An Analytical Study of Fundamental Mobility Properties for Encounter-based Protocols

Thrasyvoulos Spyropoulos, Apoorva Jindal, and Konstantinos Psounis

***Abstract*— Traditionally, mobility in ad hoc networks was considered a necessary evil that hinders node communication. However, it has recently been recognized that mobility can be turned into a useful ally, by making nodes carry data between disconnected parts. Yet, this model of routing requires new theoretical tools to analyse its performance. A *mobility-assisted* or *encounter-based* protocol forwards data only when appropriate relays encounter each other. To be able to evaluate the performance of mobility-assisted routing schemes, it is necessary to know the statistics of various quantities related to node encounters.**

In this paper, we present an analytical methodology to calculate a number of useful encounter-related statistics for a general class of mobility models. We apply our methodology to derive accurate closed form expressions for popular mobility models like Random Direction and Random Waypoint, as well as for a more sophisticated mobility model that better captures behaviors observed in real traces. Finally, we show how these results can be used to analyze the performance of mobility-assisted routing schemes or other processes based on node encounters. We demonstrate that derivative results concerning the delay of various routing schemes are very accurate, under all mobility models examined.

I. Introduction

Traditionally, ad hoc networks have been viewed as a connected graph over which end-to-end routing paths need to be established. This view, albeit successfully applied in wired networks, does not always hold in wireless environments. Wireless signals are subject to multi-path propagation, fading, and interference making links unstable and lossy. Additionally, frequent node mobility (e.g. as in vehicular ad hoc networks—VANETs [1]) significantly reduces the time a "good" link exists, and constantly changes the network connectivity graph. As a result, wireless connectivity is volatile and usually intermittent, as nodes move in and out of range from access points or from each other, and as signal quality fluctuates.

What is more, there has been a growing interest in the past few years in wireless applications that can operate over networks that are disconnected for some or most of the time. Sensor networks can significantly increase their lifetime by powering down nodes often, or by using very low power radios [2], [3]. Tactical networks may also choose to operate in an intermittent fashion for LPI/LPD reasons (low probability of interception and low probability of detection) [4]. Finally, operation over disconnected networks may be desirable for economic reasons, as for example in the case of low-cost Internet provision in remote or developing communities [5]–[7], or to extend and sometimes bypass access point connectivity to the Internet [8]–[10]. These new networks are often referred to collectively as Delay Tolerant Networks (DTN [11]).

T. Spyropoulos is with the Communication Systems Group, ETH, Zurich, Switzerland and A. Jindal and K. Psounis are with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA, 90089 USA. E-mail: (spyropoulos@tik.ee.ethz.ch,apoorvaj,kpsounis@usc.edu).

To overcome the lack of end-to-end connectivity common in DTNs, *encounter-based* or *mobility-assisted* protocols have been proposed, where messages get carried by mobile nodes between disconnected parts of the network [11], [12]. Nodes carry a set of messages, possibly for long periods of time, until they encounter another node to which they can forward messages. During this encounter or *contact* they exchange messages according to a specific protocol, and continue their trip until a new contact occurs [13]–[16].

Since messages can be forwarded only during such a contact, the statistics of node encounters are of particular importance. First, the time until a new encounter (i.e. forwarding opportunity) occurs is an important component (if not the dominant one) of the queueing delay of a message that is carried by that node, and thus of the end-to-end delivery delay, as well. Thus, one needs to know the statistics of the *arrival process of such contacts* in order to analyze the behavior of any encounter-based protocol. Second, when such a contact occurs, it is usually of limited *duration*. Whether or not all messages in the queue that need to be forwarded will get a chance to, depends on how long this encounter will last. Also, if more than one node compete for the shared channel, the probability that an encounter is "lost" due to contention and interference depends on this duration as well.

Inter-contact times and contact durations have been the focus of investigation for some recent trace-based studies [17]–[19]. Nevertheless, the debate regarding whether these statistics follow a power-law [17] or have an exponential tail [19] is ongoing. Further, when it comes to synthetic, "random" mobility models, only few of the mobility properties that are relevant to node contacts have been studied. To enable a complete analytical treatment of various encounter-based schemes, a number of statistical properties regarding encounter times and encounter durations need to be derived, which largely depend on the mobility model in hand.

With this in mind, in this paper we first present the mobility properties that are necessary to analyze the performance of encounter-based protocols. We then provide a methodology to analyze the statistics of these properties for *epoch-based* mobility models, that is, for models according to which a node's movement consists of a sequence of "atomic trips" in the network. This includes simple, popular models like the Random Direction [20] and Random Waypoint [21] mobility, but also some more sophisticated models, like the Community-based model of [22], [23], which have been shown to better capture the behaviors observed in traces. (In [24] we studied the same mobility properties for a random walk on a lattice.) The reason we focus on such epoch-based mobility models is twofold: first, they are easier to analyze and demonstrate the basic methodology for deriving encounter-related properties, and second, synthetic epoch-based models like the Community-based can successfully capture real-life mobility properties [19], [23].

Finally, we demonstrate how our various encounter time results can be readily used in a general analytical framework for mobility-assisted routing, using the delay of epidemic routing [13] as a case study. By simply "plugging in" the respective encounter-related quantities into generic expressions about the performance of different algorithms (e.g. derived using Markov Chains [25], [26], fluid models [14], etc.), we show that derivative results based on these expressions are very accurate, under various mobility models. As a final note, even though the focus of this paper is on mobility-assisted routing, the analytical methods and expressions derived here could be applicable to other processes that are based on node encounters, like virus spread through wireless devices [27] or the reception of broadcast channels [28].

In the next section we introduce our problem setting, the various encounter-related quantities we're interested at, and the methodology to derive them. Then, in Sections III and IV, we derive accurate closed form expression for these quantities for simple "epoch-based" mobility models, a popular class of synthetic mobility models. Further, to show how our methodology can be extended for more sophisticated mobility models, in Section V we derive analytically the same encounter-related properties for a more realistic mobility model, as well. Section VI incorporates the various expressions into a general analytical framework that can be used to predict the performance of mobility-assisted routing under various mobility models. Finally, Section VII discusses some related work and Section VIII concludes the paper.

II. Encounter-Related Statistics for Epoch-based Mobility Models

In this section we look at what mobility properties are relevant to encounter-based protocols, with our focus on encounter-based (or mobility-assisted) routing. In encounter-based protocols, most events of interest (e.g. forwarding) occur only when two nodes are in contact. Consequently, we argue that the following statistical properties of contacts must be available, under a given mobility model, in order to analyze the performance of a protocol in this setting.

Hitting and Meeting Times: The first quantities of interest are *hitting* and *meeting* times, and their expected values. The expected meeting time under a given mobility model is essentially the time until a new contact occurs between two chosen nodes A and B (if we start looking at them at a random point in time), and thus the expected time until these nodes can "interact". (The hitting time corresponds to the case where one of the nodes is static.) Furthermore, if we are interested instead in the next contact between A and a *subset of nodes* in the network (this is commonly the case, as for example in the "time until any of x relays encounters the destination of a message"), then we would like to find the minimum of a set of meeting times (between A and each of the nodes in the subset). To be able to calculate this, we need the hitting/meeting time distribution, as well, or at least the distribution's tail.

We give a formal definition of the hitting and the meeting time here. $X_i(t)$ denotes the position in the network of mobile node i at time t, X_i the position of a static node i, and K the transmission range of a node.

Definition 2.1 (Hitting Time): Let a node i move according to mobility model "mm", and start from its stationary distribution at time 0. Let j be a static node with uniformly chosen X_j, then the hitting time (T_{mm}) is defined as the time it takes node i to first come within range of node j, that is $T_{mm} = \underset{t}{min}\{t : \|X_i(t) - X_j\| \leq K\}$.

Definition 2.2 (Meeting Time): Let nodes i and j move according to a mobility model "mm" and start from their stationary distribution at time 0. The meeting time (M_{mm}) between the two nodes is defined as the time it takes them to first come within range of each other, that is $M_{mm} = \underset{t}{min}\{t : \|X_i(t) - X_j(t)\| \leq K\}$.

Contact Duration and Inter-Meeting Time: Knowing the hitting and meeting times allows one to calculate the delay of various mobility-assisted schemes *under ideal conditions of infinite bandwidth and buffer space* [22], [26], [29]. Although this might be a useful approximation for low traffic scenarios or low-resource protocols [16], it is inaccurate when resources are rather limited, e.g. in sparse sensor networks for wildlife tracking [2], or when the protocols utilize a lot of resources, e.g. when epidemic protocols are used which lead to a lot of contention and overuse the available resources [13], [30], [31].

In such a scenario, forwarding opportunities can be lost due to: (i) lack of buffer space at the next hop; message gets dropped, (ii) limited bandwidth; there is not enough time to forward all messages in the queue while the two nodes are in range (contact duration), (iii) MAC contention; more than one nodes *within range* are trying to access the media at the same time, (iv) interference; ongoing communications in the surrounding area contribute to the noise level. If the network is not very sparse, traffic loads are high, or nodes tend to concentrate in some locations (infostations, cafeteria, etc.), one or more of the above events may often occur, even in the context of DTNs. Thus, even if a node comes in contact with a potential relay or even the destination, it might not be able to transfer the packet during that encounter.

To be able to analyze situations that include contention, one needs to calculate: (i) the average time two nodes have to exchange data during an encounter (*contact time*), and (ii) the next time that these two nodes will have another opportunity to exchange data (*inter-meeting time*), if the current one is "lost" due to lack of bandwidth, buffer space, or a collision[1]. Contact and inter-meeting times are formally defined as follows:

Definition 2.3 (Contact Time): Let nodes i and j move according to a mobility model "mm". The nodes are initially out of range, and assume they come within range of each other at time 0. The contact time τ_{mm} is defined as the time they remain in contact with each other before moving out of the range of each

[1] It is important to note that inter-meeting and meeting times are *not* the same quantity and do *not* generally follow the same statistics, even though they are sometimes used inter-changeably. It happens that for some mobility models, including one model we treat here, the expected values for these quantities are *approximately* equal, under some assumptions. Yet, in other important cases, they follow very different statistics (e.g. Random Walk on a lattice [19], [32]).

other, that is $\tau_{mm} = \min_t\{t-1 : \|X_i(t) - X_j(t)\| > K\}$.

Definition 2.4 (Inter-meeting Time): Let nodes i and j move according to a mobility model "mm". Let the nodes start from within range of each other at time 0 and then move out of the range of each other at time t_1, that is $t_1 = min_t\{t : \|X_i(t) - X_j(t)\| > K\}$. The inter-meeting time (M^+_{mm}) of the two nodes is defined as the time it takes them to first come within range of each other again, that is $M^+_{mm} = \min_{t>t_1}\{t - t_1 : \|X_i(t) - X_j(t)\| \le K\}$. (Note that τ_{mm} defined in Definition 2.3 is the same as $t_1 - 1$.)

A. Assumptions and Notation

Here, we will analyze these statistics for a particular class of mobility models, namely "epoch-based" mobility models. These include, for example, the popular Random Waypoint (RWP) [21] and Random Direction (RD) [20] models. We first look at these two mobility models, as representative simple epoch-based models, in order to describe our methodology. Then, we look into a more realistic Community-based mobility model [22], [23] that introduces the concept of "communities" to capture some characteristics often observed in real traces. We will show how our methodology can be applied to this model also to derive the respective statistics.

We introduce next some useful definitions and notation and state the assumptions we'll be making throughout the remaining of the paper. Table I summarizes our notation.

(a) All nodes exist in area U of size $\|U\| = N$, and have a transmission range equal to K. The position of node i at time t is denoted as $X_i(t)$ or X_i if it is static.

(b) All the mobility models we deal with are *epoch-based*; An *epoch* is a given period of time during which a node moves towards the same direction and with the same speed; Each node's trajectory is a sequence of epochs.

(c) The length L of an epoch, measured as the distance between the starting and finishing points of it, is a random variable with expected value $\overline{L} = O(\sqrt{N})$. (The assumption that $\overline{L} = O(\sqrt{N})$ ensures fast mixing of the corresponding process and simplifies analysis as it will become clear in the proof of Theorem 3.1. For the RWP model this assumption is satisfied by definition. The assumption is also inline with the spirit of the RD model since it has been introduced as a close alternative to RWP. Last, note that when $\overline{L}$ is small then an epoch-based mobility model behaves similarly to a random walk model, whose properties we have studied in [15].)

(d) The speed v of a node during an epoch is randomly chosen from $[v_{min}, v_{max}]$, with $v_{min} > 0$, $v_{max} < \infty$ and average speed $\overline{v}$.

(e) At the end of each epoch a node pauses for a random amount of time chosen from $[0, T_{max}]$, with average pause time $\overline{T}_{stop}$.

(f) The expected duration of an epoch (without the pause time) is denoted as $\overline{T} = E[\frac{L}{v}]$.

(g) Let $\vec{v}_i$ denote the velocity of node i and $\overline{v}_{mm} = \|\vec{v_i} - \vec{v_j}\|$ be the mean relative speed between two nodes i and j when both are moving according to mobility model *mm*. Then we define the *normalized relative speed* $\hat{v}_{mm}$ as $\hat{v}_{mm} = \frac{\overline{v}_{mm}}{\overline{v}}$.

Remark: Before we proceed, note that we are primarily interested in easy-to-use analytical formulas rather than exact results. With this in mind, we make a number of assumptions and approximations to keep the analysis tractable and simple. During the derivation of each of the results we state the conditions under which the approximations hold. Furthermore, we compare our results against simulations to show that the error introduced by these approximations is always small for scenarios of interest.

TABLE I
NOTATION

T_{mm}	hitting time under "mm"
M_{mm}	meeting time under "mm"
M^+_{mm}	inter-meeting time under "mm"
τ_{mm}	contact time under "mm"
N	size of network area
K	transmission range
$\overline{L}$	expected epoch length
$\overline{v}$	average node speed
$\overline{T}_{stop}$	average pause time after an epoch
$\overline{T}$	expected epoch duration
$\hat{v}_{mm}$	normalized relative speed under "mm"
EX	expectation of any other random variable X

III. Encounter Statistics for Random Direction

Although the Random Waypoint model was the first epoch-based model to be proposed, it was quickly recognized that it can result in a non-uniform stationary node distribution. This is not only in discord with the common assumption of uniformity made in many studies, but also complicates the analysis. To overcome this, the Random Direction model, which induces a uniform stationary node distribution, has been proposed [20]. The following gives a formal description of the Random Direction mobility model:

Definition 3.1 (Random Direction): In the Random Direction (RD) model each node moves as follows: (i) choose a direction θ uniformly in $[0, 2\pi)$; (ii) choose a speed according to assumption (d); (iii) choose a duration T of movement from an exponential distribution with average $\frac{\overline{L}}{\overline{v}}$; (iv) move towards θ with the chosen speed for T time units; [2] (v) after T time units pause according to assumption (e) and go to step (i).

