Wireless Contention in Mobile Multi-hop Networks

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ABSTRACT

Wireless contention is the most important and least studied characteristic of multi-hop networks. It is the most important because not only does it have a drastic effect on the performance but also it is the fundamental property which makes these networks very different from traditional wired and single-hop wireless networks. Finite bandwidth is only one of the sources of contention. In addition, there is extensive interference among nearby links irrespective of whether they have any common nodes. This is usually alleviated by employing a scheduling mechanism. Moreover, even transmissions from quite distant nodes outside the scheduling area may interfere with a link due to multipath fading. Contention is one of the least studied properties because of its complicated nature. This forces the researchers to use simple, intuitive, but often unrealistic models to keep the analysis tractable, which may lead to significantly inaccurate conclusions.

This paper introduces a mathematical framework to analyze wireless contention that models all three manifestations of contention, namely, finite bandwidth, local scheduling, and interference from transmissions outside the scheduling area. The framework can be used with any realistic channel and mobility model, allows the derivation of exact, rather than asymptotic, results, and it is quite accurate. As a case study, we use the framework to compute the expected packet delay of two popular routing schemes for mobile ad hoc networks.

1. INTRODUCTION

Wireless contention is one the most important properties of mobile multi-hop networks because of the following two reasons: (i) It has a drastic impact on the performance of these networks. For example, [11] shows that the capacity of a wireless ad hoc network does not scale because wireless contention limits the maximum number of possible simultaneous transmissions. (ii) Contention makes these networks very different from traditional wired and wireless single-hop networks. As a result, the protocols which were originally proposed for these traditional networks, perform badly when deployed in wireless mobile multi-hop networks, see, for example, [7, 21].

Wireless contention is also a much more complicated phenomenon than contention in wired and single-hop wireless networks. Wired networks have to deal only with bandwidth contention, that is, with the situation where a number of packets need to be transferred via a link, but the finite link bandwidth prevents them from being transferred instantly. In addition to finite bandwidth, single-hop wireless networks also suffer from contention for the medium around the base station/access point as only one node can communicate to the base station at a given time. Collisions between the transmissions from nodes to the common base station/access point are avoided by deploying a scheduling mechanism like TDMA, FDMA or CSMA-CA. The situation for mobile multi-hop networks is even more complex than in single-hop ones. First, in a mobile network, it's not even certain that all the packets waiting to be transferred via a wireless link will make it before the link breaks. Also, traffic via other nearby wireless links may interfere directly with the link under consideration, even when there are no common nodes between the two links. In order to avoid collisions, the scheduling mechanism will prevent such links from transmitting simultaneously. Finally, even transmissions from quite distant nodes that lie outside the scheduling area may interfere with the link under discussion due to multipath fading [1,40]. Thus, contention in wireless multi-hop networks manifests itself in three ways: (i) finite bandwidth, (ii) scheduling of transmissions, and (iii) interference from distant transmissions outside the scheduling area.

Inspite of its importance, wireless contention in a mobile multi-hop network has not been studied properly because of its complicated nature. Most researchers do not even incorporate wireless contention in their analytical studies. Instead, they use simulations to study the performance of these networks with contention. The few analytical studies which incorporate contention use models which make intuitive sense and are simple enough to keep the analysis tractable, for example the protocol model [9,11,17,27], physical model [11], cell-partitioned model [22], conflict graph [6, 13] etc. But, using simple models like the ones mentioned above can lead to significantly inaccurate conclusions.

To illustrate the inaccuracy of simple models, in Figure 1 we plot the expected end-to-end delay between a source and a destination when flooding is used for routing in a mobile multi-hop network, using different contention models. We also plot the delay obtained from simulating the network with IEEE 802.11 scheduling at the MAC layer, Rayleigh fading at the channel, and receivers that declare the transmission successful only when the signal to interference ratio is greater than a threshold. (Note that flooding generates a lot of contention in the network and hence, clearly brings out the inaccuracies resulting from unrealistic models.) Its easy to see that these simple models predict results very different from the results obtained by the simulation[1]. Even more disturbing is the fact that using different contention models not only yield different constants but can also lead to different asymptotic results. For example, under a cell partitioned model, [22] obtained a delay capacity tradeoff of $\lambda \leq \Theta\left(\frac{D}{n}\right)$ where D is the average packet delay, n is the number of nodes and λ is the average per node throughput; while [17] obtained a tradeoff of $\lambda^3 \leq \Theta\left(\frac{D log^3 n}{n}\right)$ for the protocol model. (Other than the contention model, the scenario in both papers is the same.) Additionally, the analytical techniques used in all the previous papers depend on the specific mobility and channel model at hand and cannot be easily generalized. [10] and [27] show that using different channel and mobility models can yield different results too. Hence, analytical studies should use more realistic models to be able to predict more accurate results.

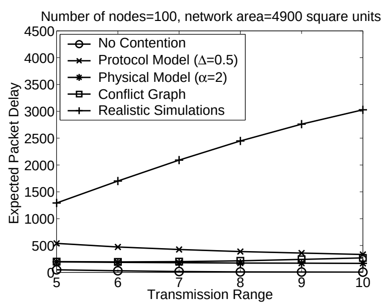


Figure 1: Expected packet delay for flooding in mobile multi-hop networks.

Its not that researchers do not use realistic models in their analysis because they are unaware of which models accurately represent reality. It is well-known that the scheduling scheme used in most real networks is CSMA-CA. Trace analysis has shown that interference models should be based on comparing the signal-to-interference ratio to a threshold before declaring the transmission successful [30]. [1, 40] show that the wireless channel suffers from multipath fading and hence channel models like Rayleigh fading and log normal shadowing are more accurate than the disk model or the distance based attenuation model. Traces also show that Random Waypoint and Random Walk mobility models, even though are the most commonly used mobility models, do not closely resemble mobility involving humans [12, 18]. Based on the intuition derived from these works, more accurate community based models have been proposed to model node mobility [33]. Taking all these factors in the analysis makes it intractable which precludes their use in analytical studies. On a different point, inspite of using simpler, less accurate models, most of the previous papers present asymptotic results only. Although asymptotic results provide valuable insights into the scalability of a family of protocols, explicit results are often necessary to design and compare practical schemes.

[1]The two main reasons why results from these contention models are so inaccurate are: (i) To keep the analysis simple, they assume TDMA scheduling, even though in practice, TDMA is not used in the context of such networks, and (ii) these models do not incorporate the effect of multipath fading.

The main contribution of this paper is the introduction of a realistic yet analytically tractable framework to model wireless contention. The framework incorporates all the three manifestations of contention: finite bandwidth, scheduling, and interference from nodes outside the scheduling area. The framework can be used analyze any CSMA-CA like or TDMA scheduling mechanism. It incorporates interference from nodes outside the scheduling area by comparing the signal to interference ratio at the receiver to a threshold before declaring the transmission to be successful. The analytical methodology used in the framework works for all the fading channel models, like Rayleigh fading, Rician fading, and log normal shadowing [25], and all the mobility models which have a steady state node location distribution. Further, it allows the derivation of explicit performance results rather than asymptotic values, and works for a wide variety of practical routing strategies. To keep the analysis tractable, instead of using simplified, less accurate models, we make simplifying and justifiable approximations during the course of the analysis. We use simulations to verify that these approximations do not have a significant impact on the accuracy of the analysis.

To demonstrate how the framework can be used to derive performance metrics of interest, we find the expected end-to-end delay for two different routing schemes. First, we find the expected delay for shortest path routing in dense networks. Then, we analyze epidemic routing [35] proposed to route packets in sparse networks (also referred to as Delay Tolerant Networks or Intermittently Connected Mobile Networks [14, 34, 39]). Delay analysis with contention in a wireless multi-hop network is a challenging problem. Analyzing delay in sparse networks is even more challenging as the routing schemes for these networks require the relay nodes to store, carry and forward packets. To the best of our knowledge, this is the first paper to derive highly accurate expected delay expressions for routing schemes with contention in a multi-hop network.

The outline of the paper is as follows: Section 2 presents the related work and points out the differences between the proposed framework and the models used by prior works. Section 3 presents the contention framework. Section 4 demonstrates how the framework can be used to find performance metrics of interest for different routing schemes. Section 5 presents simulation results to verify that the approximations made during the course of the analysis do not have a significant effect on the accuracy of the analysis. Finally, in Section 6 we conclude and discuss future work.

