2. Different two-edge topologies: (a) Coordinated stations, (b) Near hidden edges, (c) Asymmetric topology, (d) Far hidden edges.](image-url)
yields the result. $p_{e_2,CoS}^{e_1}$ is derived using the same argument. 

Approximating $p_{w_0}^{e_1}$ by its upper bound is accurate when there are few collisions and data losses at the physical layer, otherwise approximating it with its lower bound will be more accurate. So we make the following approximation, $p_{w_0}^{e_1} = \begin{cases} \frac{2}{e^{w_0+t_1}} & \text{if } p_{e_0}^{e,CoS} \leq p_{\text{cut-off}} \\ \frac{2}{e^{w_0+t_1}} & \text{if } p_{e_0}^{e,CoS} > p_{\text{cut-off}} \end{cases}$ where $p_{\text{cut-off}}$ is the value of the DATA/ACK exchange loss probability which results in the lower and upper bound yielding the same error. (Its value for the default parameters of Table I is equal to 0.8.) This approximation is not introducing significant inaccuracies for the following reason. Assumption 2 implies that the probability of an RTS collision at some edge $e$ due to another edge with which it forms a coordinated stations topology is rather small (since the upper bound is small). On the other hand, the probability of RTS collisions due to edges with which $e$ forms an asymmetric or far hidden edges topology (Sections III-B.3 and III-B.4) is much larger, and it dominates the calculation of the overall RTS collision probability. Finally, if there are only coordinated stations in $e$’s neighborhood, the effect of the backoff counter being frozen due to carrier sensing will dominate over RTS collisions (see Equation (1)). Section V verifies that making this approximation has no significant impact on the accuracy of the analysis.

Finally, we derive the value of $p_{\text{idle}}^{e_1,CoS}$ in the next lemma. We use the following variable in this derivation. Let $K_{e,T}$ denote the expected number of DATA transmissions per packet at edge $e$ in topology $T$ including the extra transmissions due to unsuccessful DATA-ACK exchange. Using elementary probability, $K_{e,T} = \sum_{i=1}^{m-1} i \left(1 - p_{l,i}^{e,T} \right) \left(\prod_{k=1}^{i-1} p_{l,k}^{e,T} \right) + \left(\prod_{i=1}^{m-1} p_{l,i}^{e,T} \right) \left(m - 1 + \frac{1}{1 - p_{l,m}^{e,T}} \right)$.

**Lemma 3:** (i) $p_{\text{idle}}^{e_1,CoS} = 1 - K_{e_2,CoS}^{e_1} \lambda_{e_1} T_s - \lambda_{e_1} T_s$.
(ii) $p_{\text{idle}}^{e_2,CoS} = 1 - K_{e_2,CoS}^{e_1} \lambda_{e_2} T_s - \lambda_{e_2} T_s$.

**Proof:** The backoff counter for edge $e_1$ is frozen when a transmission at edge $e_2$ is going on given that no successful transmission is going on at edge $e_1$. The net rate at which packets are transmitted at edge $e_2$ is equal to $K_{e_2,CoS}^{e_1} \lambda_{e_2}$ and $T_s$ is the expected service time of one packet. Hence, the probability that there is a transmission ongoing at edge $e_2$ is equal to $K_{e_2,CoS}^{e_1} \lambda_{e_2} T_s$. Notice that this derivation ignores the extra RTS-CTS traffic generated by an unsuccessful RTS-CTS exchange, but this is fully justified by the assumption that $T_{RSTS} \ll T_s$ (Assumption 1). Similarly, the probability that a successful packet transmission is going on at $e_1$ is equal to $\lambda_{e_1} T_s$. Putting everything together yields the result. $p_{\text{idle}}^{e_1,CoS}$ is derived using similar arguments.

Note that if there is no interference link between $T_{e_1}$ and $R_{e_2}$ in Figure 2(a), then the probability of RTS collision at $e_2$ will be equal to $0$ instead of $\lambda_{e_2} E[S_{e_2}] p_{w_0}^{e_2}$. Similarly, absence of the interference link between $T_{e_2}$ and $R_{e_1}$ will result in the probability of RTS collision at $e_1$ to be equal to $0$.

2) Near Hidden Edges (NH): Figure 2(b) shows the topology belonging to this category. $T_{e_1}$ and $T_{e_2}$ do not interfere with each other, however, there is an interference link between $T_{e_1}$ and $R_{e_2}$ as well as $T_{e_2}$ and $R_{e_1}$. The values of $p_{e_1,NH}^{e_1}, p_{e_1,NH}^{e_2}$ and $p_{\text{idle}}^{e_1,NH}$, $0 \leq i \leq m, j = 1, 2$, are derived in a manner similar to the derivation of the corresponding probabilities for coordinated stations. The only difference is that now $T_{e_1}$ ($T_{e_2}$) will freeze its backoff counter only when a CTS sent from $R_{e_2}$ ($R_{e_1}$) is successfully received at $T_{e_1}$ ($T_{e_2}$). So, the RTS transmitted by $T_{e_1}$ ($T_{e_2}$) can now collide in the following four scenarios: (i) both $T_{e_1}$ and $T_{e_2}$ start transmitting an RTS in the same slot duration, (ii) $T_{e_1}$ ($T_{e_2}$) starts transmitting an RTS and $R_{e_2}$ ($R_{e_1}$) starts transmitting a CTS in the same slot duration, (iii) $T_{e_1}$ ($T_{e_2}$) starts transmitting an RTS while $T_{e_2}$ ($T_{e_1}$) is still sending an RTS, and (iv) the CTS from $R_{e_2}$ ($R_{e_1}$) is lost due to physical layer errors at $T_{e_1}$ ($T_{e_2}$).

3) Asymmetric Topology (AS): Figure 2(c) shows an example of the topology belonging to this category. $T_{e_1}$ and $T_{e_2}$ as well as $T_{e_1}$ and $R_{e_2}$ do not interfere each other, but $T_{e_2}$ and $R_{e_1}$ are within each other’s interference range. The main characteristic of this topology is that $T_{e_2}$ is aware of the channel state as it can hear the CTS from $R_{e_1}$, but $T_{e_1}$ is totally unaware of the channel state as it can hear neither the RTS nor the CTS from the transmission on $e_2$.

We first derive the collision and idle probabilities for edge $e_1$.

The following lemma derives the value of $p_{e_1,AS}^{e_1}$.

**Lemma 4:** $p_{e_1,AS}^{e_1} = 1 - \left( p_{\text{DAT_A}}^{e_1} \times p_{\text{ACK}}^{e_1} \right) \left(1 - p_{\text{CDMA}}^{e_1} \lambda_{e_2} E[S_{e_2}] \right) - \left( p_{\text{DAT_A}}^{e_1} \times p_{\text{CTS}}^{e_1} \right) \left(1 - p_{\text{CDMA}}^{e_1} \lambda_{e_2} E[S_{e_2}] \right) - 1 - p_{\text{RTS/CTS}}^{e_1} (K_{e_2,AS}^{e_1} \lambda_{e_2} T_s)$. $0 \leq i \leq m$. The expression for $K_{e,T}$ was derived in Section III-B.1.