The following two Theorems calculate the expected hitting and meeting times for the Random Direction model.

Hitting Time (Random Direction): Hitting times are useful when some of the nodes (including the destination) are static (see for example [3] or [12]). Also the meeting time generally depends on the hitting time. Our methodology is based on calculating the expected number of epochs until a static or mobile destination, respectively, is encountered.

Theorem 3.1: The expected hitting time ET_{rd} for the Random Direction model is given by:

$$ET_{rd} = \left(\frac{N}{2K\overline{L}}\right)\left(\frac{\overline{L}}{\overline{v}} + \overline{T}_{stop}\right). \quad (1)$$

Proof: Let a node A perform RD movement, starting from its stationary distribution. A's movement consists of a sequence of randomly and independently chosen epochs. Let further a second node B be static with uniformly chosen position, and

[2] If the boundary is reached, the node either reflects back or re-enters from the opposite side of the network (torus).

let us calculate the probability that node A encounters node B during a given epoch i of length L_i. This epoch will "cover" an area of size $2KL_i$. If B lies anywhere within this area, then A "hits" B during this specific epoch. Furthermore, it is easy to see by the definition of the RD model, that the specific area of the network an epoch will cover is uniformly distributed around the whole network. Hence, the probability p_i of an epoch of length L_i hitting B is equal to $p_i = \frac{2KL_i}{N}$.

Let us now denote as N_{hit} the number of epochs until A hits B, and $P(N_{hit} > n)$ the probability that B has not been encountered after n epochs. Let further E_i, $i = 1 \ldots n$ denote the event that A doesn't hit B at the i^{th} epoch given that the length of the epoch equals l_i, and $f_L(l_1, l_2, \ldots, l_n)$ denote the joint probability density function of the lengths of these first n epochs. Then:

$$P(N_{hit} > n) = \int \cdots \int P(E_1) \cdots P(E_n|E_1 \ldots E_n) f_L(l_1, \ldots, l_n) dl_1 \ldots dl_n.$$

Although consecutive epochs are not independent (the end of one epoch is the beginning of the next one), their lengths are i.i.d. and we can use the statistics of one epoch to describe all epochs. Further, while in general E_i is not independent of E_j, $j < i$ (to see this, consider very small epoch lengths, in which case RD resembles a random walk where the probability that epoch i covers the same area as epoch j is high), we have assumed that the epoch lengths are large, specifically $O(\sqrt{N})$, such that the process mixes very fast (similar to RWP where the mixing time is exactly one epoch). Hence, E_i's are (approximately) independent (a similar argument has been made for RWP in [33]) and

$$P(N_{hit} > n) = \left(\int \left(1 - \frac{2Kl}{N}\right) f_L(l) dl \right)^n = \left(1 - \frac{2K\overline{L}}{N}\right)^n.$$

Consequently, the number of epochs needed till A hits B is geometrically distributed with average $\frac{N}{2K\overline{L}}$. Finally, the expected duration of each epoch is equal $\overline{T} + \overline{T}_{stop}$ (see assumptions (e),(f)), where $\overline{T} = \frac{\overline{L}}{\overline{v}}$ in the case of RD. ∎

Remark: One might argue that counting epochs to calculate hitting times may not capture intra-epoch behavior. However, as can be inferred from the proof of Theorem 3.1, if $K \ll \sqrt{N}$ then the probability of an epoch to hit the destination is small (order of $1/\sqrt{N}$), it takes a large (order of $\sqrt{N}$) number of epochs to hit the destination, and the relative error introduced is at most $\frac{1}{\sqrt{N}}$.

Meeting Time: We now turn our attention to the case where both nodes are moving.

Theorem 3.2: The probability distribution of the meeting time M_{rd} for the Random Direction model has an approximately exponential tail and expected value

$$EM_{rd} = \frac{ET_{rd}}{p_m \hat{v}_{rd} + 2(1 - p_m)}, \qquad (2)$$

where $\hat{v}_{rd}$ is the normalized relative speed for RD, and $p_m = \frac{\overline{T}}{\overline{T} + \overline{T}_{stop}}$ is the probability that a node is moving at any time.

Proof: Let us first assume again that only one of the two nodes, let A, performs RD movement, while the second one, let B, is static. We will re-calculate the expected hitting time of Theorem 3.1 in a slightly different manner. Let's assume that node A performs RD movement *in discrete steps* of unit size, and let p_m denote the probability that A is moving at any of these steps. Then, with probability p_m any given step covers on average a new area of size $2K\overline{v}$, and with probability $1 - p_m$ it stands still and covers no new area. Thus, on average, each node step has an independent probability of finding ("hitting") the destination equal to $\frac{p_m 2K\overline{v}}{N}$, where to claim independence we are using as before that the average length of each epoch is proportional to the network dimension to ensure fast mixing. This implies a geometric distribution for the total number of unit steps until the destination is found with an expected value equal to

$$ET'_{rd} = \frac{N}{p_m 2K\overline{v}}.$$

Note that this method of calculating the hitting time is equivalent to that of Thoerem 3.1, i.e. $ET'_{rd} = ET_{rd}$.[3] Furthermore, because the duration of the time unit is much smaller than the expected hitting time (for $K \ll N$) the distribution of the hitting can be approximated by an exponential in continuous time.

Now, to calculate the meeting time, we need to take into account that both A and B move *concurrently*. Specifically, we will assume that node B is fixed, but *node A is moving at each step with a speed vector equal to the relative speed between A and B*. (This *compound movement* can be shown to be statistically equivalent for our purposes to the original case, by defining an appropriate martingale and using a similar argument as in [32]: Ch.3 – Proposition 3.)

It is known that, for *generic* random walks on graphs, the meeting time between two walks is $\frac{1}{2}$ the respective hitting time of a single walk on the same graph [32]. This holds, because the relative movement of the nodes at consecutive steps are independent of each other. However, in the RD model a node keeps moving in the same direction for the duration of an epoch. The *relative* movement at consecutive steps is not independent, so the denominator is expected to be smaller than 2. We thus need to calculate the expected relative speed $\|\vec{v_A} - \vec{v_B}\|$ between A and B. Due to the uniform choice of direction at every epoch, and the toroidal structure of the network, we can assume without loss of generality that the direction of $\vec{v_A}$ is fixed. In other words, $\vec{v_A} = (v_A, 0)$ and $\vec{v_B} = (v_B \cos\theta, v_B \sin\theta)$. If we assume, for simplicity, that $\vec{v_A} = \vec{v_B} = \overline{v}$, this gives us

$$\|\vec{v_A} - \vec{v_B}\| = \frac{\overline{v}}{2\pi} \int_0^{2\pi} \sqrt{(1 + \cos\theta)^2 + \sin^2(\theta)} d\theta,$$

which is equal to $1.27\overline{v}$. (A little more calculus gives the general case for random speeds.) Thus, the normalized relative speed for this RD model is $\hat{v}_{rd} = \frac{\|\vec{v_A} - \vec{v_B}\|}{\overline{v}} = 1.27$.

$\hat{v}_{rd}\overline{v}$ is the relative speed between the nodes when *both* nodes are moving, which occurs with probability p_m^2. However, with probability $2p_m(1-p_m)$ only one of the node moves with relative speed $\overline{v}$, and with probability $(1 - p_m)^2$ none of the nodes is moving. Consequently, the expected number of steps until the two walks meet equals

$$EM_{rd} = \frac{N}{2K(p_m^2 \hat{v}_{rd}\overline{v} + 2p_m(1 - p_m)\overline{v})} = \frac{ET'_{rd}}{p_m \hat{v}_{rd} + 2(1 - p_m)}.$$

∎

[3] One can see this by replacing p_m with its value $\frac{\overline{T}}{\overline{T} + \overline{T}_{stop}}$, which then gives the expected hitting time in the familiar form of $\frac{N}{2K\overline{L}}\left(\frac{\overline{L}}{\overline{v}} + \overline{T}_{stop}\right)$.

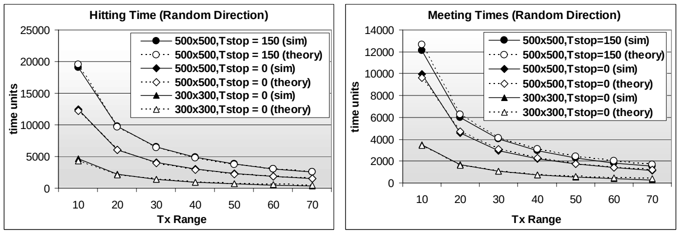


Fig. 1. Comparison of theoretical and simulations results for the expected hitting and meeting times under the Random Direction model.

Figure 1 compares analytical and simulation results for the expected hitting and meeting times, under the Random Direction model, as the transmission range (Tx) increases. (Note that in this and all other plots throughout the paper we normalize the average speed to $\overline{v} = 1$.) It is evident that analytical and simulation results are a close match.

Inter-meeting time: The next theorem finds the expected inter meeting time for the Random Direction mobility model.

Theorem 3.3: The expected inter meeting time EM_{rd}^{+} for the Random Direction model is approximately equal to EM_{rd}.

Proof: Let us approximate the Random Direction movement by a discrete time Markov Chain in which a state represents the location of the node (at the end of the epoch). When a node A starts from within range of another node B, these two nodes are *coupled* [34]. The *mixing time*, that is the time until A reaches again the stationary distribution, can be bounded in terms of the second largest eigenvalue in magnitude of the transition matrix of this Markov Chain [35], [36]. This implies that mixing occurs within $c = O(1)$ number of epochs. Further, each epoch is of length $O(\sqrt{N})$, which implies that the mixing time is also $O(\sqrt{N})$. The additional time it takes for the two nodes to meet after getting mixed is equal to one meeting time which is $O(N)$ (Eq. 2). Consequently, as N becomes large, the total intermeeting time ($mixing + meeting$) converges to the meeting time. Thus, $EM_{rd}^{+} = EM_{rd}$ and the tail of the inter-meeting time is equal to the tail of the meeting time (approx. exponential).

The only step missing is to show that the probability that the two nodes meet within the constant number of time epochs (denoted by c) is always less than $1 - \left(1 - \max\left\{\frac{2K\hat{v}_{rd}\overline{v}p_m^2}{N}, \frac{4K\overline{v}p_m(1-p_m)}{N}\right\}\right)^c$ where p_m is the probability that a node is moving at any time. This probability is negligible because $K << N$. ∎

Contact Duration: We finish this section by deriving the expected contact time for the Random Direction model.

When two nodes come within range of each other, one of the following is true: (a) Both the nodes are moving or (b) Only one of the nodes is moving and the other is paused. Let $E[\tau_{rd}^1]$ denote the expected contact time given both nodes were moving when they came within range of each other and let $E[\tau_{rd}^2]$ denote the expected contact time given only one of the nodes was moving when they came within range. We derive their values in the Appendix and assume for the rest of this discussion that they are known.

Theorem 3.4: The expected contact time $E\tau_{rd}$ for the Random Direction model is given by

$$E\tau_{rd} = \frac{p_m^2}{p_m^2 + 2p_m(1-p_m)}E[\tau_{rd}^1] + \frac{2p_m(1-p_m)}{p_m^2 + 2p_m(1-p_m)}E[\tau_{rd}^2],$$

where $p_m = \frac{\overline{T}}{\overline{T}+\overline{T}_{stop}}$ is the the probability that a node is moving at any time.

Proof: The probability that both nodes are moving is equal to p_m^2. The probability that only one of the nodes is moving is equal to $2p_m(1-p_m)$. For two nodes to come within range from out of range, at least one of the nodes has to be moving. Hence, to find $E\tau_{rd}$, we have to condition over the fact that at least one of the two nodes is moving. Applying the law of total probability gives the result. ∎

Accuracy of the Analysis: We compare analytical and simulation results for the expected contact time (Figure 2(a)) and the distribution of the meeting time and inter meeting time of the Random Direction model for some sample values (Figures 2(b) and 2(c)). Despite some approximations we made during the derivations, it is evident that there is a good match for both the expected contact time values, and the geometric/exponential tail for the meeting and inter-meeting times. Note that it is the number of epochs to hit/meet which are geometrically distributed (Theorems 3.1 and 3.2), hence the distributions of hitting, meeting and inter-meeting times will be geometric/exponential when they are much larger than one epoch time. This explains the deviation from the exponential distribution for smaller values of meeting and inter-meeting times in Figures 2(b) and 2(c).

IV. Encounter Statistics for Random Waypoint

Definition 4.1 (Random Waypoint): In the Random Waypoint (RWP) model, each node moves as follows [21]: (i) choose a point X in the network uniformly at random, (ii) choose a speed v uniformly in $[v_{min}, v_{max}]$ with $v_{min} > 0$ and $v_{max} < \infty$. Let $\overline{v}$ denote the average speed of a node, (iii) move towards X with speed v along the shortest path to X, (iv) when at X, pause for T_{stop} time units where T_{stop} is chosen from a geometric distribution with mean $\overline{T}_{stop}$, (v) and go to Step (i).

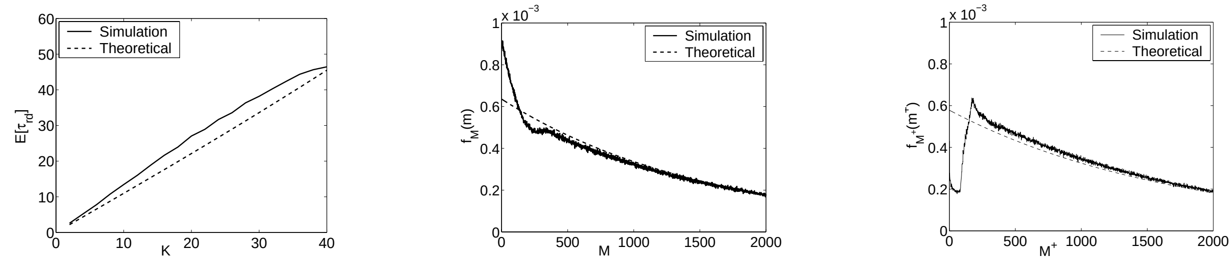
Random Waypoint on a rectangle leads to a non-uniform stationary node distribution, however Random Waypoint on a torus converges to an uniform stationary node distribution [37]. Here, we will study the properties of random waypoint mobility on a torus and not incorporate the complications arising due to the non-uniform stationary distribution.

We first state a lemma deriving the average distance covered by a node in one epoch.

Lemma 4.1: Let L be the length of an epoch, measured as the distance between the starting and the finishing points of the epoch. Then $EL_{rwp} = 0.3826\sqrt{N}$.