2. RELATED WORK

Gupta and Kumar [11] introduced two interference models: a protocol model that assumes interference to be an all or nothing phenomenon, and a physical model that considers the impact of interfering transmissions on the signal-to-noise ratio. These two models have been widely used in subsequent papers [5, 9, 17, 27]. Another commonly used

N	Area of the 2D space
M	Number of nodes in the network
K	The transmission range
Θ	The desirable SIR ratio
s_{BW}	Bandwidth of links in units of packets per time slot

Table 1: Notation used throughout the paper.

contention model is the generalized physical model [20, 29] wherein the data rate is a function of the signal to interference ratio at the receiver. [22] uses a cell partitioned model for contention wherein the entire network is divided into geographical cells and only one node can transmit successfully within a cell at a given time. [6,13] use a conflict graph to model interference. The conflict graph indicates which group of links mutually interfere and hence cannot be active simultaneously. All these previous papers use TDMA scheduling and assume that the schedule employed ensures that no packet is lost due to interference. In a recent paper, Bader et al [3] model interference by comparing the signal to interference ratio to a desired threshold to decide if the transmission was successful or not. Unlike previous papers, it uses the Rayleigh fading model for the channel, and hence is more accurate. But, this paper also uses TDMA scheduling which allows it to assume that a given hop distance (distance covered in one hop) is achievable given the average interference in the network. Moreover, its analytical technique can be applied to static topologies only.

The previous models and analytical techniques suffer from at least one of the following drawbacks: (i) The papers which use the protocol model, the physical model, or the generalized physical model for contention present asymptotic results only. (ii) The papers which use a conflict graph to model contention require the solution of a multicommodity optimization problem and, as a result, they are not directly applicable to mobile networks. (iii) The methodologies which derive results for mobile networks assume a specific mobility model, usually the random walk or random waypoint model, and it is quite hard, if at all possible, to extend them for more general, realistic mobility models. (iv) The analytical techniques presented in these papers cannot be used to analyze CSMA-CA like networks. (v) With the exception of [3], all the other papers assume that interference from nodes outside the scheduling area will not lead to packet loss. The use of an ideal physical layer model allows this assumption. Hence, these models are not applicable to channels with multipath fading.

The proposed framework does not suffer from any of the drawbacks of the previous analytical methodologies. In particular, it models interference from distant nodes, works for all the fading channel models, can be used to analyze any CSMA-CA like or TDMA scheduling mechanism, allows the derivation of explicit performance results rather than asymptotic values, and is applicable to mobile networks under any general mobility model with a steady state node location distribution, without the analysis becoming overly complicated.

3. CONTENTION ANALYSIS

3.1 Notation and Assumptions

We first introduce our notation and state the assumptions we will be making throughout the remainder of the paper.

1. M nodes move in a two dimensional space of area N.
2. **Radio Model:** The signal to interference ratio should be greater than a desired threshold, which we call Θ, for the transmission to be successful. For ease of analysis, we assume that two nodes will try to transmit to each other only if the link between them is in the connected region (not in the transitional or grey region). [1, 40] show that this is equivalent to assuming that the nodes will transmit to each other when the distance between them is less than K. (The value of K depends on the transmit power.) Note that this does not imply that transmissions from nodes at a distance greater than K are not going to interfere with the ongoing transmission or that the ongoing transmission will always be successful.
3. **Traffic Model:** The arrival process at each node is governed by a stationary stochastic process.
4. **Channel Model:** The analysis works for any channel model.
5. **Mobility Model:** The analysis works for any mobility model in which the process governing the mobility of nodes is stationary and the movement of each node is independent of each other. To simplify the presentation so that the underlying intuition does not get lost in the analytical complexity, we will first present the framework for a mobility model which has a uniform node location distribution. Commonly used mobility models like the Random Walk model, the Random Direction model and the Random Waypoint on a torus satisfy this assumption as shown in [2, 4, 33] Then, in section 3.4 we show how does the framework extend to mobility models with a non-uniform location distribution of nodes by presenting the framework for the Community-based mobility model [33] (which has a non-uniform location distribution of nodes and a number of other properties that make it more realistic).

3.2 Three Manifestations of Contention

Finite Bandwidth: When two nodes meet, they might have more than one packet to exchange. Say two nodes can exchange s_{BW} packets during a unit of time. If they move out of the range of each other, they will have to wait until they meet again to transfer more packets. The number of packets which can be exchanged in a unit of time is a function of the packet size and the bandwidth of the links. We also assume that the s_{BW} packets to be exchanged are randomly selected from amongst the packets the two nodes want to exchange[2].

Scheduling: The framework is applicable to any CSMA-CA like scheduling mechanism. For ease of presentation, in

[2]The instantaneous unfinished work in the queue in bits will be the same for any work conserving queue service discipline like FIFO, random queueing and LIFO. Hence, for constant size packets, the throughput and the expected delay will also remain the same for all the three schemes.

this paper we will analyze a variant of CSMA-CA scheduling. In particular, the standard CSMA-CA algorithm prohibits any transmission within one hop of the transmitter and the receiver to solve the hidden terminal problem. Here, we assume that the scheduling mechanism prohibits any transmission within two hops from the transmitter irrespective of whether it is within one hop from the receiver or not. (Note that nodes within one hop of the transmitter are less than K distance away, and thus two hops away are within $2K$ distance from the transmitter.)

For ease of analysis, we also assume that time is slotted. At the start of the time slot, all node pairs contend for the channel and once a node pair captures the medium, it retains the medium for the entire time slot.

Interference: Even though the scheduling mechanism is ensuring that no simultaneous transmissions are taking place within a distance $2K$ of each other, there is no restriction on simultaneous transmissions taking place separated by a distance more than $2K$. These transmissions act as noise for each other and hence can lead to packet corruption.

In the absence of contention, two nodes would exchange all the packets they want to exchange whenever they come within range of each other. Contention will result in a loss of such transmission opportunities. This loss can be caused by either of the three manifestations of contention. In general, these three manifestations are not independent of each other. We now propose a framework which uses conditioning to separate their effect and analyze each of them independently.

3.3 The Framework

Lets look at a particular packet, label it packet A. Suppose two nodes i and j are within range of each other at the start of a time slot and they want to exchange this packet. Let p_{txS} denote the probability that they will successfully exchange the packet during that time slot. First, we look at how the three manifestations of contention can cause the loss of this transmission opportunity.

Finite Bandwidth: Let E_{bw} denote the event that finite link bandwidth allows nodes i and j to exchange packet A. The probability of this event depends on the total number of packets which nodes i and j want to exchange. Let there be a total of S distinct packets in the system at the given time (label this event E_S). Let there be s, $0 \leq s \leq S-1$, other packets (other than packet A) which nodes i and j want to exchange (label this event E_s^S). If $s \geq s_{BW}$, then the s_{BW} packets exchanged are randomly selected from amongst these $s+1$ packets. Thus, $P(E_{bw})$ is equal to $\sum_S P(E_S)\left(\sum_{s=0}^{s_{BW}-1} P(E_s^S) + \sum_{s=s_{BW}}^{S-1} \frac{s_{BW}}{s+1} P(E_s^S)\right)$. To simplify the analysis, we make our first approximation here by replacing the random variable S by its expected value in the expression for $P(E_{bw})$[3]. (Note that simulations results presented in Section 5 verify that this approximation does not have a drastic effect on the accuracy of the analysis.)

(i)	**Finite Bandwidth**
E_{bw}	Event that finite link bandwidth allows exchange of packet A
E_s^S	Event that i and j want to exchange s other packets given there are S distinct packets in the system
p_{ex}	Probability that nodes i and j want to exchange a particular packet
(ii)	**Scheduling**
E_{sch}	Event that scheduling mechanism allows i and j to exchange packets
E_a	Event that there are a nodes within a distance $2K$ from the transmitter
E_c	Event that there are c nodes in the $2K < d \leq 3K$ ring from the transmitter
$t(a,c)$	Expected number of possible transmissions whose transmitter is within $2K$ distance from the transmitter
p_{pkt}	Probability that two nodes have at least one packet to exchange
(iii)	**Interference**
E_{inter}	Event that transmission of packet A is not corrupted due to interference
E_{M-a}	Event that packet A is successfully exchanged inspite of the interference from $M-a$ nodes outside the scheduling area
$\overline{x}$	Average number of interfering transmissions
r_{avg}	Average distance between the transmitter of the interfering transmission and the desired receiver

Table 2: Notation used in Section 3.3

Scheduling: Let E_{sch} denote the event that the scheduling mechanism allows nodes i and j to exchange packets. The scheduling mechanism prohibits any other transmission within $2K$ distance of the transmitter. Hence, to find $P(E_{sch})$, we have to determine the number of transmitter-receiver pairs which have at least one packet to exchange and are contending with the i-j pair. Let there be a nodes within a distance $2K$ of the transmitting node (label it event E_a) and let there be c nodes in the $2K < d \leq 3K$ ring from the transmitter (label it event E_c). The nodes in the $2K < d \leq 3K$ ring have to be accounted for because a node at the edge of the $2K$ circle can be within the transmission range of these nodes and will contend with the desired transmitter. Let $t(a,c)$ denote the expected number of possible transmissions whose transmitter lies within $2K$ distance of the desired transmitter. Due to the backoff mechanism, by symmetry all the contending nodes are equally likely to capture the channel. So, $P(E_{sch} \mid E_a, E_c)$ is equal to $1/t(a,c)$.