**Proof:** The DATA packet send by $T_{e_1}$ will collide if one of following two events happen: (i) If $T_{e_2}$ starts transmitting an RTS and $R_{e_2}$ starts transmitting a CTS in the same slot duration. (ii) The CTS from $R_{e_1}$ is not recovered at $T_{e_2}$ due to physical layer errors, and $T_{e_2}$ starts a transmission as it is not aware of the ongoing transmission at $e_1$.

We next derive the value of $p_{e_1,AS}^{e_1}$ in the following sequence of lemmas. The first lemma directly follows from the following observation: if $T_{e_1}$ transmits an RTS while a transmission at edge $e_2$ is going on, it will collide. As before, note that this lemma ignores the extra RTS traffic generated at $e_2$ by an unsuccessful RTS-CTS exchange, which is not a problem since $T_{RSTS} \ll T_s$ (Assumption 1).

**Lemma 5:** $p_{\text{idle}}^{e_1,AS} = 1 - (p_{\text{RTS}}^{e_1} \times p_{\text{RTS}}^{e_1} (1 - K_{e_2,AS} \lambda_{e_2} T_s))$.

Now, let’s look at what happens after the first RTS collision. The RTS collision will cause the backoff window at $T_{e_1}$ to increase to $W_1$ and a new backoff counter is chosen uniformly at random between $(0, W_1)$. If the remaining transmission time at edge $e_2$ is more than the new backoff counter, the second transmission at $e_1$ will collide with the same transmission. (Note that multiple RTS exchanges on $e_1$ can collide with the same DATA transmission on $e_2$, see Figure 3.) And if the remaining transmission time at edge $e_2$ is lower than the new backoff counter, then the probability of RTS collision is equal to $K_{e_2,AS} \lambda_{e_2} T_s$. So, $P_{\text{RTS/CTS}}^{\text{ex}}$ is unsuccessful at the end of second backoff | a collision occurred at the end of the first backoff = $(1 - p_{0}^{1} + p_{0}^{1} p_{\text{idle}}^{e_1,AS})$, where $p_{0}^{1}$ is the probability that the transmission at $e_2$ which collided with the first RTS transmission by $T_{e_1}$ (when the backoff window at $T_{e_1}$ was $W_0$) ends before the second backoff counter at $T_{e_1}$ expires (when the backoff window at $T_{e_1}$ is $W_1$). To evaluate $p_{e_1,AS}^{e_1}$, note that the backoff window also increments if the first RTS-CTS exchange went through but the subsequent DATA or ACK
packet was lost, in which case the RTS collision probability after the second backoff counter expires is equal to \( K_{2, AS} \lambda_{e_2} T_s \).

Putting everything together yields \( p_{e_1, AS} = 1 - \left( p_{R_{RTS}} \times p_{CTS} \left( 1 - \left( 1 - p_{R_{RTS}} \right) p_{c,0} + p_{R_{RTS}} \left( 1 - p_{0} \right) + p_{0} p_{c,0} \right) \right) \), where

\[
p_{R_{RTS}} = \frac{K_{2, AS} \lambda_{e_2} T_s}{p_{c,0} \left( 1 - p_{c,0} \right) + 1 - p_{R_{RTS}}}
\]

is the probability that an RTS collision occurred at the end of the first backoff given that either the RTS/CTS exchange or the DATA/ACK exchange was unsuccessful at the end of the first backoff.

\[
p_{c,i} \text{, for } i \geq 1
\]

\[
\text{is the probability that an RTS collision occurred at the end of the (i + 1)th backoff given that either the RTS/CTS exchange or the DATA/ACK exchange was unsuccessful at the end of the (i + 1)th backoff. If there is no RTS collision at the end of the (i + 1)th backoff, then the probability of RTS/CTS exchange being unsuccessful at the end of the next backoff ((i + 2)th backoff) is equal to \( p_{e_0, AS} \). (b) Let \( p_{R_{RTS_{n=0,i}}} \), \( 0 \leq i \leq m \), denote the probability that an RTS collision occurred at the end of the (i + 1)th backoff given that (i) either the RTS/CTS exchange or the DATA/ACK exchange was unsuccessful at the end of the (i + 1)th backoff, and (ii) the collision occurred with a transmission on \( e_2 \) which started when the backoff window at \( T_{e_1} \) was \( W_i \), that is, the colliding transmission on \( e_2 \) started while the backoff counter at \( T_{e_1} \) was decremented during the (i + 1)th backoff. This probability indicates the start of a new transmission on \( e_2 \) which might collide with the subsequent RTS exchanges. (c) Let \( E_{j,i} \) denote the event that an RTS collision occurred at \( e_1 \) when the backoff window at \( T_{e_1} \) was \( W_i \), with a transmission on \( e_2 \) which had started when the backoff window at \( T_{e_1} \) was \( W_j \). This event indicates the start of the ongoing transmission on \( e_2 \). (d) Finally, let \( p_{j} \) denote the probability that a transmission on \( e_2 \), which started when the backoff window at \( T_{e_1} \) was \( W_j \), ends when the backoff window at \( T_{e_1} \) is \( W_i \) given that it had not ended when the backoff window was \( W_{i-1} \). This probability is used to count the number of RTS exchanges on \( e_1 \) which collide with the same transmission on \( e_2 \).

**Lemma 6:**

\[
p_{e_1, AS} = 1 - \left( p_{R_{RTS}} \times p_{CTS} \left( 1 - \left( 1 - p_{R_{RTS}} \right) p_{c,0} + p_{R_{RTS}} \left( 1 - p_{0} \right) + p_{0} p_{c,0} \right) \right)
\]

\[
- \sum_{j=0}^{i-1} \left( \prod_{j+1}^{i} \left( 1 - p_{j} \right) + p_{0} p_{c,0} \right) \right) \leq 1 \leq m.
\]

**Proof:** Given event \( E_{j,i-1} \) occurs, the probability that an RTS collision occurs when the backoff window at \( T_{e_1} \) is \( W_i \) is equal to \( \left( 1 - p_{j} + p_{c,0} \right) \). On the other hand if there is no RTS collision when the backoff window at \( T_{e_1} \) was \( W_{i-1} \), the probability of RTS collision when the backoff window at \( T_{e_1} \) is \( W_i \) is equal to \( p_{c,0} \). Combining everything together using the law of total probability yields the result. To complete the derivation of \( p_{e_1, AS} \), we state the values of \( P \left( E_{j,i}, p_{R_{RTS}}, p_{R_{RTS_{new}}} \right) \) and \( p_{j} \)'s in Appendix I.

The only remaining variable to be derived for edge \( e_1 \) is \( p_{e_1, AS} \). To derive its value, we use the fact that \( T_{e_1} \) cannot hear the transmission on \( e_2 \), and hence the channel at \( T_{e_1} \) is always idle.

**Lemma 7:** \( p_{e_1, AS} = 1 \).

The next lemma states the value of the collision and idle probabilities for edge \( e_2 \). The proof directly follows from the following two observations: (i) no transmission from \( e_1 \) can collide at \( T_{e_2} \), and (ii) a CTS transmission from \( e_1 \), if successfully received by \( T_{e_2} \), will free the backoff counter at \( T_{e_2} \) due to virtual carrier sensing.

**Lemma 8:**

(i) \( p_{e_2, AS} = 1 - \left( p_{R_{DATA}} \times p_{R_{ACK}} \right), 0 \leq i \leq m \),

(ii) \( p_{e_1, AS} = 1 - \left( p_{R_{RTS}} \times p_{R_{CTS}} \right), 0 \leq i \leq m \).