Proof: The current position as well as the destination picked is uniformly distributed on the torus. The pdf of L_{rwp} can be easily evaluated using geometrical arguments to be $f_{L_{rwp}}(l) = \begin{cases} \frac{2\pi l}{N} & l \leq \frac{\sqrt{N}}{2} \\ \frac{4l}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2l}\right)\right) & \frac{\sqrt{N}}{2} \leq l \leq \frac{\sqrt{N}}{\sqrt{2}} \end{cases}$. Then $EL_{rwp} = \int_0^{\frac{\sqrt{N}}{\sqrt{2}}} f_{L_{rwp}}(l)dl = 0.3826\sqrt{N}$. ∎

Hitting and Meeting Times: We next state expressions for the expected hitting and meeting times for the Random Waypoint



(a) (b) (c)

Fig. 2. Random Direction Mobility Model: (a) Comparison of the theoretical and simulation results for the expected contact time for parameters $N = 100\times100, \overline{T} = 300, \overline{v} = 1, \overline{T}_{stop} = 50$. (b) Meeting time distribution with parameters $N = 300\times300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$. (c) Inter-meeting time distribution with parameters $N = 300\times300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$.

mobility model. The derivation of these expressions is similar to the derivation of the corresponding variables for the Random Direction mobility model. So, we skip the derivation of the following theorems.

Theorem 4.1: The expected hitting time ET_{rwp} for the Random Waypoint model is given by:

$$ET_{rwp} = \left(\frac{N}{2KEL_{rwp}}\right)\left(\frac{EL_{rwp}}{\overline{v}} + \overline{T}_{stop}\right). \tag{3}$$

Theorem 4.2: The expected meeting time EM_{rwp} for the Random Waypoint model is given by:

$$EM_{rwp} = \frac{ET_{rwp}}{p_m \hat{v}_{rwp} + 2(1-p_m)}, \tag{4}$$

where $\hat{v}_{rwp} = 1.27$ is the normalized relative speed for RWP, and $p_m = \frac{EL_{rwp}/\overline{v}}{EL_{rwp}/\overline{v}+\overline{T}_{stop}}$ is the probability that a node is moving at any time.

Inter-meeting time: The next theorem derives the expected inter-meeting time for the Random Waypoint mobility model.

Theorem 4.3: The expected inter meeting time EM^+_{rwp} for the Random Waypoint model is approximately equal to EM_{rwp}.

Proof: When the nodes move out of the range of each other, they pick up a destination uniformly at random in the torus. After reaching their destination, they are fully mixed (back in their stationary distribution) and the additional time it takes for them to meet again is equal to the meeting time. In general, since an epoch time is much less than the expected meeting time, $EM^+_{rwp} = EM_{rwp}$. ■

Contact Duration: Now we find the expected contact time for the Random Waypoint mobility model. The approach is exactly the same as for the Random Direction model. Let $E[\tau^1_{rwp}]$ denote the expected contact time given both nodes were moving when they came within range of each other and $E[\tau^2_{rwp}]$ denote the expected contact time given only one of the nodes was moving when they came within range. We derive their values in Appendix II and assume for the rest of this discussion that they are known.

Theorem 4.4: The expected contact time $E[\tau_{rwp}]$ for the Random Waypoint model is given by

$$E[\tau_{rwp}] = \frac{p_m^2}{p_m^2 + 2p_m(1-p_m)}E[\tau^1_{rwp}] + \frac{2p_m(1-p_m)}{p_m^2 + 2p_m(1-p_m)}E[\tau^2_{rwp}].$$

where $p_m$is the the probability that a node is moving at any time.

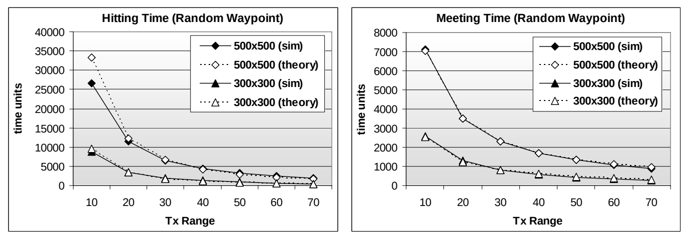


Fig. 3. Comparison of theoretical and simulations results for the expected hitting and meeting times under the Random Waypoint model.

Proof: The proof runs along similar lines as the proof of Theorem 3.4. ■

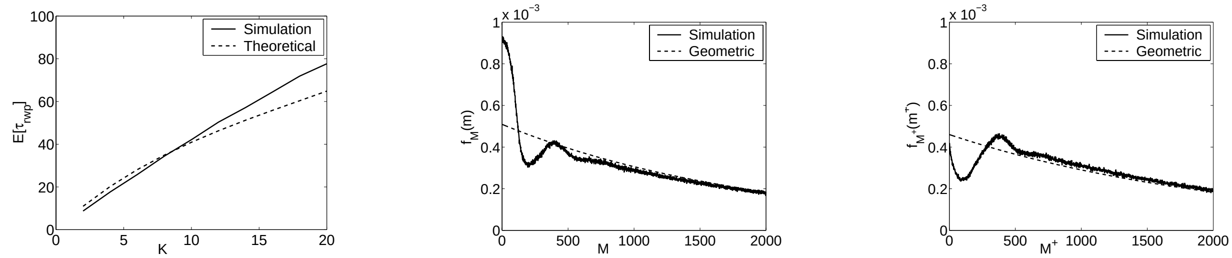
Accuracy of the Analysis: We compare analytical and simulation results for the expected hitting and meeting time in Figure 3, expected contact time in Figure 4(a) and the distribution of the meeting time and inter meeting time in Figures 4(b) and 4(c) for the Random Waypoint mobility model for some sample values. As can be seen, theory matches simulations quite closely.

V. Encounter Statistics for Community-based Mobility

So far we have dealt with simple epoch-based mobility models, like the Random Direction model. Despite their usefulness in theoretical analysis these models have been found to often be unrealistic. Specifically, various collected traces [10], [38] consistently confirm that real life mobility exhibits *location preference* and considerable *heterogeneity* in behavior, not captured by popular models like Random Direction, Random Waypoint and Random Walk. To capture these characteristics, a number of synthetic mobility models have been proposed based on real traces [23], [39], [40].

One of these models, the “Community-based Mobility Model” [22], [23], is still epoch-based in nature, but introduces the concept of communities (and time-dependency) to better capture real life mobility characteristics. In its simplest version, the model consists of two states only, a “local” state where the node moves inside a small community, representing a location of high preference (e.g. office), and a “roaming” state where the node may go anywhere in the network. This can be modeled by a simple two-state Markov Chain.

Definition 5.1 (Community-based Model): Nodes move inside the network as follows:



(a) (b) (c)

Fig. 4. Random Waypoint Mobility Model: (a) Comparison of the theoretical and simulation results for the expected contact time for parameters $N = 100 \times 100, \overline{v} = 1, \overline{T}_{stop} = 150$. (b) Meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$. (c) Inter-meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$.

- each node i has a local community C_i of size $\|C_i\| = c^2N, c \in (0, 1]$; a node's movement consists of a sequence of local and roaming epochs.
- a **local epoch** is a Random Direction movement[4] restricted inside area C_i with average epoch length $\overline{L}_c$ equal to the expected distance between two points uniformly chosen in C_i.
- a **roaming epoch** is a Random Direction movement in the entire network with expected length $\overline{L}$.
- *(local state L)* if the previous epoch of node i was a local one, the next epoch is a local one with probability $p_l^{(i)}$, or a roaming epoch with probability $1 - p_l^{(i)}$.
- *(roaming state R)* if the previous epoch of node i was a roaming one, the next epoch is a roaming one with probability $p_r^{(i)}$, or a local one with probability $1 - p_r^{(i)}$.

The locality of movement is captured by the existence of a community inside which each node spends a configurable amount of its time. Further, each node may have different $p_r^{(i)}$ and $p_l^{(i)}$ parameters modeling a large range of different mobility characteristics per node. Finally, different nodes may have communities of different sizes, or may have more than one community. These together allow for a large range of node heterogeneity to be captured. The realism of this model has been further confirmed in [23], where a somewhat "richer" version of the model (multi-tiered communities and time-dependent behavior) has been shown to closely match existing traces. However, to simplify our exposition, we will focus here on the "vanila" version of the model described above.

Lemma 5.1 calculates some useful probabilities, and follows easily from elementary probability theory.

Lemma 5.1: Let us denote as $\pi_l^{(i)}$ and $\pi_r^{(i)}$ the probability that a given epoch of node i is a local or a roaming one, respectively. Let us further denote the probability that, at any time, the node is: (a) moving in local epoch as $p_{ml}^{(i)}$, (b) moving in roaming epoch as $p_{mr}^{(i)}$, (c) pausing after a local epoch as $p_{pl}^{(i)}$, (d) pausing after a roaming epoch as $p_{pr}^{(i)}$. Then:

[4]Note that each node could also perform Random Waypoint movement or some other i.i.d. movement in each epoch, instead of Random Direction.

TABLE II

ADDITIONAL NOTATION FOR SECTION V

C_i	community of node i: $\|C_i\| = c^2N, c \in (0, 1]$
p_l	probability that next epoch is local, given that previous epoch was local
p_r	probability that next epoch is roaming, given that previous epoch was roaming
π_l	probability that a given epoch is a local one
π_r	probability that a given epoch is a roaming one
p_{mr}	probability that a node is in roaming state and moving
p_{ml}	probability that a node is in local state and moving
p_{pr}	probability that a node is in roaming state and pausing
p_{pl}	probability that a node is in local state and pausing
$\overline{L}_c$	expected length of local epoch
$\overline{T}^l_{stop}$	expected pause time for a local epoch
$\overline{T}_l$	expected local epoch duration ($\overline{L}_c/\overline{v} + \overline{T}^l_{stop}$)
$\overline{T}_r$	expected roaming epoch duration ($\overline{L}/\overline{v} + \overline{T}_{stop}$)

$$\pi_l^{(i)} = \frac{1 - p_r^{(i)}}{2 - p_l^{(i)} - p_r^{(i)}}, \qquad \pi_r^{(i)} = \frac{1 - p_l^{(i)}}{2 - p_l^{(i)} - p_r^{(i)}},$$

$$p_{ml}^{(i)} = \frac{\pi_l^{(i)} \frac{\overline{L}_c}{\overline{v}}}{\pi_l^{(i)}\overline{T}_l + \pi_r^{(i)}\overline{T}_r}, \qquad p_{mr}^{(i)} = \frac{\pi_r^{(i)} \frac{\overline{L}}{\overline{v}}}{\pi_l^{(i)}\overline{T}_l + \pi_r^{(i)}\overline{T}_r},$$

$$p_{pl}^{(i)} = \frac{\pi_l^{(i)}\overline{T}^l_{stop}}{\pi_l^{(i)}\overline{T}_l + \pi_r^{(i)}\overline{T}_r}, \qquad p_{pr}^{(i)} = \frac{\pi_r^{(i)}\overline{T}_{stop}}{\pi_l^{(i)}\overline{T}_l + \pi_r^{(i)}\overline{T}_r}.$$

Table II summarizes some additional notation related to the community model. We will focus here on the case where each node i has its own community C_i, but all nodes have the same mobility characteristics, that is, $p_l^{(i)} = p_l$ and $p_r^{(i)} = p_r$, $\forall i$ (i.e. drop the (i) from all probabilities). The heterogeneous case is only a straightforward extension of this, see [22].

Hitting Time: Let's assume that a node A with community C_A moves according to the Community-based model, until it encounters a node B that is static with uniformly chosen position. If B's position is outside C_A, then A can only encounter B during a roaming epoch. Otherwise, if B lies inside C_A, A is expected to encounter B much faster, since it tends to move preferentially inside C_A. The following two Lemmas calculate the expected hitting time for each of these two subcases.

Lemma 5.2: The expected hitting time $ET_{comm}^{(out)}$ until a node A, moving according to the Community model, encounters a static node B, who lies outside A's community, is given by:

$$ET_{comm}^{(out)} = ET_{rd} + \frac{1-p_r}{1-p_l}\frac{N}{2K\overline{L}}\overline{T}_l. \quad (5)$$

Proof: Let N_l and N_r denote the number of times A visits the local state (L) and roaming state (R), respectively, before it finds B. Furthermore, let $N_{hit} = N_l + N_r$ denote the total number of epochs of any kind. Then, according to the law of large numbers, when $N_{hit} \to \infty$, $N_l \to \pi_l N_{hit}$ and $N_r \to \pi_r N_{hit}$.

Since B does not lie inside A's community, B can only be encountered while A is in the roaming state.[5] The expected number of roaming epochs needed until such a destination is met was found in Theorem 3.1, to be equal to $\frac{2K\overline{L}}{N}$. This implies that A visits state R $EN_r = \frac{2K\overline{L}}{N}$ number of times before it meets B. The sum of the duration of these epochs is equal to ET_{rd}. Additionally, according to the previous argument based on the law of large numbers, A also visits state L *on average*

$$EN_l = \frac{\pi_l}{\pi_r}EN_r = \frac{1-p_r}{1-p_l}EN_r$$

times, before it meets B (given that A starts from its stationary distribution). The average time spent at state L, each time it is visited, is equal to $\frac{L_c}{\overline{v}} + \overline{T}^l_{stop}$. Putting everything together gives us Eq.(5). ∎

Otherwise, if B lies inside C_A, A is expected to encounter B much faster, since it tends to move preferentially inside C_A. Lemma 5.3 calculates the expected hitting time for this case.

Lemma 5.3: The expected hitting time $ET_{comm}^{(in)}$ until a node A, moving according to the Community model, encounters a static node B, who lies inside A's community, is given by:

$$ET_{comm}^{(in)} \simeq \frac{1}{1-\left[(1-p^l_{hit})^{\pi_l}(1-p^r_{hit})^{\pi_r}\right]}(\pi_l\overline{T}_l + \pi_r\overline{T}_r), \quad (6)$$

where $p^r_{hit} = \frac{2K\overline{L}}{N}$ and $p^l_{hit} = \frac{p^r_{hit}}{c}$.

Proof: Let us count the number of steps in the Markov chain corresponding to the community model until B is found. Let further N_l and N_r denote again the number of local and roaming epochs elapsed, respectively, before B is encountered, and let $N_{hit} = N_l + N_r$ denote the total number of epochs. Finally, let $P(N_l, N_r)$ denote the probability that *at least* N_l local and N_r roaming epochs elapse before B is found. Then, $P(N_l, N_r) = (1-p^l_{hit})^{N_l}(1-p^r_{hit})^{N_r}$. According to the law of large numbers, when $N_{hit} \to \infty$, $N_l \to \pi_l N_{hit}$ and $N_r \to \pi_r N_{hit}$. Consequently, for large n

$$P(N_{hit} > n) = (1-p^l_{hit})^{\pi_l n}(1-p^r_{hit})^{\pi_r n}.$$

This implies that the probability distribution of the total number of epochs N_{hit} (local or roaming) has a geometric tail with parameter

$$p_{hit} = 1 - \left[(1-p^l_{hit})^{\pi_l}(1-p^r_{hit})^{\pi_r}\right].$$

Hence, when the average number of epochs necessary to find B is not too small, we can approximate the pdf of the total epochs with a geometric distribution with the above parameter p_{hit}. For this to occur we require that the transmission range is much smaller than the network dimensions, which is the case indeed in most situations of interest (i.e. when mobility is required to deliver a message). In this case, the expected number of epochs until B is encountered EN_{hit} is equal to $\frac{1}{p_{hit}}$. Finally, each of these epochs is a local one with probability π_l or a roaming one with probability π_r, and with duration $\overline{T}_l$ and $\overline{T}_r$, respectively. ∎

We can now go ahead and calculate the hitting time for the case where the destination's position is uniformly chosen over the entire network area.