Interference: Let E_{inter} denote the event that the transmission of packet A is not corrupted due to interference given that nodes i and j exchanged this packet. Let there be $M-a$ nodes outside the transmitter's scheduling area (this is equivalent to event E_a). If two of these nodes are within the transmission range of each other, then they can exchange packets which will increase the interference for the transmission between i and j. Lets label the event that packet A is successfully exchanged inspite of the in-

[3]We incorporate the arrival process through $E[S]$ in the analysis. $E[S]$ depends on the arrival rate through Little's Theorem. Thus, after deriving the expected end-to-end delay for a routing scheme in terms of $E[S]$, Little's Theorem can be used to express the delay in terms of only the arrival rate.

terference caused by these $M-a$ nodes as E_{M-a}. Then, $P(E_{inter} \mid E_a)$ is equal to $P(E_{M-a})$.

Packet A will be successfully exchanged by nodes i and j only if the following three events occur: (i) the scheduling mechanism allows these nodes to exchange packets, (ii) nodes i and j decide to exchange packet A from amongst the other packets they want to exchange, and (iii) this transmission does not get corrupted due to interference from transmissions outside the scheduling area. Thus,

$$p_{txS} = P(E_{bw}) \times \sum_{a,c} P(E_a, E_c) P(E_{sch} \mid E_a, E_c) P(E_{inter} \mid E_a)$$

$$= \left(\sum_{s=0}^{s_{BW}-1} P(E_s^{E[S]}) + \sum_{s=s_{BW}}^{E[S]-1} \frac{s_{BW} P(E_s^{E[S]})}{s+1} \right) \times \sum_{a,c} \frac{P(E_a) P(E_c \mid E_a) P(E_{M-a})}{t(a,c)}. \quad (1)$$

Next, we find expressions for the unknown values in Equation (1).

3.3.1 Finite Bandwidth

To account for finite bandwidth, we have to find $P(E_s^{E[S]})$ (the probability that nodes i and j have s other packets to exchange given there are $E[S]$ distinct packets in the system). Let p_{ex} be the probability that nodes i and j want to exchange a particular packet. Now, since there are $E[S]-1$ packets other than packet A in the network, $P(E_s^{E[S]}) = \begin{pmatrix} E[S]-1 \\ s \end{pmatrix} p_{ex}^s \left(1-p_{ex}\right)^{E[S]-s-1}$.

The value of p_{ex} depends on the routing mechanism at hand because which packets should the two nodes exchange is dictated by the routing policy. Note that this is the only term affected by the routing mechanism in the analysis. We will derive its value for two different routing mechanisms in Section 4.

3.3.2 Scheduling

To account for scheduling, we have to figure out $P(E_a)$ (the probability that there are a nodes within a distance of $2K$ from the desired transmitter), $P(E_c \mid E_a)$ (the probability that out of the remaining $M-a$ nodes, there are c nodes in the $2K < d \leq 3K$ ring from the transmitter) and $t(a,c)$ (the expected number of possible transmissions competing with the i-j pair).

Each of the other $M-2$ nodes (other than i and j) are equally likely to be anywhere in the two dimensional space because the mobility model has a uniform stationary distribution. So, we use geometric arguments to figure out how many transmissions contend with the transmission between i and j.

LEMMA 3.1. $P(E_a) = \begin{pmatrix} M-2 \\ a-2 \end{pmatrix} (p_1)^{a-2}(1-p_1)^{M-a}$ *where* $p_1 = \frac{4\pi K^2}{N}$ *is the probability that a particular node lies within* $2K$ *distance of the transmitter.*

Proof: The node is equally likely to be anywhere in the two dimensional space. Consequently, $p_1 = \Pr$[a particular node is within a distance $2K$ of the transmitting node] $= \int_0^{2K} \frac{2\pi r}{N} dr = \frac{4\pi K^2}{N}$. Recall that nodes i and j are within $2K$ distance of the transmitter. So, $P(E_a) = \begin{pmatrix} M-2 \\ a-2 \end{pmatrix} (p_1)^{a-2}(1-p_1)^{M-a}$. □

COROLLARY 3.1. $P(E_c \mid E_a) = \begin{pmatrix} M-a \\ c \end{pmatrix} (p_2)^c (1-p_2)^{M-a-c}$ *where* $p_2 = \frac{5\pi K^2}{N}$ *is the probability that a particular node lies in the* $2K < d \leq 3K$ *ring from the transmitter.*

Proof: The corollary can be derived in a manner similar to the proof of Lemma 3.1. □

LEMMA 3.2. $t(a,c) = \left(1 + p_a p_{pkt} \left(\begin{pmatrix} a \\ 2 \end{pmatrix} - 1 \right)\right) + \left(\frac{acp_c p_{pkt}}{2}\right)$ *where* $p_a = \left(\frac{1}{16} + \frac{A}{4\pi K^2}\right)$ *is the probability that two nodes are within a distance* K *of each other given that both of them are within* $2K$ *distance of the transmitter,* $p_c = \left(\frac{3}{20} - \frac{A}{5\pi K^2}\right)$ *is the probability that two nodes are within a distance* K *of each other given that one of them is within* $2K$ *distance of the transmitter and the other node is in the* $2K < d \leq 3K$ *ring from the transmitter,* $p_{pkt} = 1-(1-p_{ex})^{E[S]}$ *is the probability that two nodes have at least one packet to exchange, and* A *is a constant equal to* $\int_K^{2K} \frac{x}{2K^2} \left[K^2 cos^{-1}\left(\frac{x^2-3K^2}{2Kx}\right) + 4K^2 cos^{-1}\left(\frac{x^2+3K^2}{4Kx}\right) - \frac{1}{2}\sqrt{(x^2-K^2)(9K^2-x^2)} \right] dx.$

Proof: See Appendix. □

3.3.3 Interference

The interference caused by other nodes depends on the number of simultaneous transmissions and the distance between the transmitters of these simultaneous transmissions and the desired receiver. Given that there are $M-a$ nodes outside the scheduling area (event E_a), let there be x interfering transmissions at a distance of $r_1, r_2, \ldots, r_x$ from the desired receiver. Then, using the law of total probability, we get

$$P(E_{M-a}) = \sum_x \sum_{r_1, r_2, \ldots, r_x} P(E_{M-a} \mid x, r_1, r_2, \ldots r_x) \times P(r_1, r_2, \ldots, r_x \mid x) P(x). \quad (2)$$

While it is possible to calculate both $P(x)$ and $P(r_1, r_2, \ldots, r_x \mid x)$ to substitute in Equation (2), the resulting expressions would be very complicated. Motivated by this, we replace x and the r_i's with their expected values. (Simulations results presented in Section 5 verify that this approximation does not have a drastic effect on the accuracy of the analysis.) Since each node is moving independently of each other, $E[r_1] = E[r_2] = \ldots = r_{avg}$. We label $E[x]$ as $\overline{x}$.

First, we compute r_{avg}. Let $f(r)$ denote the probability density function of the distance between any two nodes.

The expression for $f(r)$ depends on the shape of the area in which nodes are moving. The following equation states its value for a torus of area N:

$$f(r) = \begin{cases} \frac{2\pi r}{N} & r \le \frac{\sqrt{N}}{2} \\ \frac{4r}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2r}\right)\right) & \frac{\sqrt{N}}{2} < r < \frac{\sqrt{N}}{\sqrt{2}} \end{cases}. \quad (3)$$

Hence, $r_{avg} = \int_{2K}^{\frac{\sqrt{N}}{\sqrt{2}}} rf(r)dr = \frac{\pi\sqrt{N}}{3\sqrt{2}} - \frac{16\pi K^3}{3N} - \frac{\sqrt{N}}{6}\left(\sqrt{2}\,(\pi-1) - \log\left(\sqrt{2}+1\right)\right)$.