(iii) \( p_{idle} = \left( 1 - \left( 1 - p_{R_{CTS}} \right) T_e K_{1, AS} \lambda_{e_1, T_e} \right) \).

**For Hidden Edges (FH):** Only \( e_1 \) and \( e_2 \) are within each others’ range in this topology. Figure 2(d) shows the topology belonging to this category. For this topology, an RTS sent by a transmitter will not receive a CTS back if a transmission is going on at the other edge because of virtual carrier sensing at the receiver. Thus, \( p_{e_1, AS} = 1 \), \( 0 \leq i \leq m, j = 1, 2 \), is derived in a manner similar to the derivation of \( p_{e_1, AS} \). The only difference occurs when the CTS from \( T_{e_2} (R_{e_1}) \) is lost at \( T_{e_1} (R_{e_2}) \) causing \( T_{e_2} (R_{e_1}) \) to be unaware of the channel state at \( e_1 (e_2) \) and sending a CTS back in response to the RTS from \( T_{e_1} (T_{e_2}) \). Hence, the probability of RTS collision is equal to the probability that there is a transmission ongoing at the other edge conditioned on the event that the CTS was correctly received. The probability of the event that the CTS is not correctly received is derived during the derivation of \( p_{idle} \).

We next derive the value of the probability of DATA collisions. DATA on edge \( e_1 (e_2) \) will collide if \( R_{e_2} (R_{e_1}) \) transmits a CTS or an ACK. \( R_{e_2} (R_{e_1}) \) will send back a CTS only if it had not correctly received the CTS exchanged on \( e_1 (e_2) \). For this topology, DATA packets will not collide with ACK packets as the preceeding RTS/CTS exchange on the other edge will cause the DATA to collide, and hence the receiver will not send back an ACK packet. We now have to determine the events which can cause \( R_{e_2} (R_{e_1}) \) to not correctly receive the CTS exchanged on \( e_1 (e_2) \).
sequence of events following event $E_{1,FH}$. (a) The transmission of RTS on $e_1$ ($e_2$) will succeed and $R_{e_1}$ ($R_{e_2}$) will send back a CTS. This CTS will collide with the RTS transmission on $e_2$ ($e_1$) at $R_{e_2}$ ($R_{e_1}$). This collision results in $R_{e_2}$ ($R_{e_1}$) not receiving both the packets. (b) DATA transmission will commence on $e_1$ ($e_2$) while $T_{e_2}$ ($T_{e_1}$) backs off. (c) Backoff counter at $T_{e_2}$ ($T_{e_1}$) expires and an RTS is transmitted on $e_2$ ($e_1$), $R_{e_2}$ ($R_{e_1}$) responds back with a CTS. (d) If the DATA transmission on on $e_1$ ($e_2$) has not ended, the CTS transmission by $R_{e_2}$ ($R_{e_1}$) in step (c) will collide with the DATA transmission at $R_{e_1}$ ($R_{e_2}$). (e) $T_{e_3}$ ($T_{e_2}$) backs off and DATA transmission commences on $e_2$ ($e_1$). (f) The backoff counter at $T_{e_1}$ ($T_{e_2}$) expires and an RTS is transmitted on $e_2$ ($e_1$), $R_{e_2}$ ($R_{e_1}$) sends back a CTS. (g) If the DATA transmission on $e_2$ ($e_1$) has not ended, the CTS transmission by $R_{e_3}$ ($R_{e_2}$) will collide with the DATA transmission at $R_{e_3}$ ($R_{e_2}$). (h) This process goes on till at least one of the DATA packets get successfully exchanged.\(^5\) $p^i_{e_1,FH}$, $E_{1,FH}$ and $E_{2,FH}$ are similarly defined for edge $e_2$.

The value of $p^i_{e,FH}$, $0 \leq i \leq m$, is stated in the next lemma, whose proof follows directly from the discussion above. We define the following additional variables for ease of presentation.

(a) Let $P_{D,i}^e$ denote the probability that a DATA collision occurs on $e_i$ due to events $E_{1,FH}^e$ or $E_{2,FH}^e$ having occurred during previous exchanges, given the current backoff window at $T_{e_i}$ is $W_i$ and either the RTS/CTS or the DATA/ACK exchange was unsuccessful when the backoff window value at $T_{e_i}$ was $W_0, \ldots, W_{i-1}$. If the DATA/ACK loss does not occur due to events $E_{1,FH}^e$ or $E_{2,FH}^e$ having occurred during previous exchanges, the probability of DATA collision after the next backoff is equal to $p_{i,0}^{e,FH}$. (b) Let $P_{D_E,i}^e$ ($p^e_{D_E,i}$) denote the probability that event $E_{1,FH}^e$ ($E_{2,FH}^e$) occurs during the current data exchange given that the current backoff window at $T_{e_i}$ is $W_i$ and either the RTS/CTS or the DATA/ACK exchange was unsuccessful when the backoff window value at $T_{e_j}$ was $W_0, \ldots, W_{j-1}$. Event $E_{1,FH}^e$ ($E_{2,FH}^e$) may be followed with a sequence of DATA collisions.

Lemma 9: For $j = 1, 2$,

\[(i) p_{i,0}^{e,FH} = 1 - P_{D_A}^{e,e_i} P_{A}^{e,A} \left(1 - P_{D,C}^{e,e_i} P_{C}^{e,C} \right) \]

\[(ii) p_{i,j}^{e,FH} = 1 - \left( P_{D_A}^{e,e_i} P_{A}^{e,A} \right) \left(1 - P_{D,E}^{e,e_i} P_{E}^{e,E} \right) \]

\[
\begin{align*}
&= P_{D_E,i}^{e,e_i} \left( P_{c,u}^{e} + \left(1 - P_{c,u}^{e} \right) P_{i,u}^{e} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times P_{D_E,i_1}^{e_1,FH} \left( P_{c,u}^{e_1} + \left(1 - P_{c,u}^{e_1} \right) P_{i,u}^{e_1} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \sum_{k=0}^{i-1} \left( P_{D,E}^{e,k} P_{e,k}^{e,FH} \right) \left( P_{c,u}^{e} + \left(1 - P_{c,u}^{e} \right) P_{i,u}^{e} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \left( P_{D,E}^{e_1,FH} P_{e_1,FH} \right) \left( P_{c,u}^{e_1} + \left(1 - P_{c,u}^{e_1} \right) P_{i,u}^{e_1} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \left( P_{D,E}^{e_2,FH} P_{e_2,FH} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \left( P_{D,E}^{e,FH} P_{e,FH} \right) \left( P_{c,u}^{e} + \left(1 - P_{c,u}^{e} \right) P_{i,u}^{e} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \left( P_{D,E}^{e_1,FH} P_{e_1,FH} \right) \left( P_{c,u}^{e_1} + \left(1 - P_{c,u}^{e_1} \right) P_{i,u}^{e_1} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right) \\
&\quad \times \left( P_{D,E}^{e_2,FH} P_{e_2,FH} \right) \left( P_{c,u}^{e_2} + \left(1 - P_{c,u}^{e_2} \right) P_{i,u}^{e_2} \right), \quad 1 \leq i \leq n.
\end{align*}
\]

The values of $p_{i,j}^{e,FH}$ and $p_{D_E,i}^{e,e_i}$ are stated in Appendix II.