Theorem 5.1: The expected hitting time ET_{comm} under the Community-based Mobility Model is given by:

$$ET_{comm} = (1-c^2)ET_{comm}^{(out)} + c^2 ET_{comm}^{(in)}. \quad (7)$$

Proof: With probability $\frac{\|U\|-c^2\|U\|}{\|U\|} = 1-c^2$ B's position is outside A's community C_A. In that case, B can only be encountered during a roaming phase, and the expected time until this occurs is given is $ET_{comm}^{(out)}$ (Lemma 5.2). Similarly, with probability c^2 B lies inside C_A, in which case the expected hitting time is given by Lemma 5.3. ∎

Meeting Time: The proof for the community meeting time follows in a similar manner as that of Theorem 3.2. The main difference is that here we need to consider two cases: (i) *non-overlapped communities*, which refers to the case where the communities of the two nodes under study are disjoint, and (ii) *overlapped communities*, which refers to the case where the communities of the two nodes are the same.

Theorem 5.2: The probability distribution of the meeting time M_{comm} under the Community-based mobility model can be approximated by the weighted sum of two exponential distributions, with expected value:

$$EM_{comm} = (1-c^2)EM_{comm}^{(out)} + c^2 EM_{comm}^{(in)}. \quad (8)$$

where,

$$\begin{aligned} EM_{comm}^{(out)} &= \Big[\frac{2K\overline{v}}{N}\left((2p_{mr}(p_{pr}+p_{pl})) + (2p_{ml}p_{pr}) + \hat{v}_{rd}\right. \\ &\quad \left.((p_{mr}+p_{ml})^2 - p_{ml}^2)\right)\Big]^{-1} \\ EM_{comm}^{(in)} &= \Big[\frac{2K\overline{v}}{N}\Big(\frac{\hat{v}_{rd}p_{ml}^2}{c^2} + \frac{2p_{ml}p_{pl}}{c^2} + (2p_{mr}(p_{pr}+p_{pl})) + \\ &\quad (2p_{ml}p_{pr}) + \hat{v}_{rd}((p_{mr}+p_{ml})^2 - p_{ml}^2)\Big)\Big]^{-1} \end{aligned}$$

are the expected meeting time for nodes with non-overlapping and overlapping communities, respectively.

Proof: If the communities do not overlap, the nodes can only meet when at least one of them is out of the community (i.e., roaming). The first and the second terms in the expression for $EM_{comm}^{(out)}$ correspond to the scenario when one node is moving and the other is not. In the first term, the moving node is in roaming state and the non-moving node can be in either local or roaming state. The moving node covers $2K\overline{v}$ new area each time unit. Since it performs a roaming movement, it meets with the other node with probability $\frac{2K\overline{v}}{N}$ as it does not have a priori knowledge about where the paused node is. In the second term the moving node performs a local movement and the paused node in roaming epoch happens to pause within the community of the moving node, which happens with probability $\frac{c^2}{N}$. Since the moving node moves locally, it meets with the other node with probability $\frac{2K\overline{v}}{c^2}$.

[5] Recall that we have assumed that the transmission range K of nodes is much smaller than the total network area N, and thus the probability that B is near the edge of C_A and thus can be encountered even while A is inside its community goes to 0 as $N \to \infty$.

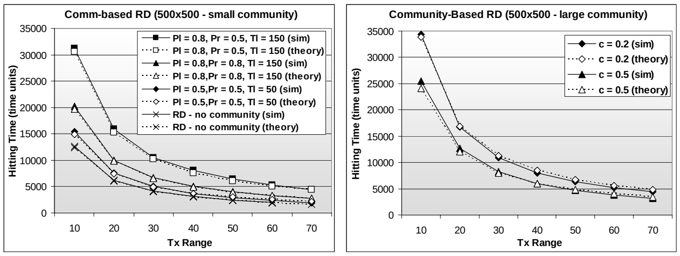


Fig. 5. Hitting Times under the Community-based RD model for small (left) and large (right) communities.

The third term corresponds to the scenario when both nodes are moving. Note that the two nodes cannot meet if they both perform local movement, hence we have to multiply the meeting probability by the factor of $(p_{mr} + p_{ml})^2 - p_{ml}^2)$ (that is, at least one of them is moving in roaming epoch).

If the communities overlap, the nodes meet with higher probability when they both perform local movements. Here we make the simplifying assumption that the two communities are perfectly overlapped. As we show later in this section, the theory is reasonably close to the simulation despite such simplification. The first two terms in the expression of $EM_{comm}^{(in)}$ correspond to the scenario when both nodes are in local epochs. Under such scenario, the new area covered by a moving node contains the other node with probability $\frac{2K\overline{v}}{c^2}$. The first term captures the scenario when both nodes move locally and the second term captures the scenario when only one node moves, with similar reasonings as above. The remaining terms correspond to the scenario when at least one of the nodes is in roaming epoch. They are exactly the same as in the sub-cases with non-overlapped communities.

Finally, we take a weighted average over the two cases to find the expected meeting time as we did for the expected hitting time. ■

Inter-meeting Time: To calculate the inter-meeting times, we condition on the two subcases of overlapping and non-overlapping communities. We first look at the simpler case of non-overlapping communities.

Lemma 5.4: The expected inter-meeting time for nodes with non-overlapped communities is $EM_{comm}^{+(out)} = EM_{comm}^{(out)}$.

Proof: Since both nodes are moving according to the Random Direction mobility model, the location distribution will converge to the stationary distribution within a few time epochs (as discussed in Theorem 3.3). Further, the Markov Chain which describes the transition of nodes between local and roaming states can be shown to converge to within ϵ of its stationary distribution within $\max\left\{\frac{log\left(\frac{1-p_l}{\epsilon(2-p_l-p_r)}\right)}{2-p_l-p_r}, \frac{log\left(\frac{1-p_r}{\epsilon(2-p_l-p_r)}\right)}{2-p_l-p_r}\right\}$ time units (we bound the mixing time in terms of the second largest eigenvalue of the transition matrix [35], [36]). Thus, it takes only *a constant number of epochs* until two nodes that just met reach again their stationary distribution, and we can show that the probability that the two nodes meet during this time is negligible if $K \ll N$ (similar to Theorem 3.3). After this, the additional time it takes for them to meet is equal to one meeting time. ■

When the communities of the two nodes overlap, then the situation becomes slightly more complicated. Specifically, if the two nodes meet within their community, there is a high probability that they will meet again quite fast.

Lemma 5.5: The expected inter-meeting time for nodes with overlapping communities is

$$EM_{comm}^{+(in)} = p_1^+ E[M_1^+] + p_2^+ E[M_2^+] + (1 - p_1^+ - p_2^+) EM_{comm}^{(in)}, \quad (9)$$

where (i) p_1^+ is the probability that when the two nodes met, both were in their local states and only one of the nodes was moving, and $E[M_1^+]$ is the expected inter-meeting time for this case, (iii) p_2^+ is the probability that when the two nodes met, both were in their local states and moving, and $E[M_2^+]$ is the expected inter-meeting time for this latter case.
The expressions for $p_1^+, E[M_1^+], p_2^+$ and $E[M_2^+]$ are contained in the proof of the lemma which is presented in Appendix III.

We next state the value of the expected inter-meeting time, EM_{comm}^+, in terms of $EM_{comm}^{+(out)}$ and $EM_{comm}^{+(in)}$ in the following theorem.

Theorem 5.3: The expected inter-meeting time of the Community-based mobility model is

$$EM_{comm}^{+} = (1 - c^2) EM_{comm}^{+(out)} + c^2 EM_{comm}^{+(in)}. \quad (10)$$

Proof: The proof is similar to Theorem 5.1. ■

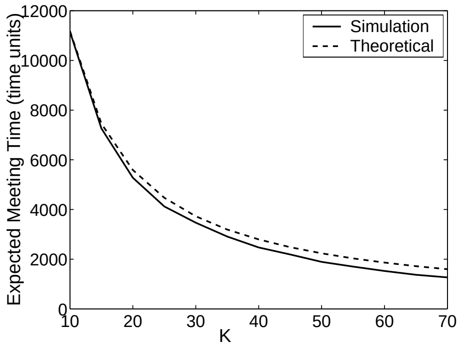
Contact Duration: The expected contact time is also derived after conditioning on the two subcases of overlapping and non-overlapping communities. Let $E\tau_{comm}^{(out)}$ and $E\tau_{comm}^{(in)}$ denote the expected contact time for nodes with non-overlapped and overlapped communities respectively. Appendix IV discusses how to derive their values. The following theorem states the value of the expected contact time, $E\tau_{comm}$, and is derived in a manner similar to the derivation of Theorem 5.1.

Theorem 5.4: $E\tau_{comm} = (1 - c^2) E\tau_{comm}^{(out)} + c^2 E\tau_{comm}^{(in)}$.

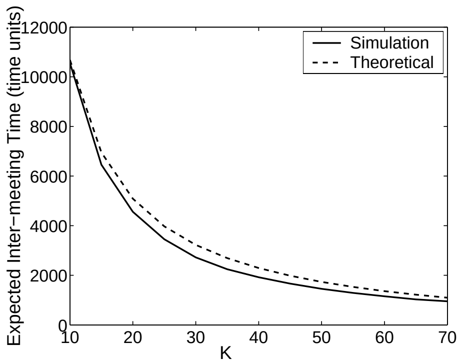
Accuracy of the Analysis: Figure 5 compares analytical and simulation results for the expected hitting time under the Community-based mobility model, for small and large communities (for the large community case all pause times are zero and $p_l = 0.9, p_r = 0.5$). Figures 6(a) and 6(b) compare the analytical and simulation results for the expected meeting and inter-meeting times under the Community-based mobility model. As can be seen, theory matches simulations quite closely.

Small Communities: As a special case, in some real-life situations each node tends to move most of the time in a very small area that may be common for some of the nodes, and could be entirely covered by the node's antenna, while the network might be much larger, like several office buidling on a campus or several conference rooms in a hotel. We now discuss how the preceeding analysis changes for such small communities.

If two nodes have non-overlapping communities, then if two nodes are within range, then both of them will be in their roaming states. Incorporating this observation in the preceeding derivations yield the expressions for expected meeting, inter-meeting and contact times. Also, for non-overlapping small communities, the tail of the distribution of the meeting and inter-meeting times is expoenential as their relative movement is similar to the relative movement of two nodes moving according to Random Direction mobility.



(a)



(b)

Fig. 6. Simulation and analytical results for the Community-based mobility model. (a) Meeting time. (b) Inter-meeting time. Network parameters: $N = 500 \times 500, \overline{L} = 150, c = 0.1, p_l = 0.9, p_r = 0.5, \overline{v} = 1.0, \overline{T}_{stop} = \overline{T}^{l}_{stop} = 0$.

If two nodes have overlapping communities, then if these two nodes are within their community, then the probability that they are within each other's range is equal to one. Incorporating this observation in the preceeding derivations yield the expressions for expected meeting, inter-meeting and contact times. In general, for overlapping small communities, the tail of the distribution of the meeting and inter-meeting times is not exponential. However, for the special case when most of the nodes remain within their own community such that the probability that both the nodes under consideration are in their roaming states is small, the distribution of the meeting and inter-meeting times will be exponential as their distribution is governed by the node in the roaming state returning back to the local state.

VI. Delay of Mobility-Assisted Routing

In this section, our goal is to demonstrate how the mobility properties we have derived thus far can be readily used in various analytical expressions of interest related to encounter-based protocols. As an example, we will show how our results regarding the various encounter times fit into a general theoretical framework that can be used to analyze the performance of mobility-assisted routing. Note that the derived encounter-related expressions could also be useful for the calculation of various other interesting results such as: capacity results [41], end-to-end delay and resource usage results based on Markov Chains and Random Walk theory [15], [16], [25], [26], fluid models [14], wireless virus spread models [27], PeopleNet [9], etc. By "plugging" our basic encounter-related results into more generic equations, we derive similarly accurate performance results under a specific mobility model, in closed form, without resorting to simulations or curve fitting.

Our focus here will be the delay of two well known mobility-assisted routing algorithms: Direct Transmission and Epidemic Routing [29]. We will first look into a scenario with idealized conditions (infinite bandwidth and buffer space), and derive upper and lower bounds for the delay. Then, we address a more realistic situation, where nodes contend for access to limited resources like bandwidth.

We use a custom simulator described in [42] to get the simulation values we compare our theoretical results to. The simulator avoids excessive interference by implementing a scheduling scheme which prohibits simultaneous transmissions within two hops of each other. It incorporates interference by adding the received signal from other simultaneous transmissions (outside the scheduling area) and comparing the signal to interference ratio to the desired threshold. The simulator allows the user to choose from different physical layer, mobility and traffic models.

A. Mobility-Assisted Routing under no Contention

We will first assume that all nodes have infinite buffer space, and the available bandwidth per contact is much larger than the amount of data to be sent. These assumptions are valid, when traffic is low, the network is sparse and the spatial distribution of nodes does not have large peaks. In this case, the probability that many nodes will try to access the same "wireless area" at the same time is small. We can thus safely ignore contention or queueing and concentrate on the important effect on delay of storing and carrying a message.

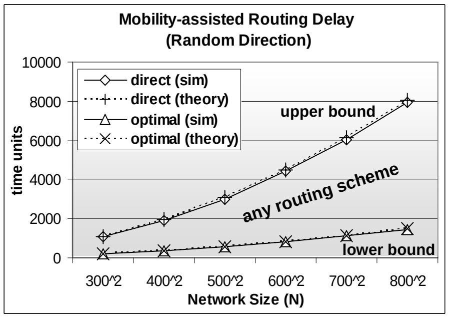
In Direct Transmission, since the source of a message holds on to it until it comes within range of the destination itself [29], [43], its delay (under no contention) is equal to the expected meeting time under the given mobility model, and is also an upper bound on the delay of any other (non-adversarial) mobility-assisted routing scheme [29]. Epidemic Routing, on the other hand, has the minimum expected delivery delay under the assumption of no contention, being equivalent with an optimal "oracle-based" scheme that knows all future connectivity [6], [15]. The properties of the optimal algorithm have been widely studied [14], [25], [26], [29], [44]. The following Lemma gives the delay of the two routing schemes as a function of the expected meeting time for a given mobility model, and thus it also gives bounds for the expected delay of any mobility-assisted routing scheme, under a given mobility model (proofs can be found in [29], and the lemma is reproduced here for completeness).