Next, we compute $\overline{x}$. For a pair of nodes to interfere with the transmission between i and j, they should be within range of each other, have at least one packet to exchange, and the scheduling mechanism should allow them to exchange packets. Lets define p_m to be the probability that two nodes are within a distance K of each other. Since the stationary node location distribution is uniform, $p_m = \frac{\pi K^2}{N}$. These pair of nodes will have at least one packet to exchange with probability p_{pkt} (defined in Lemma 3.2). The probability that the scheduling mechanism allows a pair of nodes to exchange packets equals $\sum_{a,c} \frac{1}{t(a,c)} P(E_a)P(E_c \mid E_a)$. The values of $t(a,c)$, $P(E_a)$ and $P(E_c \mid E_a)$ were derived in Section 3.3.2. Since there are $\left(\begin{array}{c} M-a \\ 2 \end{array}\right)$ possible pairs of nodes, the expected number of interfering transmissions equals $p_m p_{pkt} \left(\sum_{a,c} \frac{1}{t(a,c)} P(E_a)P(E_c \mid E_a)\right)\left(\begin{array}{c} M-a \\ 2 \end{array}\right)$.

$P(E_{M-a} \mid x, r_1, r_2, \ldots r_x)$ is the complement of the outage probability and depends on the channel model. The channel model only affects this term in the entire analysis. The outage probabilities have been calculated for several realistic channel models including the Rayleigh-Rayleigh fading channel [16] (both the desired signal and the interfering signal are Rayleigh distributed), the Rician-Rayleigh fading channel [37] (the desired signal has Rician and the interfering signal has Rayleigh distribution), the log normal shadowing channel [26] and the superimposed Rayleigh fading and log normal shadowing channel [36]. The results from these papers can be directly used here to make the framework work for any of these channel models.

LEMMA 3.3. *For the Rayleigh-Rayleigh fading channel model,* $P(E_{M-a}) = \left(\frac{1}{1+\frac{\Theta\left(\frac{2K}{3}\right)^4}{r_{avg}^4}}\right)^{\overline{x}}$.

Proof: Kandukuri et al [16] evaluated the outage probability for the Rayleigh-Rayleigh fading channel to be $1 - \prod_{i=1}^{x} \frac{1}{1+\frac{\Theta P_i^R}{P_0^R}}$, where P_0^R is the received power from the desired signal and P_i^R is the received power from the i^{th} interferer. Assuming all the nodes are transmitting at the same power level and $\alpha = 2$ in the distance attenuation model, $P(E_{M-a} \mid x, r_1, r_2, \ldots r_x) = \prod_{i=1}^{x} \frac{1}{1+\frac{\Theta r_0^2}{r_i^2}}$, where r_0 = the distance between nodes i and j. Replacing r_0, x and the r_i's with their expected values gives the result. □

Note that the preceding analysis ignores the interference from a node outside the scheduling area transmitting to a node within this area. (Section 3.3.2 takes care of the opposite.) But, since the number of such transmissions are very few as compared to the other transmissions taking place outside the scheduling area, despite their relative proximity, their effect on the total interference is negligible. The simulation results in Section 5 verify this.

Now, we have all the components to put together to find p_{txS} in Equation (1). In Section 4, we present case studies to demonstrate how the framework is used for performance analysis of routing schemes.

3.4 Extension: The Framework for a Mobility Model with Non-Uniform Node Location Distribution

The preceding analysis assumes a mobility model with a uniform steady state node location distribution. Real world mobility traces indicate that this assumption is not realistic [12,18]. Nodes usually have some locations where they spend a large amount of time. Additionally, node movements are not identically distributed. Different nodes visit different locations more often, and some nodes may be more mobile than others. Based on this intuition, Spyropoulos et al [33] proposed a more realistic and analytically tractable community-based mobility model. To demonstrate the applicability of the contention framework to any mobility model, we rederive the terms in Equation (1) which depend on the mobility model, for the community based mobility model.

We first define the family of Community-Based Mobility models: The model consists of two states, namely the 'local' state and the 'roaming' state. The model alternates between these two states. Each node inside the network moves as follows: (i) Each node i has a local community C_i of size $\| C_i \| = c^2 N, c \in (0,1]$. A node's movement consists of local and roaming epochs. (ii) A **local epoch** is a Random Direction movement restricted inside area C_i with average epoch length $\overline{L}_c$. (iii) A **roaming epoch** is a Random Direction movement inside the entire network with expected length $\overline{L}$. (iv) (Local state L) If the previous epoch of node i was a local one, the next epoch is a local one with probability p_l^i, or a roaming epoch with probability $1 - p_l^i$. (v) (Roaming state R) If the previous epoch of node i was a roaming one, the next epoch is a roaming one with probability p_r^i, or a local one with probability $1-p_r^i$. (Note that nodes are more likely to be found within the community than outside the community.)

The Community-based mobility model can be used to model a large number of scenarios by tuning its parameters. For ease of exposition, we choose a specific scenario where all the nodes belong to the same community, the p_l^i and p_r^i for all the nodes i are the same and equal to p_l and p_r, and the community is very small (which is the case for a conference or an office building scenario). All nodes within the community are within range of each other.

Equation (1) is independent of the mobility model, and hence still holds. But, the values of $P(E_a)$, $P(E_c \mid E_a)$,

$t(a,c)$ and $P(E_{M-a})$ will have to be re-derived. In general, these expressions can be evaluated after conditioning on the current transmitter and receiver location. Then, the law of total probability would be used to remove the condition. For the community-based mobility model, we condition over whether both the transmitter and the receiver or one of them or none of them, lie within the community. The following Lemma finds p_{txS} when both the transmitter and the receiver are in the community.

LEMMA 3.4. *When the transmitter and the receiver are within the community, the probability that they successfully exchange the packet in one time slot is* $p_{txS} = \Big(\sum_{s=0}^{s_{BW}-1} P(E_s^{E[S]}) + \sum_{s=s_{BW}}^{E[S]-1} \frac{s_{BW}}{s+1} P(E_s^{E[S]})\Big) \times \Big(\sum_{k=2}^{M} \sum_{a,c} Pr(E_k) \frac{1}{t(a,c,k)} P(E_a \mid E_k) P(E_c \mid E_a, E_k) P(E_{M-a} \mid E_k)\Big)$, *where:*

- (a) E_k *is the event that there are* k *nodes in the community.* $P(E_k) = \begin{pmatrix} M-2 \\ k-2 \end{pmatrix} \pi_l^{k-2} \pi_r^{M-k}$ *where* $\pi_l = \frac{1-p_r}{2-p_l-p_r}$ *is the probability that a particular node is in the local state and* $\pi_r = \frac{1-p_l}{2-p_l-p_r}$ *is the probability that a particular node is in the roaming state.*
- (b) $P(E_a \mid E_k)$ *is the probability that out of the* $M-k$ *nodes in the roaming state,* a *of them are within a distance* $2K$ *of the transmitter and is equal to* $\begin{pmatrix} M-k \\ a \end{pmatrix} (p_1)^a (1-p_1)^{M-k-a}$*. The value of* p_1 *was derived in Lemma 3.1.*
- (c) $P(E_c \mid E_a, E_k)$ *is the probability that there are* c *nodes within the* $2K < d \leq 3K$ *ring of the transmitter and is equal to* $\begin{pmatrix} M-k-a \\ c \end{pmatrix} (p_2)^c (1-p_2)^{M-a-k-c}$*. The value of* p_2 *was derived in Corollary 3.1.*
- (d) $t(a,c,k) = 1 + p_{pkt} \left(\left(\begin{pmatrix} k \\ 2 \end{pmatrix} - 1 \right) + p_a \begin{pmatrix} a \\ 2 \end{pmatrix} + \frac{acp_c}{2} \right)$*. The values of* p_a, p_c *and* p_{pkt} *were derived in Lemma 3.2.*
- (e) $P(E_{M-a} \mid E_k)$ *is the probability that the packet exchange does not get corrupted due to interference from other transmissions given that there are* $M-k-a$ *nodes at a distance of more than* $2K$ *from the transmitter. For the Rayleigh-Rayleigh fading model, the expression derived in Lemma 3.3 still holds, except the expected number of interfering transmissions is now equal to* $\overline{x} = \frac{\pi K^2}{N} p_{pkt} \Big(\sum_{a,c} \frac{1}{t(a,c)} P(E_a \mid E_k) P(E_c \mid E_a, E_k) \Big) \begin{pmatrix} M-k-a \\ 2 \end{pmatrix}$.

Sketch of Proof: The Lemma can be proved using geometric and combinatorial arguments similar to the ones made in the proofs in Sections 3.3.2 and 3.3.3. The main difference here is that now only $M-k$ nodes are moving according to the Random Direction mobility model over the entire network while the rest of the k nodes are within the transmission range of each other. $\square$

The previous Lemma finds p_{txS} when both the transmitter and the receiver are within the community. Similar Lemmas can be derived when only one of them is within the community and when none of them is within the community. Combining everything together and using the law of total probability yields the unconditioned value of p_{txS}.