The only remaining variable to be derived is $P_{d,e}^{e,FH}$. To derive its value, we use the fact that both the transmitters cannot oversee each other.

Lemma 10: $P_{d,e}^{e,FH} = 1, j = 1, 2.$

C. Determining the Achievable Edge-Rate Region in any Multi-hop Topology

To determine the edge-rate region for a given multi-hop topology $T$, recall that we first have to determine the expected service time at each edge which in turn requires the values of $p_{e,T}^{e_1,FH}$, $p_{e,T}^{e_2,FH}$ and $p_{e,T}^{e_3,FH}$ for each edge $e$. To derive these probabilities for an edge, we will decompose the local topology around the edge into a number of two-edge topologies, then find these probabilities for each two-edge topology, and finally find the net probability by appropriately combining the individual probabilities from each two-edge topology. We will use the Flow in the Middle topology (Figure 6(a)) as an example throughout the section.

The algorithm to decompose the local topology around an edge is conceptually straightforward. For example, the local topology around edge $4 \rightarrow 5$ can be decomposed into the following two-edge topologies: (i) Coordinated Stations: $5 \rightarrow 6$, (ii) Near Hidden Edges: None, (iii) Asymmetric Topology: $2 \rightarrow 3$ and $8 \rightarrow 9$, and (iv) Far Hidden Edges: $1 \rightarrow 2$ and $7 \rightarrow 8$. The previous section discussed how to find the collision and idle probabilities for each individual two-edge topology. This section focusses on how to combine the probabilities obtained from each individual two-edge topology.

Combining these probabilities must account for possible dependencies between the neighboring edges. For example, the transmitters of edges $1 \rightarrow 2$ and $2 \rightarrow 3$ in the Flow in the Middle topology, which are both interfering with edge $4 \rightarrow 5$, can hear each other. Hence, DATA transmission on these two edges will not occur simultaneously. Thus, the collision probabilities due to these two edges cannot be combined independently to find the aggregate collision probabilities at $4 \rightarrow 5$.

We first present the scenarios where probabilities can be independently combined, and then discuss the scenarios where the dependencies have to be carefully accounted for. The RTS and DATA collision probabilities can be independently combined if they are caused by two (or more) transmitters / receivers starting transmission in the same slot duration. For example, the RTS collision probability due to coordinated stations, and the DATA collision probability due to asymmetric topologies (if the CTS is received correctly at the other edge) can be independently combined. (For a complete list of events which can be independently combined, see the discussion following Lemmas 12 and 13.)

When the computation of any probability (either collision or idle probabilities) depends on the probability of the event that
there is no ongoing transmission among a set of edges, $\mathcal{N}$, dependencies have to be carefully accounted for and combining probabilities is more involved. For example, the computation of the RTS collision probability due to far hidden edges and asymmetric topologies, and the computation of the DATA collision probability due to asymmetric topologies (if the CTS is not received correctly at the other edge) belong to this category. Also, the computation of the idle probability for coordinated stations, near hidden edges and asymmetric topologies belongs to this category. To understand how to compute the probability that there is no ongoing transmission among edges belonging to $\mathcal{N}$, it is helpful to distinguish between two type of dependencies which can exist between these edges.

Consider edge $4 \rightarrow 5$ in the Flow in the Middle topology (Figure 6(a)). In this topology, edges $1 \rightarrow 2$ and $2 \rightarrow 3$ interfere with edge $4 \rightarrow 5$ but do not interfere with each other, whereas $2 \rightarrow 3$ and $8 \rightarrow 9$ interfere with both $4 \rightarrow 5$ and each other. Generalizing, (i) if two edges interfere with each other, then they will not be simultaneously scheduled (ignoring the extra RTS traffic due to the event that a colliding RTS transmission is taking place on both the edges, which is easily justified by Assumption 1), and (ii) if two edges do not interfere with each other, then they can be independently scheduled given that none of the edges which interfere with both are transmitting. For example, edges $2 \rightarrow 3$ and $8 \rightarrow 9$ will be independently scheduled given there is no transmission ongoing at edges $4 \rightarrow 5$ and $5 \rightarrow 6$. We now state a lemma which finds the probability that there is an ongoing transmission on at least one of the edges in the given set $\mathcal{N}$. The lemma is derived using concepts from basic probability. In what follows, let $X_e$ denote the event that there is a transmission going on at edge $e$ and note that $P(X_e) = K_e T \lambda_e T_s$.

**Lemma 11:**

\[
P(\bigcup_{e \in \mathcal{N}} X_e) = \sum_{i \in \mathcal{N}} P(X_e) - \sum_{i,j \in \mathcal{N}} P(X_e \cap X_j) + \ldots + (-1)^{|\mathcal{N}| - 1} P(\bigcap_{i \in \mathcal{N}} X_i),
\]

where $\mathcal{N} \subseteq \mathcal{N}$, $P(\bigcap_{i \in \mathcal{N}} X_i) = 0$, if any two edges in $\mathcal{N}$ interfere with each other \(\{ (\prod_{i \in \mathcal{N}} P(X_i)) / (1 - P(\bigcap_{i \in \mathcal{N}} X_i)) \}^{|\mathcal{N}| - 1}\), otherwise where $S_{\mathcal{N}}$ denotes the set of edges in $E$ which interfere with all the edges in $\mathcal{N}$.

Based on the previous discussion, we can derive the collision and idle probability for each edge in a given multi-hop network. For completeness, we state the value of each probability in the next three lemmas. The individual expressions are large because we combine the effect of each two-edge topology. However, each term in the expression can be traced to a term derived for one of the two-edge topologies.

We first define the notation used in these lemmas. Denote by $\mathcal{N}^c$ the set of edges which interfere with the edge under study $e$. Any edge $e_{\in} \in E \setminus e$ which either forms a coordinated station or asymmetric topology or near hidden edge or far hidden edge with $e$ belongs to this set. We subdivide the edges in $\mathcal{N}^c$ into subsets corresponding to the four two-edge topologies, and the coordinated station topologies and asymmetric topologies are further subdivided into two, giving us the following six sets: (i) $\mathcal{N}^c_1$: edges which form a coordinated station with $e$ and interfere with the receiver of edge $e$, (ii) $\mathcal{N}^c_2$: edges which form a coordinated station with $e$ and do not interfere with the receiver of edge $e$, (iii) $\mathcal{N}^c_3$: edges which form a near hidden edge with $e$, (iv) $\mathcal{N}^c_4$: edges which form an asymmetric topology with $e$ being the edge with an incomplete view of the channel state, (v) $\mathcal{N}^c_5$: edges which form an asymmetric topology with $e$ being the edge which has the complete view of the channel state, and (vi) $\mathcal{N}^c_6$: edges which form a far hidden edge with $e$. Edges in the set $\mathcal{N}^c_1$, $\mathcal{N}^c_2$, $\mathcal{N}^c_3$, $\mathcal{N}^c_4$ and $\mathcal{N}^c_6$ effect the RTS collision probabilities, edges in the set $\mathcal{N}^c_5$ and $\mathcal{N}^c_6$ effect the DATA collision probability and edges in the set $\mathcal{N}^c_1$, $\mathcal{N}^c_2$, $\mathcal{N}^c_3$ and $\mathcal{N}^c_5$ effect the proportion of idle time at the transmitter of $e$.