Lemma 6.1: Let M nodes move according to a given mobility model with exponentially distributed meeting times. Then, the expected message delivery time of any routing algorithm ED_{mm} under mobility model "mm" is

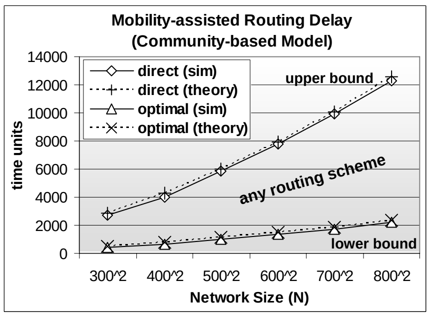
$$\frac{H_{M-1}}{(M-1)} EM_{mm} \leq ED_{mm} \leq EM_{mm} \tag{11}$$

where H_n is the n^{th} Harmonic Number, i.e, $H_n = \sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n)$.

We can replace the values we calculated for the meeting times under different mobility models in Equation (11) and



(a)



(b)

Fig. 7. Upper and lower bounds on the delay of any mobility-assisted routing scheme under Random Direction (a) and Community-based mobility (b).

derive closed-form expressions for the delays. Note that these expressions hold for mobility models with exponential tails for the meeting time distribution, and thus the epoch-based mobility models we have seen, such as Random Direction, Random Waypoint or Community mobility with small communities. In the case of large communities, Equation (11) has to be slightly modified. These equations also hold for the case of random Walk mobility whose meeting time has been derived in [29].

In Figure 7(a) we compare our analytical results, based on Lemma 6.1 and the expressions derived in Sections III and V, to simulation results, for the Random Direction model. Figure 7(b) does the same for the Community-based model with small non-overlapping communities and parameter values $p_l = 0.8$, $p_{roam} = 0.5$, $\overline{T}_{stop} = 0$, and $\overline{T}^{l}_{stop} = 150$. For the non-contention case, we turned off all interference and scheduling modules in the simulator, and route only a single message in each run. As can be seen by both plots, our theoretical results for the optimal delay match very closely with simulations results. This implies not only that our meeting time expressions for different mobility models are accurate, but that *derivative delay expressions based on these meeting times, and pertaining to the delay of more complicated mobility-assisted routing schemes are also accurate*.

B. Mobility-Assisted Routing under Contention

The simple approach of Section VI-A that takes into consideration only the expected meeting times fails to take into account contention for the shared channel, as described in Section II. This can produce too optimistic delay results for resource-demanding protocols like epidemic routing (see [16] and Fig. 8(b)).

When contention occurs during the whole duration of a given contact, a "loss" of a forwarding opportunity may occur. Such a loss can be modeled by a loss probability, which is a function of contact duration, propagation environment, and traffic load. Note that different routing protocols induce different load for the same amount traffic, since they use different degrees of data replication. Hence, the loss probability also depends on the routing protocol. [42] discusses how to find the value of the loss probability in terms of these network parameters. [6] We will not reproduce these result here, but instead discuss how to find the delay given this loss probability. In the rest of this section we denote by p_{txS} the probability of a successful transmission, and by $1 - p_{txS}$ the loss probability.

The two nodes will remain in contact with each other for one contact time. If contention causes loss of every transmission opportunity in one *contact time*, then the two nodes will move out of each other's range without being able to exchange the packet. As a result, they will have to wait for one *inter-meeting time* to be able to meet each other again. Thus, when contention is significant, knowing the statistics of these two properties in addition to meeting times is necessary and sufficient to be able to analyze the delay of any mobility-assisted routing scheme.

We first analyze the performance of Direct Transmission under contention. Although this scheme is somewhat "trivial" and not very likely to be used in a real implementation, it is very useful to demonstrate how the various encounter statistics all fit together, and will serve as the building block for more complex protocols.

Theorem 6.1: Let ED_{dt} denote the expected delay and $1 - p^{dt}_{txS}$ denote the loss probability under direct transmission. Then,

$$ED_{dt} = EM_{rd} + \frac{(1 - p^{dt}_{success})EM^{+}_{rd}}{p^{dt}_{success}} \simeq \frac{EM_{rd}}{p^{dt}_{success}},$$

where EM_{rd} is the expected meeting time of the Random Direction mobility model and $p^{dt}_{success} = 1 - \left(1 - p^{dt}_{txS}\right)^{E\tau_{rd}}$ is the probability that when two nodes come within range of each other, they successfully exchange the packet before going out of each other's range (within the contact time τ_{rd}).

Proof: The expected time it takes for the source to meet the destination for the first time is EM_{rd} (the expected meeting time). Then, with probability $1 - p^{dt}_{txS}$, the source and the destination are unable to exchange the packet in one time slot, where p^{dt}_{txS} is given in [42]. Since these nodes are within range of each other for $E\tau_{rd}$ number of time slots, $\left(1 - p^{dt}_{txS}\right)^{E\tau_{rd}}$ is the probability that the source fails to deliver the packet to the destination when they came within range of each other. (We are making an approximation here by replacing τ_{rd} by its expected value.) Thus, $p^{dt}_{success} = 1 - \left(1 - p^{dt}_{txS}\right)^{E\tau_{rd}}$.

If the two nodes fail to exchange the packet when they were within range, then they will have to wait for one inter-meeting time to come within range of each other again. If they fail yet again, they will have to wait another inter-meeting time to come within range. Thus, $ED_{dt} = EM_{rd} + p^{dt}_{success}\big((1 - p^{dt}_{success})EM^{+}_{rd} + 2(1 - p^{dt}_{success})^2 EM^{+}_{rd} + \ldots\big) = EM_{rd} + \frac{(1-p^{dt}_{success})EM^{+}_{rd}}{p^{dt}_{success}}$. Since $EM^{+}_{rd} =$

[6] Note that this probability could also reflect other things too, for example, loss due to lack of buffer space or the unwillingness of a node to further forward packets.

EM_{rd} for the Random Direction mobility model, ED_{dt} evaluates to $\frac{EM_{rd}}{p^{dt}_{success}}$. ■

We will now analyze the performance of epidemic routing with contention assuming Random Direction mobility.

Theorem 6.2: Let ED_{epid} denote the expected delay of epidemic routing and $1 - p^{epid}_{txS}$ denote the induced loss probability. Then,

$$ED_{epid} = \sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} \frac{EM_{rd}}{m(M-m)p^{epid}_{success}}, \tag{12}$$

where $p^{epid}_{success} = 1 - \left(1 - p^{epid}_{txS}\right)^{E\tau_{rd}}$.

Proof: To find the expected end-to-end delay, we first find the expected time it takes for the number of nodes having a copy of the packet to increase from m to $m+1$. The expected time it takes for one of the m nodes having a copy of the packet to encounter one of the other $M-m$ nodes is equal to $\frac{EM_{rd}}{m(M-m)}$. Similar to the Direct Transmission case, each "encounter" fails with probability $p^{epid}_{success} = 1 - \left(1 - p^{epid}_{txS}\right)^{E\tau_{rd}}$, where p^{epid}_{txS} is given in [42]. Since both meeting and inter-meeting times have exponential tails, the expected time it takes for the number of nodes having a copy of the packet to increase from m to $m+1$ is equal to $\frac{EM_{rd}}{m(M-m)p^{epid}_{success}}$.

Finally, the probability that the destination is the i^{th} node to receive a copy of the packet is equal to $\frac{1}{M-1}$ for $2 \leq i \leq M$. The amount of time it takes for the i^{th} copy to be delivered is equal to $\sum_{m=1}^{i} \frac{EM_{rd}}{m(M-m)p^{epid}_{success}}$. Applying the law of total probability over the random variable i gives Eq.(12). ■

Figures 8(a) and 8(b) compare the analytical and simulation delay results for different network densities for Direct Transmission and Epidemic routing respectively. To generate contention in the network, instead of routing a single message per run (used to generate results for Section VI-A), we use the Poisson arrival process to generate traffic in our simulations. The channel model is assumed to be Rayleigh-Rayleigh fading and Random Direction mobility is used to model node mobility in the simulations. Its easy to see the analytical results closely match the simulation results. Also, the analytical delay derived without incorporating contention heavily underestimates the actual delay of Epidemic routing as flooding is used to route packets in Epidemic routing.

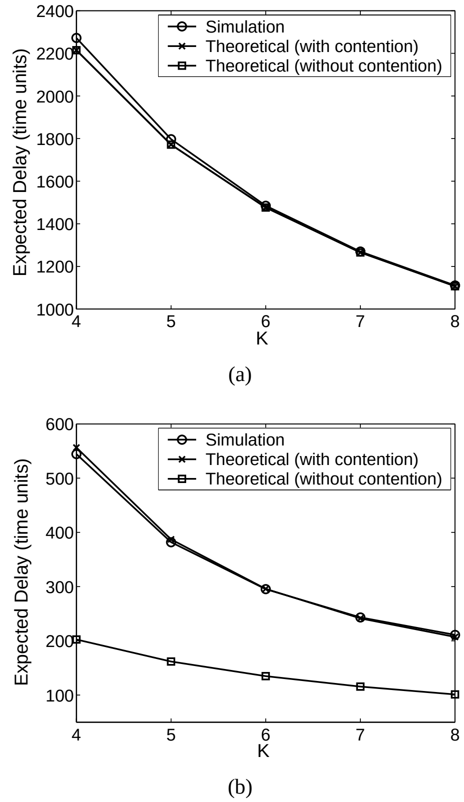


Fig. 8. Simulation and analytical results for the expected delay of (a) Direct Transmission. (b) Epidemic Routing. Network parameters: $N = 150 \times 150, \overline{v} = 1, \overline{L} = 55, M = 50, \overline{T}_{stop} = 0$.

VII. Related Work

There has been a line of work pre-dating delay tolerant networks, which also proposed the use of node mobility, but with the aim to overcome the limited capacity problem of ad hoc networks [43], [45]. A significant research thread has spawned thereafter exploring the fundamental trade-offs between the capacity and the delay of the original "2-hop" scheme and other similar algorithms (e.g. [41], [44], [46], [47]). Nevertheless, most of these results are of asymptotic nature. Although asymptotic results provide useful insight on the scalability of a given family of protocols, they are more applicable to large dense wireless networks, which do not represent our target applications.

A large number of (mobility-assisted) routing protocols have been proposed for DTN networks, some assuming known or enforced future connectivity (e.g. [6], [12]), while others assuming random connectivity and making "opportunistic" forwarding decisions (e.g [13]–[16]). A detailed list of proposals can be found in [11] or [48].

Following this, a significant amount of theoretical work has also recently emerged in the context of intermittently connected networks or DTNs [14], [25], [26], [29], [31], [44], [49]. These papers try to analyze the delay of epidemic routing [13] or other mobility-assisted protocols, in networks that are not connected for the majority of time. However, a lot of these works assume that the expected time between encounters, the basic building component in most models, is just a parameter of the mobility model that can be acquired from simulations or curve fitting [25], [26], [44]. Although this makes these results quite generic, at the same time it also reduces the usefulness of analytical expressions, as a simulation must be run beforehand to obtain some quantities necessary for the model.

Random Walk mobility is one of the mobility models, where hitting and meeting times have been analyzed extensively [29], [32] and used to derive various performance metrics for mobility-assisted routing protocols [3], [29]. Further, various statistical properties of the Random Waypoint and the Random Direction model (e.g. node distribution [33], convergence [37], [50], etc.) have been studied. Nevertheless, hitting and meeting times for these models have to our best knowledge only been treated in [51]. There, the authors use a similar methodology to the one we use, but only derive upper and lower bounds on the meeting time between two nodes performing Random Waypoint movement, and use it to calculate an asymptotic result.

To fill this important gap, in this paper we analyzed the statistics of various important encounter-related quantities for a generic class of mobility models, namely "epoch-based" models. In [22], [23] we had dealt with *expectations* for hitting and meeting times for these models. Here, we extended this work by deriving the complete probability distributions for these quantities, and also by calculating two other important statistics, namely inter-meeting times and contact durations, which are necessary to model contention (for limited bandwidth or buffer space).

VIII. Conclusions

In this paper, we have presented a methodology to analyze the encounter statistics for some commonly used ("epoch-based") mobility models. We have derived accurate closed form solutions for all the respective hitting, meeting, inter-meeting, and contact times for Random Direction and Random Waypoint mobility which are simple and popular epoch-based models. Additionally, we have applied our methodology to derive similar results for a more realistic mobility model that aims at capturing real-world mobility characteristics more accurately than many existing models. Finally, we have demonstrated how these results can be used in a more general framework to analyze the delay of different mobility-assisted routing schemes, that is, schemes that require the node to carry a message for (potentially long) periods of time. Such schemes have been recently recognized to be very helpful in improving the performance of regular wireless networks or to enable data delivery in networks that are disconnected for the majority of time. We believe that this work can help in better understanding the particular advantages and shortcomings of various protocols in different settings, and can facilitate the design of new, improved schemes.

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APPENDIX I
CONTACT TIME FOR THE RANDOM DIRECTION MOBILITY MODEL

We first derive a series of lemmas which constitute a general methodology for finding the expected contact time for any mobility model. Then, we use this methodology to derive the expected contact time for the Random Direction mobility model. This methodology will also be used in the derivation of the expected contact times for the Random Waypoint and the Community-based mobility model.

Lemma 1.1: Let the two nodes be moving when they came within range of each other. We label the two nodes as nodes 1 and 2. Let $E[\tau^1_{mm}]$ denote the expected contact time for these two nodes for the mobility model 'mm'. Then,

$$E[\tau^1_{mm}] = (1 - p^1_{mm})\frac{4K}{\pi \hat{v}^c_{mm}\overline{v}} + p^1_{mm} \left(\frac{0.6366K}{\hat{v}^c_{mm}\overline{v}} + p^{11}_{mm}E[\tau^{add1}_{mm}] + p^{12}_{mm}E[\tau^{add2}_{mm}]\right)$$

where $\hat{v}^c_{mm}$ is the normalized relative speed between the two nodes given that they moved within range of each other in the current time slot,[7] p^1_{mm} is the probability that one of the two nodes pause while they are within range of each other, p^{11}_{mm} (p^{12}_{mm}) is the probability that the paused node is node 1 (node 2), and $E[\tau^{add1}_{mm}]$ ($E[\tau^{add2}_{mm}]$) is the expected additional time the two nodes remain within range after node 1 (node 2) pauses.

Proof: When both the nodes are moving when they come within range of each other, either they move out of each other's range before any of them pauses or one of them pauses before they move out of range.