Remark: Other scenarios for Community-based mobility where communities are not small and where only some of the nodes share a community can be analyzed in a similar manner by conditioning over whether the transmitter and the receiver are in their local or roaming states. While the complexity of the analysis increases as we go towards a more generalized Community-based mobility model, the analysis remains tractable.

4. CASE STUDY: DELAY ANALYSIS IN MOBILE AD HOC NETWORKS

This section demonstrates how the framework is applied to derive expressions for performance metrics of interest for different routing schemes. Specifically, we find the expected end-to-end delay for two different routing schemes: (i) Shortest path routing in a fully connected mobile ad hoc network and, (ii) Epidemic routing in an intermittently connected mobile ad hoc network. Then, to demonstrate an application of the framework to answer a real world question, we derive the optimal parameter value for a competitive routing scheme proposed for sparse networks [28, 34]. Though the analysis will go through for any channel and traffic model, and any/more realistic mobility models like the community based ones, for ease of exposition, these case studies use the Random Waypoint mobility model on a torus for node mobility, assume that each node acts as a source sending packets to a randomly selected destination and use the Rayleigh-Rayleigh fading model as the channel model.

4.1 Shortest Path Routing in a Fully Connected Mobile Ad Hoc Network

In this section, we consider mobile ad hoc networks which are dense enough to be connected. Several different routing schemes have been proposed for such networks, for example DSR, AODV, etc [24]. All these schemes assume that at least one complete path exists between the source and the destination and they try to find the shortest path amongst all the possible paths. These schemes differ in the way they find this shortest path. So, the overhead incurred by these schemes is different. But once the shortest path is found, the delay incurred is the same. (We assume that the additional delay incurred due to the higher priority of routing control messages is insignificant because the size of routing control messages is much smaller than the packet size.)

We now analyze the performance of shortest path routing with contention in the network. We first find the value of p_{ex}^{sp} (the probability that nodes i and j want to exchange a particular packet) for shortest path[4] and then find the expected end-to-end delay.

[4]Note that p_{ex} is the only parameter in the framework which depends on the routing scheme.

LEMMA 4.1. *$p^{sp}_{ex} = \frac{2}{Mk}$, where $k = \frac{\pi K^2 M}{N}$ is the average degree of a node.*

Proof: The proof follows from simple combinatorics. □

THEOREM 4.1. *Let $E[D_{sp}]$ denote the expected delay of shortest path routing. Then, $E[D_{sp}] = \frac{\sqrt{2}d_{SD}}{Kp_{txS}cos\left(\frac{\pi}{2k}\right)}$, where $d_{SD} = \frac{\pi\sqrt{N}}{3\sqrt{2}} - \frac{\sqrt{N}}{6}\left(\sqrt{2}\left(\pi - 1\right) - log\left(\sqrt{2}+1\right)\right)$ is the expected distance between the source and the destination and k is the average degree of a node.*

Proof: For a mobility model with a uniform node distribution, [8] shows that the expected number of hops along the shortest path from the source to the destination is approximately equal to $\frac{\sqrt{2}d_{SD}}{Kcos\left(\frac{\pi}{2k}\right)}$. The packet moves one hop towards the destination in a time slot with probability p_{txS} (given by Equation (1)) and with probability $1 - p_{txS}$ it remains at the same position. Thus, the expected number of time slots it takes to deliver the packet to the destination is equal to $\frac{\sqrt{2}d_{SD}}{Kp_{txS}cos\left(\frac{\pi}{2k}\right)}$. d_{SD} is equal to $\int_0^{\frac{\sqrt{N}}{\sqrt{2}}} rf(r)dr$, where $f(r)$ denotes the probability density function of the distance between any two nodes on the torus and is given by Equation (3). □

Note that prior works have also modeled loss due to contention with a loss probability to be able to solve for performance metrics of interest [23, 28]. Our main contribution is to explicitly derive the value of this probability (p_{txS}) in terms of the network parameters.

4.2 Epidemic Routing in an Intermittently Connected Mobile Ad Hoc Network

Intermittently Connected Mobile Networks (also referred to as delay tolerant networks) [14, 34, 39] are networks where most of the time there does not exist a complete end-to-end path from the source to the destination. Even if such a path exists, it may be highly unstable because of the topology changes due to mobility and may change or break soon after it has been discovered. This situation arises when the network is quite sparse.

Epidemic routing [35] is one of the first schemes proposed to enable message delivery in such networks. The underlying idea behind epidemic routing is to flood the packet to all the nodes. Whenever two nodes meet, they exchange all the messages they don't have in common. This way, all messages are eventually spread to all nodes. The packet is delivered when the first node carrying a copy of the packet meets the destination. The packet will keep on getting copied from one node to the other node till its Time-To-Live (TTL) expires. For ease of analysis, we assume that as soon as the packet is delivered to the destination, no further copies of the packet are spread. [31, 34, 38] studied the performance of epidemic routing without contention. We now use the general framework to analyze its delay performance with contention in the network.

Before analyzing epidemic routing, we first define some properties of mobility models. We will use the statistics of these properties during the course of the analysis.

(i) Meeting Time: Let nodes i and j move according to a mobility model 'mm' and start from their stationary distribution at time 0. Let $X_i(t)$ and $X_j(t)$ denote the positions of nodes i and j at time t. The meeting time (M_{mm}) between the two nodes is defined as $min_t\{t : \|X_i(t) - X_j(t)\| \leq K\}$.

(ii) Inter-Meeting Time: Let nodes i and j start from within range of each other at time 0 and then move out of range of each other at time t_1, that is $t_1 = min_t\{t : \|X_i(t) - X_j(t)\| > K\}$. The inter meeting time (M^+_{mm}) of the two nodes is defined as $min_t\{t - t_1 : \|X_i(t) - X_j(t)\| \leq K\}$.

(iii) Contact Time: Assume that nodes i and j come within range of each other at time 0. The contact time τ_{mm} is defined as $min_t\{t - 1 : \|X_i(t) - X_j(t)\| > K\}$.

The statistics of these properties for the Random Waypoint mobility model were studied by [15] and [33]. The two important properties which we use during the course of the analysis are as follows: (i) The expected inter meeting time $E[M^+_{rwp}]$ for the Random Waypoint model is approximately equal to $E[M_{rwp}]$ and (ii) The tail of the distribution of the meeting time and the inter meeting time of the Random Waypoint model is exponential.

Now we analyze the performance of epidemic routing with contention in the network. To find the expected end-to-end delay for epidemic routing, we first find $E[D_{epidemic}(m)]$ which is the expected time it takes for the number of nodes that have a copy of the packet to increase from m to $m+1$.

LEMMA 4.2. *$E[D_{epidemic}(m)] = \frac{E[M_{rwp}]}{m(M-m)p_{success}}$, where $p_{success} = 1 - (1 - p_{txS})^{E[\tau_{rwp}]}$ is the probability that when two nodes come within range of each other, they successfully exchange the packet before going out of each other's range (within the contact time).*

Proof: $E[D^{rwp}_{epidemic}(m)]$ is the expected time it takes for the copies of a packet to increase from m to $m+1$. When there are m copies of a packet in the network, if one of the m nodes having a copy meets one of the other $M-m$ nodes not having a copy, there is a transmission opportunity to increase the number of copies by one. For sparse networks, we look at the tail of the distribution of the meeting time which is exponential for the Random Waypoint mobility model. The time it takes one of the m nodes to meet one of the other $M - m$ nodes is equal to the minimum of $m(M-m)$ exponentials, which is again an exponential random variable with mean $\frac{E[M_{rwp}]}{m(M-m)}$. Now when they meet, the probability that the two nodes are able to successfully exchange the packet is $p_{success}$. If they fail to exchange the packet, they will have to wait one inter meeting time to meet again. But, since $E[M_{rwp}] = E[M^+_{rwp}]$ and both meeting and inter meeting times have exponential tails, the expected time it takes for one of the m nodes to meet one of the other $M - m$ nodes again is still equal to $\frac{E[M_{rwp}]}{m(M-m)}$.

Hence, $E[D^{rwp}_{epidemic}(m)] = p_{success}\frac{E[M_{rwp}]}{m(M-m)} + 2p_{success}(1-p_{success})\frac{E[M_{rwp}]}{m(M-m)} + \ldots = \frac{E[M_{rwp}]}{m(M-m)p_{success}}$.