We first state the value of the DATA collision probability. We reuse the notations used in Lemmas 4 and 9. In a multi-hop topology, $P(E_{1 \rightarrow T}^c) = P(E_{2 \rightarrow T}^c) = 1 - \left( \prod_{e \in \mathcal{N}^c_1} P_{e_{\in} \cap E[S_{\mathcal{N}^c_1}]} \right) - \left( \prod_{e \in \mathcal{N}^c_2} P_{e_{\in} \cap E[S_{\mathcal{N}^c_2}]} \right)$ and $p_{D,T_{1,R_s}}$ and $p_{D,T_{2,R_s}}$ are defined and derived similarly to the corresponding variables in Section III-B.1. Also, based on the discussion in Section III-B.1, we set $p_{\text{cutoff}}^e = \frac{2 \lambda_{\in} T}{m+1}$ if $\mathcal{N}_{\in}^c \cup \mathcal{N}_{\in}^c = \phi$ and $p_{\text{cutoff}}^e \leq p_{\text{cutoff}}$ otherwise.

**Lemma 12:**

\[
\begin{align*}
(1) & \quad p_{i,T}^{e} = 1 - \left( p_{D,DATA} \times p_{\text{ACK}} \left[ 1 - P(\bigcup_{e \in \mathcal{N}^c_1} X_e \cap E[S_{\mathcal{N}^c_1}]) \right] \right), \\
(2) & \quad p_{i,T}^{e} = 1 - \left( p_{D,DATA} \times p_{\text{ACK}} \left[ 1 - \left( p_{D,DATA} - p_{D,DATA} \right) \right] \right) - \left( \sum_{k=0}^{m-1} p_{D,DATA} - p_{D,DATA} \right) \left( \prod_{e \in \mathcal{N}^c_1} P_{e_{\in} \cap E[S_{\mathcal{N}^c_1}]}) \right), \\
& \quad p_{i,T}^{e} = 1 - \left( p_{D,DATA} \times p_{\text{ACK}} \left[ 1 - \left( p_{D,DATA} - p_{D,DATA} \right) \right] \right) - \left( \sum_{k=0}^{m-1} p_{D,DATA} - p_{D,DATA} \right) \left( \prod_{e \in \mathcal{N}^c_1} P_{e_{\in} \cap E[S_{\mathcal{N}^c_1}]}) \right).
\end{align*}
\]

In the expression of $p_{i,T}^{e}$, the first term within square brackets corresponds to the situation where a DATA collision is either caused due to asymmetric topologies due to a CTS loss on the edge between the receiver of the edge under study, $R_s$, and the transmitter of a neighboring edge $e_{\in}$, $T_{\in}$, or far hidden edges due to CTS loss on the edge between $R_e$ and $R_{\in}$. And the second term within square brackets corresponds to a DATA collision due to asymmetric topologies when $T_{\in}$ and $R_{\in}$ start transmitting a CTS the same time. The third term within square brackets denotes DATA collision following event $E_{1 \rightarrow T}^c$. In the expression of $p_{i,T}^{e}$, the two terms within square brackets correspond to the events where the previous exchange was not lost or lost due to DATA collisions following the events $E_{1 \rightarrow T}^c$ or $E_{2 \rightarrow T}^c$.

The values of $p_{i,T}(E_1), p_{i,T}(E_2)$ and $p_{i,T}(E_2)$ are stated in Appendix II. Note that the events $X_{e_{\in}} \forall e_{\in} \in E$ and the CTS getting lost on an edge are independent, hence Lemma 11 is sufficient to derive $p_{i,T}^{e}$. We next state the value of the RTS collision probability. We reuse the notation used in Lemma 6. Additionally, we define the event $X_{e,T} = \left( \bigcup_{e \in \mathcal{N}^c} X_e \cap E[S_{\mathcal{N}^c_1}] \right) \cup \left( \bigcup_{e \in \mathcal{N}^c_2} X_e \cap E[S_{\mathcal{N}^c_2}] \right)$ and $E_{\frac{1}{2}}^{1,2} \frac{1}{2}$ which denotes that there is at least one ongoing transmission which will cause an RTS collision at $e$.

**Lemma 13:**

\[
\begin{align*}
(1) & \quad p_{\text{cutoff}}^e = 1 - \left( p_{D,DATA} \times p_{\text{ACK}} \left[ 1 - P(\bigcup_{e \in \mathcal{N}^c_1} X_e \cap E[S_{\mathcal{N}^c_1}]) \right] \right), \\
(2) & \quad p_{\text{cutoff}}^e = 1 - \left( p_{D,DATA} \times p_{\text{ACK}} \left[ 1 - P(\bigcup_{e \in \mathcal{N}^c_1} X_e \cap E[S_{\mathcal{N}^c_1}]) \right] \right).
\end{align*}
\]
In the expression for \( p_{e,0} \), the first term within square brackets corresponds to RTS collisions due to coordinated stations, while the second term corresponds to RTS collisions due to near hidden edges when the CTS sent by \( R_{e_0} \) is successfully received at \( T_e \). Finally, the third term corresponds to an RTS collision due to event \( X_{e,T} \). In the expression for \( p_{e,i} \), the two terms within square brackets correspond to the events where the previous exchange was not lost or lost due to the event \( X_{e,T} \) respectively.

The values of \( P(E_{j,i}) \) and \( p_j^i \) are stated in Appendix I, while the expressions for \( p_{RTS,i}^e \) and \( p_{RTSnew,i}^e \) for a multi-hop topology are stated in Appendix III.

The next lemma states the value of \( p_{edge}^e \). This lemma follows directly from the observation that any transmission on an edge belonging to \( N_2^e \cup N_2^j \) will freeze the backoff counter on \( e \), and any transmission on an edge belonging to \( N_2^j \cup N_2^e \) will freeze the backoff counter on \( e \) only if the corresponding CTS is correctly received at \( T_e \).

**Lemma 14:**

\[
p_{edge}^e = \frac{1 - p\left(\sum_{e_0=0}^i \left[1 - \sum_{j=0}^{i-1} p_{RTS,i,j}^e \right] \right)}{1 - p \left(\sum_{e_0}^i \left[1 - p_{RTS,i,j}^e \right] \right)} \cdot \Lambda_e,
\]

where \( \Lambda_e \) denotes the complements of event \( E_{RTS,T}^e \).

Equation (1) along with the expressions derived in this section enable the derivation of the expected service time at any edge in any multi-hop topology. Thus, these equations along with the constraints \( \sum_{e \in O_v} \Lambda_e \in [S] < 1 \), \( \forall v \in V \), (where \( O_v \) represents the set of outgoing edges from a node \( v \)) characterize the achievable rate region \( \Lambda_E \). (We sum over all outgoing edges from a node because the network queue for all outgoing edges at a node is the same.)

### D. Network Solution

Determining the expected service time of all edges requires solving a coupled multivariate system of equations. We adopt the following iterative procedure. During each iteration, use the values of the idle and collision probabilities computed in the previous iteration. For all the topologies considered in this paper, this procedure converged to a single global solution with different initial conditions. Proving the existence and uniqueness of a fixed point, and convergence of this iterative procedure to this fixed point is out of the scope of this paper. Conditions for the uniqueness of a fixed point solution for WLANs where all nodes are homogeneous and hence the equation describing their collision probability is the same, were derived in [25], while convergence of the iterative procedure is not well understood for even this simple setting. The proof for uniqueness and convergence is expected to be even more involved in the setting of multi-hop networks where the collision probability of nodes is given by different equations depending on the topology. [15, 17] analyze the throughput performance of 802.11-scheduled multi-hop networks for one-hop flows, and have also reported the convergence of this iterative procedure. We leave as future work to attempt to prove the convergence of the iterative procedure using fixed point theory [26].