(a) They move out of each other's range before pausing: Let one node be static and let the other node move at a speed $\vec{v}_i - \vec{v}_j$. This model is equivalent to the model when both nodes are moving at speeds $\vec{v}_i$ and $\vec{v}_j$ respectively. We will work with the former model during this proof as well as all the subsequent proofs.
So, when these two nodes come within range of each other, the angle ϕ in Figure 9 will be uniformly distributed within $[0, \pi)$. It cannot be greater than π as $\phi > \pi$ implies that the nodes were already in contact with each other. They will remain in contact with each other while the first node travels along the chord AB in Figure 9. The length of the chord AB is equal to $2Ksin(\phi)$. E[distance for which the nodes remain in contact with each other] = E[length of chord AB] = $\int_0^\pi \frac{1}{\pi}2Ksin(\phi)d\phi = \frac{4K}{\pi}$. The expected speed of the moving node is equal to $\hat{v}^c_{mm}$ (the normalized relative speed between the two nodes given that they moved within range of each other in the current time slot). Thus the expected time they remain in contact with each other is approximately equal to $\frac{4K}{\hat{v}^c_{mm}\pi\overline{v}}$.

(b) One of the nodes pauses before they move out of each other's range: We again work with the model where one of the nodes is static and the other node is moving at a speed $\vec{v}_i - \vec{v}_j$. The moving node is equally likely to pause anywhere on the chord AB in Figure 9 since the distribution of movement duration is memoryless. Let the node stop at point C which is $0 \leq x \leq 2Ksin(\phi)$ distance away from A. Thus $f_{X|\Phi}(x \mid \phi) = \begin{cases} \frac{1}{2Ksin(\phi)} & 0 \leq x \leq 2Ksin(\phi) \\ 0 & \text{otherwise} \end{cases}$.
Multiplying by $f_\Phi(\phi)$ and integrating over ϕ gives us $f_X(x)$. The expected distance that the node travels before pausing can then be evaluated to $0.6366K$. Thus, the expected time the node travels before pausing is equal to $\frac{0.6366K}{\hat{v}^c_{mm}\overline{v}}$. The probability that node 1 (node 2) pauses first is equal to p^{11}_{mm} (p^{12}_{mm}) and the additional time the two nodes spent within range of each other is equal to $E[\tau^{add1}_{mm}]$ ($E[\tau^{add2}_{mm}]$).

The values of p^1_{mm}, p^{11}_{mm} and p^{12}_{mm} depend on the mobility model. Lemma 1.4 finds their value for the Random Direction mobility model. ■

In the previous lemma, we found the expected contact time given both nodes were moving when they came within range of each other. The next lemma evaluates the expected contact time when only one node was moving when they came within range of each other.

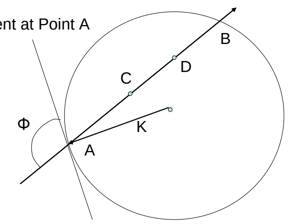

Fig. 9. The first node enters the transmission range of the second node at an angle ϕ to the tangent at A and moves along the chord AB.

[7] $\hat{v}_{mm}$ denotes the unconditioned normalized relative speed while $\hat{v}^c_{mm}$ denotes the normalized relative speed conditioned on the event that the two nodes under consideration moved within range of each other in the current time slot. This event will rule out certain relative velocities, and hence $\hat{v}_{mm}$ and $\hat{v}^c_{mm}$ will be different.

When only one node is moving, either they will move out of each other's range before the paused node restarts again and the moving node pauses, or the moving node pauses or the paused node restarts before they move out of each other's range. The derivation has to account for all the three scenarios.

Lemma 1.2: Let only one of the nodes be moving when the two nodes came within range of each other. Without loss of generality, we assume that node 1 is the moving node while node 2 is the paused node. Let $E[\tau^2_{mm}]$ denote the expected contact time for these two nodes for the mobility model 'mm'. Then,

$$E[\tau^2_{mm}] = (1 - p^2_{mm})\frac{4K}{\pi\overline{v}} + p^2_{mm} \left(\frac{0.6366K}{\overline{v}} + p^{21}_{mm}E[\tau^{add3}_{mm}] + p^{22}_{mm}E[\tau^{add4}_{mm}]\right)$$

where p^2_{mm} is the probability that the paused node restarts again or the moving node pauses before moving out of each other's range, p^{21}_{mm} is the probability that the paused node restarts before the moving node pauses, and p^{22}_{mm} is the probability that the moving node pauses before the paused node restarts. $E[\tau^{add3}_{mm}]$ and $E[\tau^{add4}_{mm}]$ are the expected additional times the two nodes remain within range after both of them start moving and after both of them are paused respectively.

Proof:

(a) Both nodes move out of the range of each other without any of them changing state: The expected time they remain in contact is $\frac{4K}{\pi\overline{v}}$. The proof goes along the same lines as in proof of Lemma 1.1 (a). Except here, the expected relative speed is $\overline{v}$.

(b) The moving node pauses or the paused node starts moving before they move out of each other's range: The expected time before one of the nodes change their state is $\frac{0.6366K}{\overline{v}}$. The proof goes along the same lines as in proof of Lemma 1.1 (b). Except here, the expected relative speed is $\overline{v}$. The probability that the moving node pauses before the paused node restarts is p^{21}_{mm} and $E[\tau^{add4}_{mm}]$ is the additional time the two nodes remain within range. The probability that the moving node pauses before the paused node restarts is p^{22}_{mm} and $E[\tau^{add3}_{mm}]$ is the additional time the two nodes remain within range.

The values of p^2_{mm}, p^{21}_{mm} and p^{22}_{mm} depend on the mobility model. Lemma 1.4 finds their value for the Random Direction mobility model. ∎

Next, we find the values of $E[\tau^{add1}_{mm}]$, $E[\tau^{add2}_{mm}]$, $E[\tau^{add3}_{mm}]$ and $E[\tau^{add4}_{mm}]$.

Lemma 1.3: $E[\tau^{add1}_{mm}]$, $E[\tau^{add2}_{mm}]$, $E[\tau^{add3}_{mm}]$ and $E[\tau^{add4}_{mm}]$ are related to each other through the following set of linear equations:

$$E[\tau^{add1}_{mm}] = (1 - p^{add1}_{mm})\frac{0.6366K}{\overline{v}} + p^{add1}_{mm}\left(\frac{4K}{3\pi\overline{v}} + p^{add11}_{mm}E[\tau^{add3}_{mm}] + p^{add12}_{mm}E[\tau^{add4}_{mm}]\right), \tag{13}$$

where p^{add1}_{mm} is the probability that one of the nodes change their state (either the paused node starts moving or the moving node pauses) before they go out of the range of each other, p^{add11}_{mm} is the probability that the paused node (node 1) starts moving before the moving node (node 2) pauses and node 1 does not change its state from roaming to local or vice versa, while p^{add12}_{mm} is equal to the probability that the moving node pauses before the paused node starts moving.

$$E[\tau^{add2}_{mm}] = (1 - p^{add2}_{mm})\frac{0.6366K}{\overline{v}} + p^{add2}_{mm}\left(\frac{4K}{3\pi\overline{v}} + p^{add21}_{mm}E[\tau^{add3}_{mm}] + p^{add22}_{mm}E[\tau^{add4}_{mm}]\right), \tag{14}$$

where p^{add2}_{mm} is the probability that one of the nodes change their state (either the paused node starts moving or the moving node pauses) before they go out of the range of each other, p^{add21}_{mm} is the probability that the paused node (node 2) starts moving before the moving node (node 1) pauses and node 2 does not change its state from roaming to local or vice versa, while p^{add22}_{mm} is equal to the probability that the moving node pauses before the paused node starts moving.

$$E[\tau^{add3}_{mm}] = (1 - p^{add3}_{mm})\frac{0.6366K}{\hat{v}^c_{mm}\overline{v}} + p^{add3}_{mm}\left(\frac{4K}{3\pi\hat{v}^c_{mm}\overline{v}} + p^{add31}_{mm}E[\tau^{add1}_{mm}] + p^{add32}_{mm}E[\tau^{add2}_{mm}]\right), \tag{15}$$

where p^{add3}_{mm} is the probability one of the nodes pause before moving out of each other's range, p^{add31}_{mm} is the probability the node 1 pauses before node 2 and p^{add32}_{mm} is the probability that node 2 pauses before node 1.

$$E[\tau^{add4}_{mm}] = \overline{T}^{stop}_{mm} + p^{add4}_{mm}\frac{0.6366K}{\hat{v}^c_{mm}\overline{v}} + \left(1 - p^{add4}_{mm}\right)\left(p^{add41}_{mm}E[\tau^{add2}_{mm}] + p^{add42}_{mm}E[\tau^{add1}_{mm}]\right), \tag{16}$$

where $\overline{T}^{stop}_{mm}$ is the expected time both the two nodes remain paused, p^{add4}_{mm} is the probability that one of the two nodes change states to move out of each other's range (p^{add4}_{mm} is needed for the community-based mobility model as nodes can move from roaming to local state or vice versa), p^{add41}_{mm} is the probability that node 2 starts moving node 1 and p^{add42}_{mm} is the probability that node 1 starts moving before node 2.

Proof:

(13) $E[\tau^{mm}_{add1}]$ is the additional time two nodes remain in contact when node 1 is paused and node 2 is moving. Either of the following can happen in the succeeding time slots:

(a) The two nodes move out of the range of each other without either of the nodes changing states: The expected distance the node travels before going out of range is $0.6366K$. (This is the expected length from a point anywhere on chord AB in Figure 9 to point B.) The expected relative speed is $\overline{v}$. Hence, the expected duration the two nodes remain in contact is $\frac{0.6366K}{\overline{v}}$.

(b) Node 1 starts moving before they move out of range. If node 1 does not change its state, the additional time both nodes spend within range of each other is $E[\tau^{mm}_{add3}]$.

(c) Node 2 pauses before they move out of range. The additional time both nodes spend within range of each other is $E[\tau^{mm}_{add4}]$.

Let $E[s]$ denote the distance travelled before one of nodes changes state for cases (b) and (c). $E[s] = E[$distance between points C and D on the chord in Figure 9 $] =$

$\int_0^{\pi} \int_0^{2Ksin(\phi)} \int_0^{x} \frac{s-x}{4\pi K^2 (sin(\phi))^2} dsdxd\phi = \frac{4K}{3\pi}$. Hence, the expected time spent before one of the two nodes change their states $= \frac{4K}{3\pi\overline{v}}$.

(14) $E[\tau_{add2}^{mm}]$ is the additional time two nodes remain in contact when node 2 is paused and node 1 is moving. This equation can be derived in a manner similar to the derivation of Equation (13).

(15) $E[\tau_{add3}^{mm}]$ is the additional time two nodes remain in contact when both the nodes are moving. Either of the following can happen in the succeeding time slots:

  (a) The two nodes move out of the range of each other without either of the nodes pausing. The expected duration the two nodes remain in contact is $\frac{0.6366K}{1.27\overline{v}}$. (The derivation is similar to the one for 13 (a).)

  (b) One of the two nodes pause before moving out of each other's range. The expected time spent before one of the node pauses $= \frac{E[s]}{1.27\overline{v}}$. If its node 1 which paused, then the additional time both nodes spend within range of each other is $E[\tau_{add1}^{mm}]$, while if its node 2 which paused, then the additional contact time is $E[\tau_{add2}^{mm}]$.

(16) $E[\tau_{add4}^{mm}]$ is the additional time two nodes remain in contact when both the nodes are paused. The expected time before one of the nodes starts moving is denoted by $\overline{T}_{stop}^{mm}$. If the node which starts moving changes state (from roaming to local or vice versa), the nodes will stay within with each other's range for a further $\frac{0.6366K}{\hat{v}_c^{mm}\overline{v}}$ time units. If the node which starts moving does not change its state, following two subcases can occur: If node 2 starts moving before node 1, the additional time spent within each other's range is $E[\tau_m^{add1}]$, while if node 1 starts moving first, the additional contact time is $E[\tau_m^{add2}]$.

The value of all the probabilities in all the four equations depend on the mobility model. Lemma 1.4 derives their value for the Random Direction mobility model. ■

The set of linear equations in Lemma 1.3 can be solved to get $E[\tau_{mm}^{add1}]$, $E[\tau_{mm}^{add2}]$, $E[\tau_{mm}^{add3}]$ and $E[\tau_{mm}^{add4}]$. Lemmas 1.1, 1.2 and 1.3 summarize the basic framework of how to find the expected contact time for two nodes moving according to a mobility model 'mm'. Now we discuss how to use these lemmas to find the expected contact time for the Random Direction mobility model, $E\tau_{rd}$. Recall that Theorem 3.4 expresses $E\tau_{rd}$ as a function of $E[\tau_{rd}^1]$ and $E[\tau_{rd}^2]$. So, what we need to determine are expressions for $E[\tau_{rd}^1]$ and $E[\tau_{rd}^2]$. Lemmas 1.1, 1.2 and 1.3 are used to derive these two. Specifically, $E[\tau_{rd}^1]$ corresponds to $E[\tau_{mm}^1]$ which is derived in Lemma 1.1 and $E[\tau_{rd}^2]$ corresponds to $E[\tau_{mm}^2]$ which is derived in Lemma 1.2. To complete the derivation, in the next lemma, we derive the value of all the variables in Lemmas 1.1, 1.2 and 1.3 which depend on the mobility model for the Random Direction mobility model.

Lemma 1.4: (a) The normalized relative speed between the two nodes given that they moved within range of each other in the current time slot, $\hat{v}_{rd}^c$, is given by,

$$\hat{v}_{rd}^c \simeq \int_K^{K+2\overline{v}} \sqrt{8(1+cos(a))}tan\left(\frac{a}{2}\right)\frac{al}{2\pi(K+1)\overline{v}P(E_A)}dl,$$

where $a = cos^{-1}\left(\frac{l^2+2\overline{v}^2-2l\overline{v}-K^2}{2\overline{v}(l-\overline{v})}\right)$ and $P(E_A) = \int_K^{K+2\overline{v}} \frac{al}{2\pi(K+1)}dl$ is the probability of the event that the two nodes were out of each other's range at time $t-1$ and were within each other's range at time t.

(b) The value of the probabilities in Lemma 1.1 which depend on the mobility model are as follows: $p_{rd}^1 \simeq \int_0^{\pi} \frac{1}{\pi}\left(1-e^{\left(\frac{-4Ksin(\phi)}{\hat{v}_{rd}^c\overline{v}\overline{T}}\right)}\right)d\phi$, $p_{rd}^{11} = p_{rd}^{12} = \frac{1}{2}$.