We now derive the value of $p_{success}$. With probability $1-p_{txS}$, the two nodes are unable to exchange the packet in one time slot. They are within range of each other for $E[\tau_{rwp}]$ number of time slots. (We are making an approximation here by replacing τ_{rwp} by its expected value.) Then $(1-p_{txS})^{E[\tau_{rwp}]}$ is the probability that the two nodes fail to exchange the packet while they are within range of each other. Thus, $p_{success} = 1-(1-p_{txS})^{E[\tau_{rwp}]}$. □

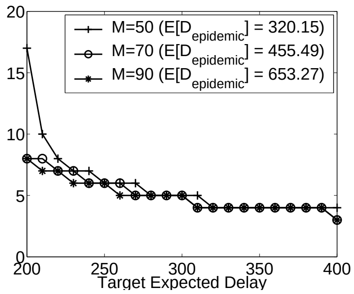


Figure 2: Minimum value of L which achieves the target expected delay for spraying based routing. Network parameters: $N = 100\times100, K = 6, \Theta = 5, E[S] = 30, \overline{T}_{stop} = 50, \overline{v} = 1, s_{BW} = 1$.

Next we find the values of $p^{epidemic}_{ex}$ for epidemic routing and then find the expected end-to-end delay.

LEMMA 4.3. $p^{epidemic}_{ex} = \sum_{m=1}^{M-1}\frac{2m(M-m)}{M(M-1)}\sum_{i=m}^{M-1}\frac{1}{M-1}\frac{\frac{1}{m(M-m)}}{\sum_{j=1}^{i}\frac{1}{j(M-j)}}$.

Proof: Let there be m copies of packet B in the network. Then the probability that node i has a copy is equal to $\frac{m}{M}$ and the probability that node j does not have a copy given that node i has one is equal to $\frac{(M-m)}{M-1}$. Thus, the probability that node i and node j want to exchange packet B given that there are m copies of packet B in the network is equal to $\frac{2m(M-m)}{M(M-1)}$.

Now, we find the probability that there are m copies of packet B in the network. The copies of a packet keep on increasing till the packet is delivered to the destination. The probability that the destination is the k^{th} node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2\le k\le M$. Packet B will have m copies in the network only if the destination wasn't amongst the first $m-1$ nodes to receive a copy. The amount of time Packet B has m copies in the network is equal to $E[D^{mm}_{epidemic}(m)]$. Hence, the probability that there are m copies of packet B in the network equals $\sum_{i=m}^{M-1}\frac{1}{M-1}\frac{E[D^{mm}_{epidemic}(m)]}{\sum_{j=1}^{i}E[D^{mm}_{epidemic}(j)]}$.

Applying the law of total probability over the random variable m and subsituting the value of $E[D^{mm}_{epidemic}(m)]$ from Lemma 4.2 gives p_{ex}. □

THEOREM 4.2. *Let $E[D_{epidemic}]$ denote the expected delay of epidemic routing. Then,*

$$E[D_{epidemic}] = \sum_{i=1}^{M-1}\frac{1}{M-1}\sum_{m=1}^{i}\frac{E[M_{rwp}]}{m(M-m)p_{success}} \quad (4)$$

Proof: The probability that the destination is the i^{th} node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2\le i\le M$. The amount of time it takes for the i^{th} copy to be delivered is equal to $\sum_{m=1}^{i}E[D_{epidemic}(m)]$. Applying the law of total probability over the random variable i and substituting the value of $E[D_{epidemic}(m)]$ from Lemma 4.2 gives Equation (4). □

4.3 Application: Spraying Small Fixed Number of Copies to Reduce Overhead of Epidemic Routing

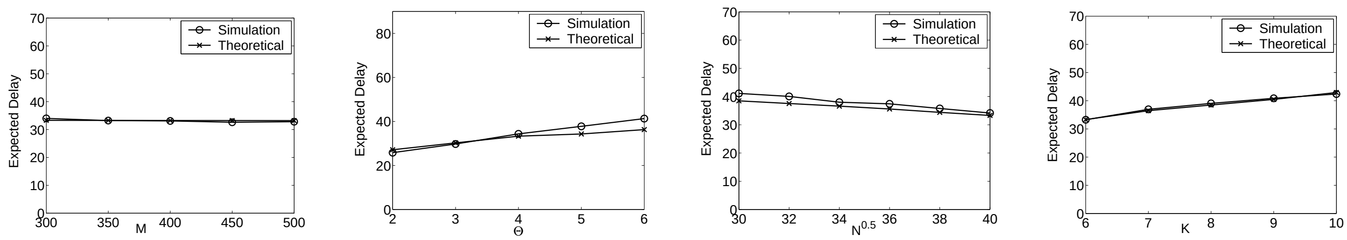
In this section, we find the optimal parameter for controlled replication or *spraying* based routing schemes [28, 32, 34] which have been proposed to reduce the overhead of epidemic routing for routing packets in sparse networks. In this approach, a small fixed number of copies are distributed to a number of distinct relays. Then, each relay carries its copy until it encounters the destination or until the TTL of the packet expires. By having multiple relays looking independently and in parallel for the destination, these protocols create enough diversity to explore the sparse network more efficiently while keeping the resource usage per message low.

In this section we analyze how to choose L (the number of copies used) in order for spraying based routing schemes to achieve a specific expected delay. We want the minimum value of L which achieves a target delay as bigger values of L consume more resources.

First we state the value of p^{spray}_{ex} for spraying based routing and then the expected delay. The derivation proceeds in a manner similar to the derivation of the expected delay of epidemic routing.

LEMMA 4.4. $p^{spray}_{ex} = \left(\frac{2Lp_{dest}(L)}{M(M-1)}\frac{E[D_{spray}(L)]}{\sum_{k=1}^{L}E[D_{spray}(k)]}\right) + \left(\frac{2}{M-1}\sum_{m=1}^{L-1}\sum_{i=m}^{L}p_{dest}(i)\frac{E[D_{spray}(m)]}{\sum_{k=1}^{i}E[D_{spray}(k)]}\right)$, *where* $E[D_{spray}(m)] = \begin{cases}\frac{E[M_{rwp}]}{(M-1)p_{success}} & 1\le m<L\\ \frac{E[M_{rwp}]}{Lp_{success}} & m=L\end{cases}$ *is the expected time it takes it takes for the copies of a packet to increase from m to $m+1$,* $p_{dest}(i) = \begin{cases}\left(\prod_{j=1}^{i-1}\frac{M-j-1}{M-1}\right)\frac{i}{M-1} & 1\le i<L\\ \left(\prod_{j=1}^{i-1}\frac{M-j-1}{M-1}\right) & i=L\end{cases}$ *is the probability that the destination is the i^{th} node to receive a copy of the packet and $p_{success} = 1-(1-p_{txS})^{E[\tau_{rwp}]}$.*

THEOREM 4.3. *Let $E[D_{spray}]$ denote the expected delay*



(a) (b) (c) (d)

Figure 3: Simulation and Analytical results for the expected delay of shortest path routing. Network Parameters: (a) $N = 40 \times 40, K = 6, \Theta = 4, E[S] = 20, T_{stop} = 0, \overline{v} = 1$ **(b)** $N = 40 \times 40, M = 300, K = 6, E[S] = 20, T_{stop} = 0, \overline{v} = 1$ **(c)** $M = 300, K = 6, \Theta = 4, E[S] = 20, T_{stop} = 0, \overline{v} = 1$ **(d)** $N = 40 \times 40, M = 300, \Theta = 4, E[S] = 20, T_{stop} = 0, \overline{v} = 1$

of spraying based routing. Then,

$$E[D_{spray}] = \sum_{i=1}^{L} p_{dest}(i) \sum_{m=1}^{i} E[D_{spray}(m)]. \quad (5)$$

The expected value of delay depends on the value of L through $p_{dest}(i)$ and $E[D_{spray}(m)]$ (see Equation (5)). $E[D_{spray}(m)]$ depends on $p_{success}$ which again depends on the value of L (see Lemma 4.4). Due to the complicated nature of the function expressing $E[D_{spray}]$ in terms of L, we numerically solve for L. Figure 2 plots the minimum value of L which achieves a target delay for different values of M. (For reference, we also show the expected delay of epidemic routing for each scenario.) Note that spraying based routing has significantly better performance than epidemic routing, because it creates far less contention. Along the same lines, it requires a surprisingly low number of copies to achieve good performance.

5. SIMULATION RESULTS

We use simulations to verify that the approximations made during the course of the analysis do not have a significant impact on the accuracy of the analysis by comparing the simulation and the analytical results. We use a custom simulator written in C++ for simulations. The simulator avoids excessive interference by implementing a scheduling scheme which prohibits simultaneous transmissions within two hops of each other. (Later we will also present simulation results that compare our model with a simulation scenario in which the standard IEEE 802.11 scheduling scheme is used.) It incorporates interference by adding the received signal from other simultaneous transmissions (outside the scheduling area) and comparing the signal to interference ratio to the desired threshold. The simulator allows the user to choose from different physical layer, mobility and traffic models. We choose the Rayleigh-Rayleigh fading model for the channel, Random waypoint model for node mobility and Poisson arrivals in our simulations.