### IV. Achievable Flow Rate Region

The achievable flow rate region of a given multi-hop network and a collection of source-destination pairs is characterized by the set of the following constraints:

\[
\begin{align*}
\lambda_e &= \sum_{f \in F} r_f^e \\
g(f) &= \sum_{e \in I_v} r_f^e, \quad \forall v \in V \\
\tilde{\lambda}_e &\leq \Lambda_E
\end{align*}
\]

where \( r_f^e \) denotes the flow rate of flow \( f \) flowing through edge \( e \), \( g(f) \) is the flow function expressed in terms of flow rates and the third is the standard flow conservation constraint. The final constraint says that the vector of edge rates \( \tilde{\lambda}_e \) induced at the edges should lie within the achievable edge-rate region.

### V. Model Verification

In this section, we verify the accuracy of the analysis by finding the achievable rate region for the four two-edge topologies and five different multi-hop topologies via simulations and comparing it to the theoretically derived achievable rate region. We also include the achievable rate region of optimal scheduling, derived using the methodology proposed by Jain et al. [1], to shed light on how far from the optimal 802.11 is. Further, motivated by prior work that has expressed concerns about the ability to achieve fair and efficient rate allocations under 802.11 [11, 15, 27], we compare the max-min rate allocation under 802.11 and under an optimal scheduler.

To ensure that the difference between 802.11 and optimal scheduling is only due to the scheduling inefficiencies of 802.11, we make the overhead imposed by control message exchange and protocol headers to be the same for both schemes. (In practice, the overhead of optimal scheduling is expected to be larger, but this is besides the point here.)

### A. Two-edge topologies

We plot the achievable edge-rate regions derived analytically and via simulations for the four two-edge topologies in Figures 5(a)-(d). We make the following observations from these figures. (i) A close match between the analytical and simulation results verifies the accuracy of the analysis. (ii) The asymmetric topology has the smallest achievable rate region amongst the four two-edge topologies, which implies that the loss in throughput with 802.11 scheduling is largest for this topology. On the other hand, the coordinated station topology has the largest achievable rate region. (iii) In the asymmetric topology, even though 802.11 is highly unfair to \( e_1 \) in saturation conditions (see arrow on the figure) as also observed in [15, 24], with rate control it is possible to achieve a max-min rate allocation of 0.277 Mbps/edge, which is not that far from the max-min rate allocation of 0.332 Mbps/edge achieved by an optimal scheduler.
they are routed interfere with each other.

1 two flows in this topology:

between simulations and analysis for the maximum rate achieved

of the analysis. We compare the error between simulations and

the following observations from this figure. (i) The analytical and

achievable rate region of these two flows in Figure 7(b). We make

Fig. 5. Capacity Regions for different two-edge topologies. The packet loss rate for a 1024 byte packet is equal to 0.2 at e1, 0.3 at e2 and 0.5 at all the interference links. (All the rates are in Mbps.) (a) Coordinated stations. (b) Near hidden edges. (c) Asymmetric topology. (d) Far hidden edges. (The error in the maximum rate achieved at e1 after fixing the rate at e2 is less than 10.1% for all the four plots.)

B. Common Topologies

The first two multi-hop topologies we consider have been used by prior works to study the performance of 802.11 in multi-hop networks: (a) Flow in the Middle topology which was used in [8, 11, 28], and (b) Chain topology which was used in [7, 27, 29].

1) Flow In the Middle Topology: Figure 6(a) shows the Flow In the Middle topology. All links are assumed to be lossless. There are three flows in this topology: 1 → 3, 4 → 6 and 7 → 9. Flows 1 → 3 and 7 → 9 do not interfere with each other, but both of them interfere with flow 4 → 6.

Since flows 1 → 3 and 7 → 9 are symmetric, we assume that they have equal rates. We plot the achievable rate of these two flows against the achievable rate for the middle flow (4 → 6) in Figure 6(b). We make the following observations from this figure. (i) The analytical and simulation curves are close to each other verifying the accuracy of the analysis. We compare the error between simulations and analysis for the maximum rate achieved by flow 4 → 6 when the rate of flows 1 → 3 and 7 → 9 is fixed. The error is less than 9%. (ii) The achievable rate region with 802.11 scheduling is not convex for this topology also. (iii) The max-min rate allocation for this topology with 802.11 is 0.09 Mbps/flow and is 0.14 Mbps/flow with optimal scheduling. Thus, 802.11 achieves 64.3% throughput as compared to optimal scheduling at the max-min rate allocation.

2) Chain Topology: Figure 7(a) shows the Chain topology. All links are assumed to be lossless. We set n = 15. There are two flows in this topology: 1 → 15 and 15 → 1. We plot the achievable rate region of these two flows in Figure 7(b). We make the following observations from this figure. (i) The analytical and simulation curves are close to each other verifying the accuracy of the analysis. We compare the error between simulations and

We say that two flows interfere with each other if any two edges over which they are routed interfere with each other.

Fig. 6. (a) The Flow in the Middle topology. (b) Achievable rate region for the Flow in the Middle topology.

Fig. 7. (a) Chain topology. (b) Achievable rate region for the Chain topology.

Fig. 8. (a) Square topology. (b) Achievable rate region for the Square topology.

analysis for the maximum rate achieved by flow 1 → 15 when the rate of flow 15 → 1 is fixed. The error is less than 12%. (ii) The achievable rate region with 802.11 scheduling is not convex for this topology also. (iii) The max-min rate allocation for this topology with 802.11 is 0.09 Mbps/flow and is 0.14 Mbps/flow with optimal scheduling. Thus, 802.11 achieves 64.3% throughput as compared to optimal scheduling at the max-min rate allocation.

C. Square Topology: Which Route

The next topology we study is the Square topology of Figure 8(a). All links are assumed to be lossless. There are two flows present in this topology: 1 → 8 and 8 → 1. There are two possible routes for each flow: 1 → 2 → 3 → 4 → 8 and 1 → 5 → 6 → 7 → 8 for flow 1 → 8, and 8 → 4 → 3 → 2 → 1 and 8 → 7 → 6 → 5 → 1 for flow 8 → 1. The purpose to study this topology is to show that 802.11 and optimal scheduling can have different optimal routes.

We plot the achievable rate region for this topology in Figure 8(b). We make the following observations from this figure. (i) Again, the simulation and analytical curves are close to each other. The error in the maximum rate achieved by flow 8 → 1 when the rate of flow 1 → 8 if fixed is less than 14%. (ii) The
maximum throughput with 802.11, when only one of the flows is on, is equal to 0.33 Mbps (point A in Figure) and is achieved by routing 0.165 Mbps along one path and 0.165 Mbps along the other path. (iii) When both flows are on, the max-min point with 802.11 (point B in Figure) is achieved by single-path routing with non-overlapping routes for the two flows, for example 1 → 8 routed along 1 → 2 → 3 → 4 → 8 and flow 8 → 1 routed along 8 → 7 → 6 → 5 → 1. However, optimal scheduling can achieve the max-min point by both single-path and multi-path routing. Thus, the optimal routing paths for 802.11 and optimal scheduling can be different. (iv) The max-min rate allocation with 802.11 is 0.18 Mbps/flow and is 0.213 Mbps/flow with optimal scheduling. Thus, 802.11 achieves 84.5% throughput as compared to optimal scheduling at the max-min rate allocation.