(c) The value of the probabilities in Lemma 1.2 which depend on the mobility model are as follows: $p_{rd}^2 \simeq \int_0^{\pi} \frac{1}{\pi}\left(1-e^{\frac{-2Ksin(\phi)}{\overline{v}}\left(\frac{1}{\overline{T}}+\frac{1}{\overline{T}_{stop}}\right)}\right)d\phi$, $p_{rd}^{21} = 1-p_{rd}^{22} = \frac{1/\overline{T}_{stop}}{1/\overline{T}+1/\overline{T}_{stop}}$.

(d) The value of the probabilities in Lemma 1.3 which depend on the mobility model are as follows: $p_{rd}^{add1} = p_{rd}^{add2} \simeq \int_0^{\pi}\int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)}\left(1-e^{\left(\frac{-x}{\overline{v}}\left(\frac{1}{\overline{T}}+\frac{1}{\overline{T}_{stop}}\right)\right)}\right)dxd\phi$, $p_{rd}^{add3} \simeq \int_0^{\pi}\int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)}\left(1-e^{\left(\frac{-2x}{1.27\overline{T}\overline{v}}\right)}\right)dxd\phi$, $p_{rd}^{add4} = 0$, $p_{rd}^{add11} = 1-p_{rd}^{add12} = p_{rd}^{add21} = 1-p_{rd}^{add22} = \frac{1/\overline{T}_{stop}}{1/\overline{T}+1/\overline{T}_{stop}}$, and $p_{rd}^{add31} = p_{rd}^{add32} = p_{rd}^{add41} = p_{rd}^{add42} = \frac{1}{2}$.

(e) Finally, the expected time both the nodes remain paused, $\overline{T}_{rd}^{stop}$ is equal to $\frac{\overline{T}_{stop}}{2}$.

Proof: (a) Let E_A denote the event that the two nodes (label them as nodes 1 and 2) were out of each other's range at time $t-1$ and came within each other's range at time t. Then, $\hat{v}_{rd}^c = \frac{E[\|\vec{v}_1-\vec{v}_2\| | E_A]}{\overline{v}}$. Recall that Theorem 3.2 evaluated the value of $\hat{v}_{rd}$ which is the unconditioned normalized relative speed. $\hat{v}_{rd}^c$ is different from $\hat{v}_{rd}$ because the fact that the two nodes are coming within range of each other rules out some relative velocities. For example, the nodes cannot be moving away from each other.

To find $\hat{v}_{rd}^c$, we make an approximation by replacing the magnitude of the node's velocities by their expected value $\overline{v}$. Let l denote the distance between the two nodes at time $t-1$. Let node 1 move along the x-axis and node 2 move at an angle $\theta = \theta_1 - \theta_2$ from the x-axis. This is equivalent to the model where both nodes are moving at an angle θ_1 and θ_2 respectively. The angle θ will satisfy the following relationship: $(l-\overline{v}-\overline{v}cos(\theta))^2 + (\overline{v}sin(\theta))^2 \le K^2 \Rightarrow \theta \ge a$ where $a = cos^{-1}\left(\frac{l^2+2\overline{v}^2-2l\overline{v}-K^2}{2\overline{v}(l-\overline{v})}\right)$. Since unconditioned θ is distributed uniformly at random between 0 and 2π, the conditioned value of θ will be uniformly distributed between $-a$ and a.

Thus, $E[\|\vec{v}_1-\vec{v}_2\| \mid E_A, l] = \int_{-a}^{a} \frac{\overline{v}}{2a}\sqrt{(1+cos(\phi))^2+sin^2(\phi)}\ d\phi = \sqrt{8(1+cos(a))}\ tan\left(\frac{a}{2}\right)$. Now, to find $E[\|\vec{v}_1-\vec{v}_2\| \mid E_A]$, we will have to remove the condition on l using the law of total probability, for which we first have to derive $f_{L|E_A}(l)$. $f_{L|E_A}(l)dl = P(L=l \mid E_A) = \frac{P(E_A|L=l)P(L=l)}{P(E_A)} = \frac{aldl}{(2K+2)\pi P(E_A)}$ where $P(E_A) = \int_K^{K+2\overline{v}} \frac{al}{2\pi(K+1)}dl$.

Now using the law of total probability, $E[\|\vec{v}_1-\vec{v}_2\| \mid E_A]$ can be derived to be $\int_K^{K+2\overline{v}} \sqrt{8(1+cos(a))}tan\left(\frac{a}{2}\right)\frac{al}{2\pi(K+1)P(E_A)}dl$.

(b) p_{rd}^1 is the probability that one of the two moving nodes pause before they move out of each other's range. As the movement duration of both the nodes is exponential with mean $\overline{T}$, p_{rd}^1 given ϕ and $\|\vec{v}_i-\vec{v}_j\|$ is equal to $1-e^{-\frac{2Ksin(\phi)}{\frac{\overline{T}}{2}\|\vec{v}_i-\vec{v}_j\|}}$.

To simplify exposition, we replace $\|\vec{v}_i - \vec{v}_j\|$ by its expected value. Hence, $p_1 \simeq \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\left(-\frac{4Ksin(\phi)}{\overline{v}^c_{rd}\overline{v}\overline{T}}\right)}\right) d\phi$ which can be evaluated numerically. Since both the nodes have the same movement duration distribution, the probability that node 1 pauses first is equal to the probability that node 2 pauses first, hence $p^{11}_{rd} = p^{12}_{rd} = \frac{1}{2}$.

(c) p^2_{rd} can be derived in a manner similar to the derivation of p^1_{rd}. The movement duration is exponentially distributed with mean $\overline{T}$ while the pause duration is exponentially distributed with mean $\overline{T}_{stop}$. Hence, the probability that the paused node restarts before the moving node pauses, p^{21}_{rd}, is equal to $\frac{1/\overline{T}_{stop}}{1/\overline{T}+1/\overline{T}_{stop}}$.

(d) p^{add1}_{rd} is the probability that either the moving node pauses or the paused node restarts before the two nodes move out of each other's range. The distance to be travelled to move out of each other's range is equal to $2Ksin(\phi)$, where ϕ is a random variable uniformly distributed between 0 and π. Since both the movement and pause distributions are uniform, the distance after which the nodes change state (denote it by x) is uniformly distributed between 0 and $2Ksin(\phi)$. Hence, $p^{add1}_{rd} \simeq \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{\overline{v}}\left(\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}\right)\right)}\right) dx d\phi$. p^{add2}_{rd} and p^{add3}_{rd} can be derived in a manner similar to the derivation of p^{add1}_{rd}. Since there are no local and roaming states in the Random Direction mobility model, $p^{add4}_{rd} = 0$. Finally, the movement and pause durations of both the nodes is exponential with means $\overline{T}$ and $\overline{T}_{stop}$ respectively, hence $p^{add11}_{rd} = p^{add21}_{rd} = \frac{1/\overline{T}_{stop}}{1/\overline{T}+1/\overline{T}_{stop}}$, and $p^{add31}_{rd} = p^{add32}_{rd} = p^{add41}_{rd} = p^{add42}_{rd} = \frac{1}{2}$.

(e) Since the pause duration of both the nodes is exponentially distributed with mean $\overline{T}_{stop}$, the expected time both the nodes remain paused is equal to $\frac{\overline{T}_{stop}}{2}$. ■

APPENDIX II
CONTACT TIME FOR THE RANDOM WAYPOINT MOBILITY MODEL

We will use the framework proposed in Appendix I to derive values for $E[\tau^1_{rwp}]$ and $E[\tau^2_{rwp}]$. $E[\tau^1_{rwp}]$ corresponds to $E[\tau^1_{mm}]$ which is derived in Lemma 1.1 and $E[\tau^2_{rwp}]$ corresponds to $E[\tau^2_{mm}]$ which is derived in Lemma 1.2. To be able to use the framework, we will first have to derive the value of all the variables in Lemmas 1.1, 1.2 and 1.3 which depend on the mobility model for the Random Waypoint mobility model.

We first derive the value of p_{rwp} which denotes the probability that a given node A pauses within the transmission range of another node B given that node A is passing through the transmission range of node B. Then, in the next lemma, we state the value of all the variables in Lemmas 1.1, 1.2 and 1.3 which depend on the mobility model. (Obviously, the value of some of these variables will depend on p_{rwp}.)

Lemma 2.1: $p_{rwp} = \frac{\frac{\pi K^2}{N}}{\frac{\pi K^2}{N}+p_{r1}+p_{r2}}$ where $p_{r1} = \frac{1}{N}\int_K^{\frac{\sqrt{N}}{2}} \frac{2\pi l}{N} \int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2rsin^{-1}\left(\frac{K}{l}\right) dr dl$ and $p_{r2} = \frac{1}{N}\int_{\frac{\sqrt{N}}{2}}^{\frac{\sqrt{N}}{\sqrt{2}}} \frac{4l}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2l}\right)\right) \int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2rsin^{-1}\left(\frac{K}{l}\right) dr dl$.

$p_{r1} + p_{r2}$ is the probability that node A will pass through the transmission range of node B but not pause within node B's transmission range.

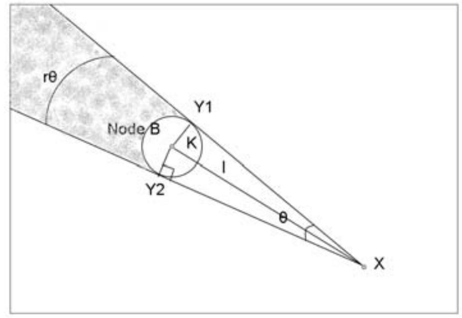

Fig. 10. Node A will pass through the transmission range of node B if and only if its destination lies in the shaded region.

Proof: Let node A start from point X shown in Figure 10. XY_1 and XY_2 are tangents from point X to the circle denoting the transmission range of node B. If the destination of node A falls within the marked region, then node A will pass within transmission radius of node B. but not pause within node B's transmission range. l denotes the distance between point X and node B. The pdf of the random variable L_{rwp} is given in the proof of Lemma 4.1(b). The angle θ shown in Figure 10 is equal to $2sin^{-1}\left(\frac{K}{l}\right)$. The sector of a circle at a distance r from node X and lying within the shaded region is equal to $r\theta$. Integrating over all possible values of r and l will give the probability that the chosen destination lies in the shaded region. p_{r1} and p_{r2} represent the probabilities when $0 \le l \le \frac{\sqrt{N}}{2}$ and $\frac{\sqrt{N}}{2} \le l \le \frac{\sqrt{N}}{\sqrt{2}}$ respectively. ■

Lemma 2.2: (a) $\hat{v}^c_{rwp} = \hat{v}^c_{rd}$.

(b) The value of the probabilities in Lemma 1.1 which depend on the mobility model are as follows: $p^1_{rwp} = 2p_{rwp} - p^2_{rwp}$, $p^{11}_{rwp} = p^{12}_{rwp} = \frac{1}{2}$.

(c) The value of the probabilities in Lemma 1.2 which depend on the mobility model are as follows: $p^2_{rwp} = p_{r3} + p_{rwp}(1 - p_{r3})$, $p^{21}_{rwp} = 1 - p^{22}_{rwp} = \frac{p_{r3}}{p_{r3}+p_{rwp}(1-p_{r3})}$, where $p_{r3} \simeq \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{\overline{T}_{stop}\overline{v}}\right)}\right) dx d\phi$.

(d) The value of the probabilities in Lemma 1.3 which depend on the mobility model are as follows: $p^{add1}_{rwp} = p^{add2}_{rwp} = p_{r3} + p_{r4}(1 - p_{r3})$, $p^{add3}_{rwp} = 2p_{r4} - p^2_{r4}$, $p^{add4}_{rwp} = 0$, $p^{add11}_{rwp} = 1 - p^{add12}_{rwp} = p^{add21}_{rwp} = 1 - p^{add22}_{rwp} = \frac{p_{r3}}{p_{r3}+p_{r4}(1-p_{r3})}$, and $p^{add31}_{rwp} = p^{add32}_{rwp} = p^{add41}_{rwp} = p^{add42}_{rwp} = \frac{1}{2}$, where $p_{r4} = \frac{\pi K^2}{N}$.

(e) Finally, the expected time both the nodes remain paused, $\overline{T}^{stop}_{rwp}$ is equal to $\frac{\overline{T}_{stop}}{2}$.

Proof: Lemma 2.2 is derived in a manner similar to the derivation of Lemma 1.4. ■

APPENDIX III
PROOF OF LEMMA 5.5

If the nodes meet within their community, then they have a higher chance of meeting again quickly because the communities are much smaller than the entire network. The probability of the event that when the nodes met, they were in their local states and only one of them was moving (denoted by p_1^+) is evaluated using Bayes' Theorem to be equal to $\frac{4K\overline{v}p_{ml}p_{pl}}{c^2Np_{m,in}}$ where $p_{m,in} = 1/EM_{comm}^{(in)}$. Similarly, the probability of the event that when the nodes met, both of them were in their local states and moving (denoted by p_2^+) is derived to be equal to $\frac{2K\hat{v}_{rd}\overline{v}p_{ml}^2}{c^2Np_{m,in}}$. Now we find $E[M_1^+]$ and $E[M_2^+]$ which are the expected inter-meeting times associated with the two cases.

$E[M_1^+]$: If both the nodes are in their local state but only one of the nodes is moving, then only after $\overline{T}_1 = \left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}_{stop}^l}\right)^{-1}$ time units, one of the nodes will change its state. With probability $1 - \left(1 - \frac{2K\overline{v}}{c^2N}\right)^{\overline{T}_1}$, the two nodes meet again within this time epoch, else one of the following three subcases occur: (i) with probability $\frac{\overline{v}\overline{T}_1}{L_c}$ the moving node pauses first and let $E[M_3^+]$ denote the additional time it takes the two nodes to meet again, (ii) with probability $\frac{(1-p_l)\overline{T}_1}{\overline{T}_{stop}^l}$ the paused node starts moving first and remains in its local state, and it takes $E[M_2^+]$ additional time units for them to meet again, and (iii) with probability $\frac{p_l\overline{T}_1}{\overline{T}_{stop}^l}$, the paused node starts moving first and moves into its roaming state and it take will take $EM_{comm}^{(in)}$ time units to meet again (as now one of the nodes is now moving over the entire network and it will mix fast). Thus,

$$E[M_1^+] = \overline{T}_1 + \left(1 - \frac{2K\overline{v}}{c^2N}\right)^{\overline{T}_1} \left(\frac{\overline{v}\overline{T}_1}{L_c}E[M_3^+] + \frac{(1-p_l)\overline{T}_1}{\overline{T}_{stop}^l}E[M_2^+] + \frac{p_l\overline{T}_1}{\overline{T}_{stop}^l}EM_{comm}^{(in)}\right). \quad (17)$$

$E[M_2^+]$: Now, if both the nodes were moving in their local states, then they will keep moving for $\frac{L_c}{2\overline{v}}$ time units before one of the nodes changes its state. With probability $1 - \left(1 - \frac{2K\hat{v}_{rd}\overline{v}}{c^2N}\right)^{\frac{L_c}{2\overline{v}}}$, the two nodes meet within this time epoch, else one of the nodes pauses within its local state and it takes $E[M_1^+]$ additional time units for them to meet again. Thus,

$$E[M_2^+] = \frac{L_c}{2\overline{v}} + \left(1 - \frac{2K\hat{v}_{rd}\overline{v}}{c^2N}\right)^{\frac{L_c}{2\overline{v}}} E[M_1^+]. \quad (18)$$

$E[M_3^+]$: If both the nodes are paused in their local states, then they will remain paused for $\frac{\overline{T}_{stop}^l}{2}$ time units. Once one of the nodes changes states, one of the following two subcases can occur: (i) With probability p_l, the node which starts moving will move into the roaming state and it will take $EM_{comm}^{(in)}$ time units for them to meet again, else (ii) With probability $1-p_l$, the node which starts moving will remain in its local state and it will take $E[M_1^+]$ additional time units for them to meet again. Thus,

$$E[M_3^+] = \frac{\overline{T}_{stop}^l}{2} + p_lEM_{comm}^{(in)} + (1-p_l)E[M_1^+]. \quad (19)$$

Equations (17), (18) and (19) form a linear set of equations which can be easily solved to find $E[M_1^+], E[M_2^+]$ and $E[M_3^+]$.