We will run simulations to test the effect of all the major approximations. The contention framework has the following two major approximations:
(i) Replacing S by $E[S]$ in the expression of $P(E_{bw})$ in Section 3.3: The effect of this approximation can be studied by varying the value of p_{ex}, which in turn depends on M for both shortest path and epidemic routing (see Lemmas 4.1 and 4.3). Figures 3(a) and 4(a) plot the expected delay for shortest path and epidemic routing obtained through analysis and simulations as a function of M. Since both the curves in both the plots are close to each other, we conclude that this approximation is accurate enough for the Poisson arrival process (which is the most commonly used process to model arrivals in a network). In general, the approximation will become more inaccurate as the variance in S will increase.
(ii) Replacing the random variables representing the number of interfering transmissions (x) and their distance from the desired receiver (r_i's) by their expected values in Section 3.3.3: The accuracy of this approximation can be studied by varying Θ. Figures 3(b) and 4(b) plot the expected delay for shortest path and epidemic routing obtained through analysis and simulations as a function of Θ. Figure 3(b) shows that even though this approximation worsens as Θ increases, yet the analysis remains accurate enough.

The delay analysis of shortest path routing uses an approximate value for the expected number of hops along the shortest path to the destination (see Theorem 4.1). The effect of this approximation can be studied by varying N and K. Figures 3(c) and 3(d) plot the expected delay for shortest path routing obtained through analysis and simulations as a function of N and K respectively. Since both the curves in both the plots are close to each other, we conclude that this approximation is accurate enough.

The delay analysis of epidemic routing makes the following two approximations: (i) replacing the contact time by its expected value in the expression of $p_{success}$ (see Lemma 4.2) and (ii) assuming the entire meeting and inter meeting time distribution to be exponential. The effect of the first approximation can be studied by varying N and K while the effect of the second can be studied by varying M. Figures 4(a), 4(c) and 4(d) plot the expected delay for epidemic routing obtained through analysis and simulations as a function of M, N and K respectively. Since both the curves in all the plots are close to each other, we conclude that the approximations are accurate enough.

All these plots show that the approximations do not induce significant inaccuracies in the analysis. Finally, to show the accuracy of the proposed framework as compared to the previous models, we add the curve obtained from our framework to the set of curves in Figure 1. Figure 5 presents the corresponding plot. The accuracy of our model as compared to previous attempts is striking.

Unlike in Figures 3 and 4, in Figure 5 we use the standard 802.11 scheduling mechanism to derive the simulation results. Thus, this curve also compares the accuracy of using a two-hop scheduling mechanism in the analysis instead of the more complex 802.11 mechanism. For this plot, the two-hop scheduling is a good approximation, but we expect that as the density of the network will increase, the performance of the two-hop scheduling mechanism will start to deviate from 802.11. As a final note, both Figures 4(d) and 5 plot the delay of epidemic routing (flooding) as a function of the transmission range (K). But, in Figure 4(d) the network is more sparse and increasing K improves the delay, whereas in Figure 5 the network is denser, and increasing K results in increased levels of contention, which, in turn, worsens the performance.

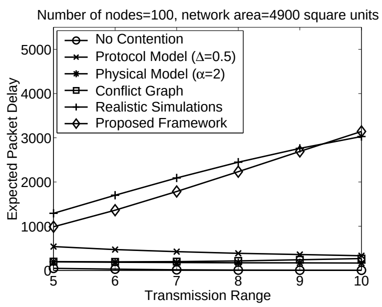


Figure 5: Expected packet delay for flooding in mobile multi-hop networks.

6. DISCUSSION

This paper presents a framework to analyze wireless contention. The proposed framework incorporates all the three manifestations of contention, namely finite link bandwidth, local scheduling and interference from nodes outside the scheduling area. It can be used to evaluate the performance of a network under a wide variety of routing strategies. The framework can be used with any realistic channel and mobility models, and allows the derivation of explicit, rather than asymptotic results. It is to be noted that the analysis assumes that the network is in steady state. As a case study, we use this framework to find the expected packet delay for shortest path routing in a dense mobile ad hoc network and epidemic routing in an intermittently connected mobile network.

The analysis presented in this paper can be easily modified for any scheduling mechanism which places restrictions on the distances between simultaneous transmissions. Most of the commonly used scheduling mechanisms, like CSMA-CA and node exclusive scheduling (used for Bluetooth networks) [19] belong to this category. Other scheduling mechanisms like TDMA and ALOHA are even easier to incorporate in the framework because they specify the probability that scheduling allows a pair of nodes to exchange packets, which directly gives $P(E_{sch})$.

Another assumption we make is that the movement of each node is independent of the other nodes. Derivation of the expressions for $P(E_a), P(E_c \mid E_a), t(a,c), \overline{x}$ and r_{avg} use this assumption. Using a mobility model with correlated movement patterns will change these derivations. These expressions can be rederived after conditioning on the location of the transmitter and receiver. The conditioning can be removed by using the law of total probability. Depending on the actual correlation structure, the resulting integrals might or might not be expressible in closed form, but one can still find their values using numerical methods.

In ongoing work, we are using the proposed framework to analyze and design more efficient routing schemes for partially connected networks.

7. REFERENCES

[1] D. Aguuayo, J. Bicket, S. Biswas, G. Judd, and R. Morris. Link-level measurements from an 802.11b mesh network. In *Proceedings of ACM SIGCOMM*, 2004.

[2] D. Aldous and J. Fill. Reversible markov chains and random walks on graphs. (monograph in preparation.). *http://stat-www.berkeley.edu/users/aldous/RWG/book.html*.

[3] A. Bader and E. Ekici. Throughput and delay optimization in interference-limited multi-hop networks. In *Proceedings of ACM MOBIHOC*, 2006.

[4] J. Y. L. Boudec and M. Vojnovic. Perfect simulation and stationarity of a class of mobility models. In *Proceedings of IEEE INFOCOM*, 2005.

[5] G. Brar, D. Blough, and P. Santi. Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks. In *Proceedings of ACM MOBICOM*, 2006.

[6] A. Brzezinski, G. Zussman, and E. Modiano. Enabling distributed throughput maximization in wireless mesh networks - a partitioning approach. In *Proceedings of ACM MOBICOM*, 2006.

[7] C. Chaudet, D. Dhoutaut, and I. Lassous. Performance issues with IEEE 802.11 in ad hoc networking. *IEEE Communications Magazine*, 43:110–116, July 2005.

[8] P. Contla and M. Stojmenovic. Estimating hop counts in position based routing schemes for ad hoc networks. *Telecommunication Systems*, 22:109–118, 2003.

[9] A. Gamal, J. Mammen, B. Prabhakar, and D. Shah. Throughput-delay tradeoff in wireless networks. In *Proceedings of IEEE INFOCOM*, 2004.

[10] D. Ganesan, B. Krishnamachari, A. Woo, D. Culler, D. Estrin, and S. Wicker. Complex behavior at scale: An experimental study of low-power wireless sensor networks. Technical Report UCLA/CSD-TR 02-0013, UCLA, 2002.

[11] P. Gupta and P. Kumar. Capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2), 2000.

[12] T. Henderson, D. Kotz, and I. Abyzov. The changing usage of a mature campus-wide wireless network. In *Proceedings of ACM MOBICOM*, 2004.

[13] K. Jain, J. Padhye, V. Padmanabhan, and L. Qiu. Impact of interference on multi-hop wireless network performance. In *Proceedings of ACM MOBICOM*, 2003.

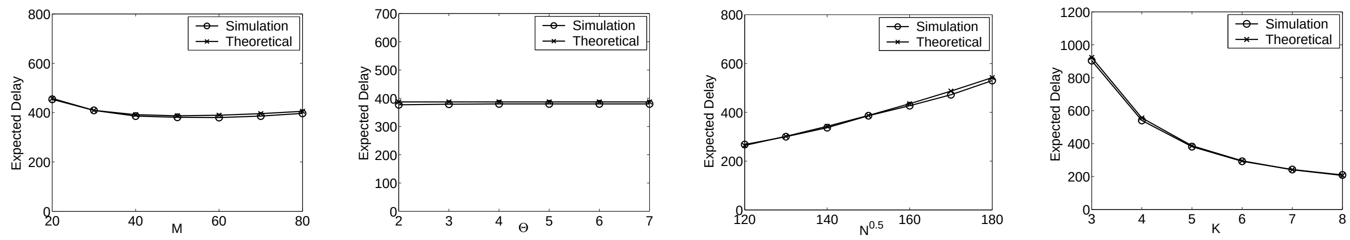
[14] S. Jain, K. Fall, and R. Patra. Routing in a delay tolerant network. In *Proceedings of ACM SIGCOMM*, Aug. 2004.