D. A Real Topology: Houston Neighborhood

![Diagram of a Houston neighborhood topology](image)

Fig. 9. Topology from the deployment in a Houston neighborhood. Arrows show the routing paths and the numerals on top of an arrow is the probability of loss of a 1024 byte packet on that link. Dashed lines represent the interference links.

The next topology we choose is the real topology of an outdoor residential deployment in a Houston neighborhood [6]. The node locations (shown in Figure 9) are derived from the deployment and fed into the simulator. The physical channel that we use in the simulator is a two-ray path loss model with Lognormal shadowing and Rayleigh fading [30]. The ETX routing metric [31] (based on data loss in absence of collisions) is used to set up the routes. Nodes 0 and 1 are connected to the wired world and serve as gateways for this deployment. All other nodes route their packets towards one of these nodes (whichever is closer in terms of the ETX metric). The resulting topology as well as the routing tree is also shown in Figure 9. The loss rates at each link are determined from the simulator by letting each node send several broadcast messages one by one and measure the number of packets successfully received at every other node. The topology information and loss rates are fed into the analytical model to find the achievable rate region for this topology. There are 16 flows in this topology. Hence, we only compare the max-min rate allocation from simulations and theory. A very good match is observed: the simulator allocates 46 Kbps/flow whereas the theory allocates 44 Kbps/flow (error = 4.4%). Optimal scheduling allocates 67.3 Kbps/flow at the max-min rate allocation. Thus, 802.11 achieves 65.3% of the throughput as compared to optimal scheduling at the max-min rate allocation.

E. Random Topology

We create the final topology by randomly placing 75 nodes in a 1000m ×1000 m area. Both transmission and interference range are set equal to 200m. We assume links used for routing packets to be lossless and assume \( p_{\text{RTS}} = p_{\text{CTS}} = 0.4 \) on all the other links as links used in routing paths typically are low loss links. We select 6 source-destination pairs at random. We compare the max-min rate allocation from simulations and theory. A very good match is observed: the simulator allocates 94 Kbps to five of the flows and 650 Kbps to the sixth flow whereas theory allocates 96Kbps to five of the flows and 600 Kbps to the sixth flow (error = 7.6%). Optimal scheduling allocates 141.7 Kbps to five of the flows and 706 Kbps to the sixth flow at the max-min rate allocation. Thus, at the max-min point, 802.11 achieves 76.35% of the total sum throughput as compared to optimal scheduling.

F. Different Network Parameters

All the previous comparisons were made for a particular set of network parameters. In this section, we investigate the accuracy of the analysis when the network parameters are modified from their default values. We compare the achievable rate region derived via simulations and theory for the Flow in the Middle topology (Figure 6(a)) for: (a) 100 byte DATA packets at 1 Mbps data rate in Figure 10(a), and (b) 1024 bytes packets at 11 Mbps data rate in Figure 10(b). The error between simulations and analysis for the maximum rate achieved by flow 4 → 6 when the rate of flows 1 → 3 and 7 → 9 is fixed is less than 15% for both scenarios. Note that for both the scenarios, Assumption 1 does not hold, and hence we see a larger error. For smaller DATA packets, the reason why Assumption 1 does not hold is obvious. However, why increasing the data rate to 11 Mbps makes this assumption invalid is not obvious as the DATA packet size is still two orders of magnitude larger than the RTS packet size. In 802.11, the PHY header contains information used to determine the data rate of the incoming transmission (to allow auto-rate adaptation [20]), and hence is always transmitted at 1 Mbps. And the PHY layer header is exchanged for both control (RTS, CTS and ACK) and DATA packets. For a data rate of 11 Mbps, the transmission time of the 1024 byte DATA packet is comparable to the transmission time of the PHY layer header which is transmitted at 1 Mbps. Hence, the transmission time of a RTS packet is comparable to the transmission time of a DATA packet, which violates Assumption 1. Note that this is a protocol issue which needs to be fixed as this violates the basic premise of protocol design that the load due to control packets should be a small fraction of the total load.

From Figures 10(a) and 10(b), we also observe that 802.11 achieves more than 84% throughput at the max-min rate allocation as compared to optimal scheduling for both the scenarios. Note that in both these examples the overhead is significantly larger than in previous scenarios.
G. Summary

We now summarize the observations made in this section. (i) Analytical results have an error less than 15%. (ii) The achievable rate region with 802.11 scheduling is non-convex. (iii) 802.11 achieves more than 64% throughput as compared to optimal scheduling at the max-min rate allocation for all the topologies studied in this paper. This is an interesting and unexpected observation. A prior work of ours [32] attempts to understand the optimality of 802.11, however characterizing the worst case performance of 802.11 is still an open question and left for future work. (iv) The optimal routing paths for 802.11 and optimal scheduling can be different.

VI. NETWORK SOLUTION WITHOUT THE ITERATIVE PROCEDURE

As discussed in Section III-D, we need an iterative procedure to solve the coupled multivariate system of equations derived in Section III. In this section, we discuss if it is possible to decouple the equations to avoid using an iterative procedure by sacrificing some accuracy in the analysis. We look at the following questions: (i) under what network conditions can the equations be decoupled without an unreasonable loss in accuracy, and (ii) what are the approximations to be made to remove the coupling.

A careful look at Lemmas 11 and 12 and the expression for $K_{r,T}^*$ derived in Section III-B.1 tells us that the equations cannot be decoupled for networks with a non-negligible probability of RTS/CTS loss on edges without a significant loss in accuracy.

For networks with a negligible probability of RTS/CTS loss, one can make the following two approximations to decouple the equations. (I) The first approximation is to replace $\lambda s_i E[S_i]$ by $\min \left( \frac{\lambda}{\lambda_{sat,n_e}}, 1 \right) s_i$ in the expressions for the following two probabilities: (i) the DATA collision probability (Lemma 12), and (ii) the RTS collision probability (Lemma 13). $\lambda_{sat,n_e}$ denotes the saturation throughput of a WLAN with $n$ transmitters transmitting to a single receiver (derived in [22]) and $n_e = |N_e|$ is the number of edges interfering with $e$. Note that $\lambda s_i E[S_i]$ is upper bounded by 1. Since approximating $\lambda s_i E[S_i]$ by its upper bound is inaccurate when $\lambda$ is small, in these situations we replace $E[S_i]$ by $1/\lambda s_i, n_e$ ($\lambda_{sat,n_e}$ as a function of $n$ flattens out rather fast [22]. As a result, even if just a few neighboring edges are saturated, $1/\lambda s_i, n_e$ would be a good lower bound since the topology that minimizes service times is the one where all nodes are within range.) (II) The second approximation is to approximate $P(\cap_{e_i \in N_e} X_{e_i}) = \left( \prod_{e_i \in N_e} P(X_{e_i}) \right) / \left( \prod_{e_i \in N_e} P(X_{e_i}) \right)$ when no two edges in $N_e$ interfere with each other in Lemma 11 with $\left( \prod_{e_i \in N_e} P(X_{e_i}) \right) / \left( \prod_{e_i \in N_e} P(X_{e_i}) \right)$.