Finally, when the nodes met, if at least one of the nodes was in its roaming state, then the probability that the two nodes meet within one mixing time is negligible and it will take one meeting time for them to meet again. Putting everything togther yields the Lemma.

APPENDIX IV
CONTACT TIME FOR THE COMMUNITY-BASED MOBILITY MODEL

In this appendix, we discuss how to derive the expressions for $E\tau_{comm}^{(out)}$ (expected contact time for nodes with non-overlapping communities) and $E\tau_{comm}^{(in)}$ ((expected contact time for nodes with overlapping communities). For ease of presentation, we define the following sets of variables. (i) $E[\tau_{comm,rr}^1]$ and $E[\tau_{comm,rr}^2]$: Expected contact time for two nodes in the roaming state when both nodes are moving and when only one node is moving respectively. (ii) $E[\tau_{comm,ll}^1]$ and $E[\tau_{comm,ll}^2]$: Expected contact time for two nodes in the local state when both nodes are moving and when only one node is moving respectively. (iii) $E[\tau_{comm,rl}^1]$ and $E[\tau_{comm,rl}^2]$: Expected contact time for two nodes with one in the roaming state and other in the local state and when both nodes are moving and when only one node is moving respectively. The framework introduced in Appendix I is used to derive expressions for these three sets of variables. The corresponding probabilities are derived in a manner similar to the derivation of Lemma 1.4 because both nodes are moving according to the Random Direction mobility model. The only difference is that now nodes can change states at the end of a pause time (from roaming to local and vice versa).

We first state a lemma needed to derive $E[\tau_{comm,rr}^1]$ and $E[\tau_{comm,rr}^2]$ using the framework. Note that $E[\tau_{comm,rr}^1]$ corresponds to $E[\tau_{mm}^1]$ (derived in Lemma 1.1) and $E[\tau_{comm,rr}^2]$ corresponds to $E[\tau_{mm}^2]$ (derived in Lemma 1.2).

Lemma 4.1: (a) $\hat{v}_{comm,rr}^c = \hat{v}_{rd}^c$.
(b) The value of the probabilities in Lemma 1.1 which depend on the mobility model are as follows: $p_{comm,rr}^1 \simeq \int_0^\pi \frac{1}{\pi}\left(1 - e^{\left(\frac{-4Ksin(\phi)}{\hat{v}_{comm,rr}^c\overline{v}\overline{T}}\right)}\right)d\phi$, $p_{comm,rr}^{11} = p_{comm,rr}^{12} = \frac{1}{2}$.
(c) The value of the probabilities in Lemma 1.2 which depend on the mobility model are as follows: $p_{comm,rr}^2 \simeq \int_0^\pi \frac{1}{\pi}\left(1 - e^{\frac{-2Ksin(\phi)}{\overline{v}}\left(\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}\right)}\right)d\phi$, $p_{comm,rr}^{21} = p_r\frac{1/\overline{T}_{stop}}{1/\overline{T}+1/\overline{T}_{stop}}$, $p_{comm,rr}^{22} = \frac{1/\overline{T}}{1/\overline{T}+1/\overline{T}_{stop}}$.
(d) The value of the probabilities in Lemma 1.3 which depend on the mobility model are as follows: $p_{comm,rr}^{add1} = p_{comm,rr}^{add2} \simeq \int_0^\pi \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)}\left(1 - e^{\left(\frac{-x}{\overline{v}}\left(\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}\right)\right)}\right)dxd\phi$, $p_{comm,rr}^{add3} \simeq \int_0^\pi \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)}\left(1 - e^{\left(\frac{-2x}{1.27\overline{T}\overline{v}}\right)}\right)dxd\phi$, $p_{comm,rr}^{add4} = 1 - p_r$, $p_{comm,rr}^{add11} = p_{comm,rr}^{add21} = p_{comm,rr}^{21}$, $p_{comm,rr}^{add12} = p_{comm,rr}^{add22} = p_{comm,rr}^{22}$, and $p_{comm,rr}^{add31} = p_{comm,rr}^{add32} = p_{comm,rr}^{add41} = p_{comm,rr}^{add42} = \frac{1}{2}$.
(e) Finally, the expected time both the nodes remain paused, $\overline{T}_{comm,rr}^{stop}$ is equal to $\frac{\overline{T}_{stop}}{2}$.

In the next lemma, we state expressions for variables needed to derive $E[\tau_{comm,ll}^1]$ and $E[\tau_{comm,ll}^2]$ using the framework. Note that $E[\tau_{comm,ll}^1]$ corresponds to $E[\tau_{mm}^1]$ (derived in Lemma 1.1) and $E[\tau_{comm,ll}^2]$ corresponds to $E[\tau_{mm}^2]$ (derived in Lemma 1.2).

Lemma 4.2: (a) $\hat{v}_{comm,ll}^c = \hat{v}_{rd}^c$.
(b) The value of the probabilities in Lemma 1.1 which depend on the mobility model are as follows: $p_{comm,ll}^1 \simeq$

$\int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\left(\frac{-4Ksin(\phi)}{\hat{v}^c_{comm,ll}L_c}\right)}\right) d\phi$, $p^{11}_{comm,ll} = p^{12}_{comm,ll} = \frac{1}{2}$.
(c) The value of the probabilities in Lemma 1.2 which depend on the mobility model are as follows: $p^2_{comm,ll} \simeq \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\frac{-2Ksin(\phi)}{\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}^l_{stop}}\right)}\right) d\phi$, $p^{21}_{comm,ll} = p_l \frac{1/\overline{T}^l_{stop}}{\overline{v}/\overline{L}_c + 1/\overline{T}^l_{stop}}$, $p^{22}_{comm,ll} = \frac{\overline{v}/\overline{L}_c}{\overline{v}/\overline{L}_c + 1/\overline{T}^l_{stop}}$.
(d) The value of the probabilities in Lemma 1.3 which depend on the mobility model are as follows: $p^{add1}_{comm,ll} = p^{add2}_{comm,ll} \simeq \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}^l_{stop}}\right)\right)}\right) dx d\phi$, $p^{add3}_{comm,ll} \simeq \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-2x}{1.27L_c}\right)}\right) dx d\phi$, $p^{add4}_{comm,ll} = 1 - p_l$, $p^{add11}_{comm,ll} = p^{add21}_{comm,ll} = p^{21}_{comm,ll}$, $p^{add12}_{comm,ll} = p^{add22}_{comm,ll} = p^{22}_{comm,ll}$, and $p^{add31}_{comm,ll} = p^{add32}_{comm,ll} = p^{add41}_{comm,ll} = p^{add42}_{comm,ll} = \frac{1}{2}$.
(e) Finally, the expected time both the nodes remain paused, $\overline{T}^{stop}_{comm,ll}$ is equal to $\frac{\overline{T}^l_{stop}}{2}$.

Finally, in the next lemma we state expressions for variables needed to derive $E[\tau^1_{comm,rl}]$ and $E[\tau^2_{comm,rl}]$ using the framework. Note that $E[\tau^1_{comm,rl}]$ corresponds to $E[\tau^1_{mm}]$ (derived in Lemma 1.1) and $E[\tau^2_{comm,rl}]$ corresponds to $E[\tau^2_{mm}]$ (derived in Lemma 1.2). The only additional observation used in proving this lemma is that the probability that the node in the roaming state is moving and the node in the local state is paused given that at least one of the nodes is paused is equal to $\frac{p_{mr}p_{pl}}{p_{mr}p_{pl} + p_{pr}p_{ml}}$.

Lemma 4.3: (a) $\hat{v}^c_{comm,rl} = \hat{v}^c_{rd}$.
(b) The value of the probabilities in Lemma 1.1 which depend on the mobility model are as follows: $p^1_{comm,rl} \simeq \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\left(\frac{-2Ksin(\phi)}{\hat{v}^c_{comm,rl}\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}}\right)\right)}\right) d\phi$, $p^{11}_{comm,rl} = 1 - p^{12}_{comm,rl} = \frac{1/\overline{T}}{\overline{v}/\overline{L}_c + 1/\overline{T}}$.
(c) The value of the probabilities in Lemma 1.2 which depend on the mobility model are as follows: $p^2_{comm,rl} \simeq \frac{p_{mr}p_{pl}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\frac{-2Ksin(\phi)}{\overline{v}}\left(\frac{1}{\overline{T}} + \frac{1}{\overline{T}^l_{stop}}\right)}\right) d\phi$ $+ \frac{p_{pr}p_{ml}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\frac{-2Ksin(\phi)}{\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}_{stop}}\right)}\right) d\phi$, $p^{21}_{comm,rl} = \frac{p_l p_{mr}p_{pl}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \frac{1/\overline{T}^l_{stop}}{1/\overline{T} + 1/\overline{T}^l_{stop}} + \frac{p_r p_{pr}p_{ml}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \frac{1/\overline{T}_{stop}}{\overline{v}/\overline{L}_c + 1/\overline{T}_{stop}}$, $p^{22}_{comm,rl} = \frac{p_{mr}p_{pl}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \frac{1/\overline{T}}{1/\overline{T} + 1/\overline{T}^l_{stop}} + \frac{p_{pr}p_{ml}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \frac{\overline{v}/\overline{L}_c}{\overline{v}/\overline{L}_c + 1/\overline{T}_{stop}}$.
(d) The value of the probabilities in Lemma 1.3 which depend on the mobility model are as follows: $p^{add1}_{comm,rl} = p^{add2}_{comm,rl} \simeq \frac{p_{mr}p_{pl}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{\overline{v}}\left(\frac{1}{\overline{T}} + \frac{1}{\overline{T}^l_{stop}}\right)\right)}\right) dx d\phi +$ $\frac{p_{pr}p_{ml}}{p_{mr}p_{pl} + p_{pr}p_{ml}} \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}^l_{stop}}\right)\right)}\right) dx d\phi$, $p^{add3}_{comm,rl} \simeq \int_0^{\pi} \int_0^{2Ksin(\phi)} \frac{1}{2\pi Ksin(\phi)} \left(1 - e^{\left(\frac{-x}{1.27\overline{v}}\left(\frac{\overline{v}}{L_c} + \frac{1}{\overline{T}}\right)\right)}\right) dx d\phi$, $p^{add4}_{comm,rl} = (1 - p_r) \frac{1/\overline{T}_{stop}}{1/\overline{T}_{stop} + 1/\overline{T}^l_{stop}} + (1 - p_l) \frac{1/\overline{T}^l_{stop}}{1/\overline{T}_{stop} + 1/\overline{T}^l_{stop}}$, $p^{add11}_{comm,rl} = p^{add21}_{comm,rl} = p^{21}_{comm,rl}$, $p^{add12}_{comm,rl} = p^{add22}_{comm,rl} = p^{22}_{comm,rl}$, and $p^{add31}_{comm,rl} = p^{add32}_{comm,rl} = p^{add41}_{comm,rl} = p^{add42}_{comm,rl} = \frac{1}{2}$.
(e) Finally, the expected time both the nodes remain paused, $\overline{T}^{stop}_{comm,rl}$ is equal to $\left(\frac{1}{\overline{T}_{stop}} + \frac{1}{\overline{T}^l_{stop}}\right)^{-1}$.

We now derive $E\tau^{(out)}_{comm}$ and $E\tau^{(in)}_{comm}$ in the following two lemmas. The proof of both the lemmas follow directly by listing all the possible cases two nodes can be in when they come within each other's range, find the probability of each case, find the expected contact time associated with each case and then combining everything together using the law of total probability.

Lemma 4.4: $E\tau^{(out)}_{comm} = p^1_{out}E[\tau^1_{comm,rr}] + p^2_{out}E[\tau^2_{comm,rr}] + p^3_{out}E[\tau^1_{comm,rl}] + p^4_{out}E[\tau^2_{comm,rl}]$ where $p^1_{out} = \frac{2Kp^2_{mr}\hat{v}_{rd}\overline{v}}{Np_{m,out}}$, $p^2_{out} = \frac{4Kp_{mr}p_{pr}\overline{v}}{Np_{m,out}}$, $p^3_{out} = \frac{4Kp_{mr}p_{ml}\hat{v}_{rd}\overline{v}}{Np_{m,out}}$, $p^4_{out} = \frac{4K(p_{mr}p_{pl} + p_{pr}p_{ml})\overline{v}}{Np_{m,out}}$, and $p_{m,out} = 1/EM^{(out)}_{comm}$.

Lemma 4.5: $E\tau^{(in)}_{comm} = p^1_{in}E[\tau^1_{comm,rr}] + p^2_{in}E[\tau^2_{comm,rr}] + p^3_{in}E[\tau^1_{comm,rl}] + p^4_{in}E[\tau^2_{comm,rl}] + p^5_{in}E[\tau^1_{comm,ll}] + p^6_{in}E[\tau^2_{comm,ll}]$ where $p^1_{in} = \frac{2Kp^2_{mr}\hat{v}_{rd}\overline{v}}{Np_{m,in}}$, $p^2_{in} = \frac{4Kp_{mr}p_{pr}\overline{v}}{Np_{m,in}}$, $p^3_{in} = \frac{4Kp_{mr}p_{ml}\hat{v}_{rd}\overline{v}}{Np_{m,in}}$, $p^4_{in} = \frac{4K(p_{mr}p_{pl} + p_{pr}p_{ml})\overline{v}}{Np_{m,in}}$, $p^5_{in} = \frac{2Kp^2_{ml}\hat{v}_{rd}\overline{v}}{c^2Np_{m,in}}$, $p^6_{in} = \frac{4Kp_{ml}p_{pl}\overline{v}}{Np_{m,in}}$, and $p_{m,in} = 1/EM^{(in)}_{comm}$.