[15] A. Jindal and K. Psounis. Fundamental properties of mobility for realistic performance analysis of mobility-assisted networks. In *Proceedings of IEEE PerCom Workshop on Intermittently Connected Mobile Ad Hoc Networks*, 2007.

[16] S. Kandukuri and S. Boyd. Optimal power control in interference-limited fading wireless channels with outage-probability specifications. *IEEE Transactions on Wireless Communications*, 1, Jan. 2002.

[17] X. Lin and N. Shroff. The fundamental capacity-delay tradeoff in large mobile ad hoc networks. In *Third Annual Mediterranean Ad Hoc Networking Workshop*, 2004.

[18] M. McNett and G. Voelker. Access and mobility of wireless PDA users. *ACM Mobile Computing and Communications Review*, 2003.



(a) (b) (c) (d)

Figure 4: Simulation and Analytical results for the expected delay of epidemic routing. Network Parameters: (a) $N = 150 \times 150, K = 5, \Theta = 5, T_{stop} = 0, \overline{v} = 1$ **(b)** $N = 150 \times 150, M = 50, K = 5, E[S] = 50, T_{stop} = 0, \overline{v} = 1$ **(c)** $M = 50, K = 5, \Theta = 5, E[S] = 50, T_{stop} = 0, \overline{v} = 1$ **(d)** $N = 150 \times 150, M = 50, \Theta = 5, E[S] = 50, T_{stop} = 0, \overline{v} = 1$

[19] B. Miller and C. Bisdikian. *Bluetooth Revealed: The Insider's Guide to an Open Specification for Global Wireless Communications*. Prentice Hall, 2000.

[20] T. Moscibroda, R. Wattenhofer, and A. Zollinger. Topology control meets sinr: The scheduling complexity of arbitrary topologies. In *Proceedings of ACM MOBIHOC*, 2006.

[21] K. Nahm, A. Helmy, and C. J. Kuo. TCP over multihop 802.11 networks: issues and performance enhancements. In *Proceedings of ACM MOBIHOC*, 2005.

[22] M. Neely and E. Modiano. Capacity and delay tradeoffs for ad-hoc mobile networks. *IEEE Transactions on Information Theory*, 51, 2005.

[23] J. Padhye, R. Draves, and B. Zill. Routing in multi-radio, multi-hop wireless mesh networks. In *Proceedings of ACM MOBICOM*, 2004.

[24] C. Perkins. *Ad Hoc Networking*. Addison-Wesley, first edition, 2001.

[25] T. Rappaport. *Wireless Communications: Principles and Practice*. Prentice Hall, 2 edition, 1996.

[26] H. Sachs. A realistic approach to defining the probability of meeting acceptable receiver performance criteria. *IEEE Transactions on Electromagnetic Compatibility*, EMC-13:3–6, 1971.

[27] G. Sharma, R. R. Mazumdar, and N. B. Shroff. Delay and capacity trade-offs in mobile ad hoc networks: A global perspective. In *Proceedings of IEEE INFOCOM*, 2006.

[28] T. Small and Z. Haas. Resource and performance tradeoffs in delay-tolerant wireless networks. In *Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN)*, 2005.

[29] P. Soldati, B. Johansson, and M. Johansson. Proportionally fair allocation of end-to-end bandwidth in STDMA networks. In *Proceedings of ACM MOBIHOC*, 2006.

[30] D. Son, B. Krishnamachari, and J. Heidemann. Experimental analysis of concurrent packet transmissions in low power wireless networks. In *Proceedings of ACM SENSYS*, 2006.

[31] T. Spyropoulos, K. Psounis, and C. S. Raghavendra. Single-copy routing in intermittently connected mobile networks. In *Proceedings of IEEE SECON*, 2004.

[32] T. Spyropoulos, K. Psounis, and C. S. Raghavendra. Spray and wait: Efficient routing in intermittently connected mobile networks. In *Proceedings of ACM SIGCOMM workshop on Delay Tolerant Networking (WDTN)*, 2005.

[33] T. Spyropoulos, K. Psounis, and C. S. Raghavendra. Performance analysis of mobility-assisted routing. In *Proceedings of ACM MOBIHOC*, 2006.

[34] T. Spyropoulos, K. Psounis, and C. S. Raghavendra. Efficient routing in intermittently connected mobile networks: The multi-copy case. *to appear in IEEE Transactions on Networking*, 2007.

[35] A. Vahdat and D. Becker. Epidemic routing for partially connected ad hoc networks. Technical Report CS-200006, Duke University, Apr. 2000.

[36] A. Williamson and J. Parsons. Outage probability in a mobile radio system subject to fading and shadowing. *IEEE Electronics Letters*, 21:622–623, 1985.

[37] Y.-D. Yao and A. Sheikh. Outage probability analysis for microcell mobile radio systems with cochannel interferers in rician/rayleigh fading enviornment. *IEEE Electronics Letters*, 26, June 1990.

[38] X. Zhang, G. Neglia, J. Kurose, and D. Towsley. Performance modeling of epidemic routing. In *Networking*, 2005.

[39] W. Zhao, M. Ammar, and E. Zegura. A message ferrying approach for data delivery in sparse mobile ad hoc networks. In *Proceedings of ACM MOBIHOC*, 2004.

[40] M. Zuniga and B. Krishnamachari. Analyzing the transitional region in low power wireless links. In *Proceedings of IEEE SECON*, 2004.

APPENDIX

Proof: (**Lemma 3.2**) It is given that there are a nodes within $2K$ distance of the transmitting node. Hence, there are $\binom{a}{2}$ pairs of these nodes. Lets choose one such pair and let $p_a = \Pr[$the nodes of this pair are within a distance K of each other$]$ and let $p_{pkt} = \Pr[$the nodes of this pair have at least one packet to exchange$]$. Out of these a nodes, i and j are within K distance of each other and have at least one packet to exchange. The rest of these $\binom{a}{2}$ pairs are within K distance of each other and have at least one packet to exchange with probability $p_a p_{pkt}$. Hence, the expected number of possible transmissions amongst these a nodes is $1 + p_a p_{pkt}\left(\binom{a}{2} - 1\right)$. To figure out the value of p_a, lets choose a pair of nodes amongst these a nodes and label the nodes u_1 and u_2. The probability that a node u_1 is at a distance x away from the transmitter is $\frac{2\pi x dx}{4\pi K^2}$. Conditioned over the fact that node u_1 is at a distance x from the transmitter, the probability that node u_2 is within K distance from u_1 is equal to the common area Y between the two circles in Figure 6 divided by total area where node u_2 can lie ($= 4\pi K^2$) . Using results from geometry, p_a can be derived to be $\frac{1}{16} + \frac{A}{4\pi K^2}$. The value of p_{pkt} can be derived from simple combinatorics to be $1 - (1 - p_{ex})^{E[S]}$.

Now, we quantify the contention due to the c nodes in the $2K < d \leq 3K$ ring. Contention arises when one of the a nodes can transmit to one of the c nodes. There are ac such pairs. Lets choose one such pair and label the corresponding nodes u_1 and u_3, where u_1 lies within $2K$

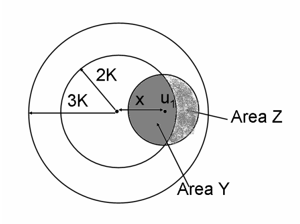


Figure 6: Node u_1 is at a distance x from the transmitter. If another nodes lies in the area Y or Z, transmissions between them can contend with the desired transmission.

distance of the transmitter while u_3 lies in the $2K < d \leq 3K$ ring. Define $p_c = \Pr[u_1$ and u_3 are within a distance K of each other$]$. Though both the nodes can transmit to each other, contention with the desired transmitter will arise only when u_1 transmits to u_3. Thus, the expected number of transmissions contending are $\frac{acp_c p_{pkt}}{2}$. To find p_c, notice that, conditioned over the fact that node u_1 is at a distance x from the transmitter, the probability that node u_3 is within K distance from u_1, is equal to the area Z in Figure 6 divided by the total area where node u_3 can lie ($= 5\pi K^2$). In a manner similar to that used for the derivation of p_a, p_c is derived to be $\left(\frac{3}{20} - \frac{A}{5\pi K^2}\right)$. $\square$