With the first approximation, the DATA collision probabilities can be derived for each edge independently. Now, given the DATA collision probabilities at each edge, with the second approximation, one can find the RTS collision probabilities and idle probability at each edge independently.

Using these approximations will introduce some inaccuracies. However for the topologies studied in this paper, the inaccuracies are not large. For example, Figures 11(a) and 11(b) compare the achievable rate region derived with these approximations with the simulation results for the Flow in the Middle topology and the Chain topology respectively. With the two approximations, the maximum error is less than 20% for both the topologies.

Fig. 11. (a) Achievable rate region for the Flow in the Middle topology with the approximations of Section VI. Error between simulations and analysis is less than 20%. (b) Achievable rate region for the Chain topology with the approximations of Section VI. Error between simulations and analysis is less than 12%.

VII. EXTENSIONS

We now discuss how to modify the analysis if some of the simplifying assumptions made on the physical layer model and packet sizes do not hold.

Different Transmission Rates and Packet Sizes: Different edges in the network can have different average transmission rates due to the automatic rate adaptation employed at the 802.11 physical layer. Moreover, there can be multiple sized packets flowing through the network. Both these events will result in different transmission times at each edge. To account for these, the pdf of the transmission time for each edge would be derived based on the packet pdfs and the automatic rate adaptation algorithm, the expected service time at each edge would be derived as a function of the transmission time at that edge, and then the law of total probability would be used to integrate out this dependence.

More Detailed Physical Layer Model: The analysis in Section III assumed a binary and pairwise interference model. However, recent measurement studies suggest that interference is neither binary [33] nor pairwise [19]. Even though our main objective is to analyze the achievable rate region under 802.11 MAC, it is important to discuss how the derivation of Section III gets modified if a more realistic interference model is used.

First lets discuss how to remove the binary assumption. [12] proposed a non-binary interference model by associating a capture and a deferral probability to model that a collision might not result in packet loss and the channel might not be sensed busy at a node inspite of the ongoing interfering transmission. For each of the two-edge topologies, incorporating the capture and deferral probabilities will change the collision and idle probabilities.

We now discuss how to modify the analysis if some of the simplifying assumptions made on the physical layer model and packet sizes do not hold.

Different Transmission Rates and Packet Sizes: Different edges in the network can have different average transmission rates due to the automatic rate adaptation employed at the 802.11 physical layer. Moreover, there can be multiple sized packets flowing through the network. Both these events will result in different transmission times at each edge. To account for these, the pdf of the transmission time for each edge would be derived based on the packet pdfs and the automatic rate adaptation algorithm, the expected service time at each edge would be derived as a function of the transmission time at that edge, and then the law of total probability would be used to integrate out this dependence.

More Detailed Physical Layer Model: The analysis in Section III assumed a binary and pairwise interference model. However, recent measurement studies suggest that interference is neither binary [33] nor pairwise [19]. Even though our main objective is to analyze the achievable rate region under 802.11 MAC, it is important to discuss how the derivation of Section III gets modified if a more realistic interference model is used.
on both the edges.

Now let’s discuss how to remove the pairwise assumption. Many simultaneous transmissions can cause deferral/collision at a node even though each of them individually might not have the same effect. [14, 18] proposed a model for this physical layer effect. For each edge $e$, there is a deferral and collision probability associated at both the receiver and the transmitter of $e$ defining its behavior if a set $S$ of edges are transmitting simultaneously. Thus, instead of considering the effect of interfering edges on $e$, we should consider the effect of interfering sets on $e$. Given that each edge in the set $S$ does not cause any interference individually, the set of edges $S$ can interact in only one of the following two ways: (i) either it causes a deferral at the transmitter, which can be analyzed using techniques developed for analyzing coordinated stations, or (ii) it does not cause a deferral at the transmitter but causes a collision at the receiver, which can be analyzed using techniques developed for analyzing asymmetric topologies.

Hence, even with a non-binary and non-pairwise interference model, the essence of the analysis in terms of decomposing a local topology around an edge into a number of interfering sets and then combining them using the results from Section III-C, remains unchanged. So, we believe that the analysis presented in this paper can be extended to a more realistic interference model.

VIII. CONCLUSIONS AND FUTURE WORK

In this paper we have characterized the capacity region of an arbitrary multi-hop wireless network with 802.11 scheduling by deriving a methodology to characterize the achievable edge-rate region. This paper is a precursor to several works which require a general and accurate characterization of the achievable rate region of 802.11-scheduled multi-hop networks. We briefly describe three such ongoing works.

Optimality of 802.11: In Section V, we observed that 802.11 achieves more than 64% throughput as compared to optimal scheduling at the max-min rate allocation for all the topologies we studied. These results serve as a motivation to understand the worst-case performance of 802.11.

Optimal Routing and Rate Allocation: The constraints characterizing the achievable flow-rate region of a given 802.11-scheduled multi-hop network (Section IV) can be fed into an optimization problem to find optimal routing and rate allocation for different utility functions.

Residual Bandwidth Estimation: The methodology of Section III can be used to find the residual bandwidth at a given edge given the edge-rates at the other edges in the network. This can be used to design interference-aware routing which routes along the path with the maximum available bandwidth [16, 34] or a congestion control algorithm which sends explicit and precise rate feedback to the sources (similar to XCP [35] and RCP [36] which were proposed for wired networks).

REFERENCES

In this appendix, we derive expressions for variables required to find the RTS collision probabilities for asymmetric topologies. We first state the values of $P_{\text{RTS},i,j}$, $P_{\text{RTS},k,j}$, and $P_{\text{RTS},i,k}$. The expressions for these variables follow directly from their definition. For $1 \leq i \leq m$,

$$P_{\text{RTS},i,j} = \left\{ \begin{array}{ll} \frac{\prod_{k=j+1}^{j} (1-p_{k,j}) p_{i,j}}{\prod_{k=j+1}^{j-1} (1-p_{k,j}) p_{i,j}} & \text{if } j < i \\ p_{j,i} & \text{if } j = i \\ \prod_{k=j+1}^{j} (1-p_{k,j}) p_{i,j} & \text{if } j > i \end{array} \right. $$

Next, we derive the value of $p_{j,i}$’s by dividing the total number of favorable cases by the total number of possible cases. Let $1 \leq i \leq m$.

$$p_{j,i} = \left\{ \begin{array}{ll} \frac{\sum_{w_{1} \leq i} \sum_{w_{2} \leq j} I \left( \left\{ \sum_{k=1}^{i} u_{k} \leq w_{1}, \sum_{k=1}^{j} u_{k} \leq w_{2} \right\} \right) \right)}{\sum_{w_{1} \leq i} \sum_{w_{2} \leq j} I \left( \left\{ \sum_{k=1}^{i} u_{k} \leq w_{1}, \sum_{k=1}^{j} u_{k} \leq w_{2} \right\} \right) \left( \sum_{k=1}^{j} u_{k} \leq T_{j} \right) \left( \sum_{k=1}^{j} u_{k} \leq T_{j} \right) } & \text{if } j > 0 \\ 0 & \text{otherwise} \end{array} \right. $$

Next, we derive the expression for the events $p_{j,i}(E_{1})$, $p_{j,i}(E_{2})$, and $p_{j,i}(E_{3})$. The expressions for these variables follow directly from their definition.