Negotiating Multichannel Sensing and Access In Cognitive Radio Wireless Networks

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***Abstract*— We investigate the following question - how should secondary users coordinate with each other to determine which channels to sense and potentially access in a cognitive radio network? If users may have a potentially different valuation of each channel and don't know of each other's valuations, then it is unclear whether there may be some benefit to explicitly exchanging this information, albeit at some cost, in order to minimize their chances of picking the same channel. We formulate and analyze a relevant 2-player 2-channel game and quantify how the cost of gathering the information affects the optimal number of rounds of negotiation.**

I. INTRODUCTION

The rapid growth of wireless devices and services continue to strain the limited spectral resource. A basic approach to efficiently utilize the radio frequency spectrum is opportunistic spectrum access (OSA), where secondary users sense channels to determine if they may safely send packets without interfering with primary users before accessing the medium [1].

An important component of OSA is a sensing strategy at the MAC layer to track spectrum opportunity. Most prior work has focused on the sensing decision from the perspective of a single secondary user [8], [3]. When there are multiple secondary users within interfering range of each other contending for opportunities, the sensing decision must take into account the possibility of collision among secondary users on good channels.

In this research, we investigate the coordination among secondary users on channel sensing strategy in cognitive wireless networks. In this problem, each secondary user has an evaluation for the sensing candidate channels and ranks the valuation in descending order. The secondary users can exchange valuation information with each other to coordinate among themselves when deciding which channel to sense.

The information exchange or negotiation process is a round-based one. In each round, each secondary user reveals his/her valuation about a particular channel with all his/her neighbors. The order in which channel information is revealed is the order in which the channels are ranked. On one hand, exchanging information reduces collisions among secondary users, hence improving the secondary user's expected throughput. On the other hand, information exchange decreases the data transmission time if the opportunities appear. There is a tradeoff between getting more information to avoid collisions among secondary users and exiting the information exchange round to obtain more data transmission time when the opportunity arises. Deciding how many rounds to participate in the information gathering to maximize the expected throughput in a time slot is the problem that is the focus of this study.

In this paper, we formulate and analyze a relevant 2-player 2-channel game and quantify how the cost of gathering the information affects the optimal number of rounds of negotiation. In this 2-player 2-channel game, each secondary user has a valuation on each channel. A channel valuation can be "high" or "low" to a user, with probability q and $1-q$ correspondingly. The users decide how many rounds to negotiate with the other user before the negotiation actually happens. The objective of the user is to maximize the expected throughput in a given time period. We show that even the 2-player 2-channel case is not trivial. We also show that users' optimal number of negotiation rounds changes with the ratio of each round's length to the given period length (which quantifies the cost of negotiation).

The paper is organized as follows: section II lists the related works and section III describes the game formulation and assumptions used in this research. Section IV presents the decision process of the secondary users and calculates the optimal negotiation rounds. This section also shows how the negotiation cost (i.e., length of each negotiation round) affects the optimal decision. Section V concludes the paper and points out some possible future directions.

II. RELATED WORKS

Game theoretic treatments of medium access and spectrum sharing have been previously presented in settings other than opportunistic access for cognitive radios. For instance, Mackenzie and Wicker analyze a slotted Aloha game in [13]. Halldorsson *et al.*[14] present a channel assignment game in the context of WiFi networks. Konorski [15] provides a game theoretic analysis of CSMA/CA protocols.

Cao and Zheng [2] consider the case where multiple conflicting users share common spectrum resource in a multi-terminal wireless ad-hoc network. They propose a bargaining mechanism between users that seeks to maximize system fairness. Unlike the negotiation game we consider in this work which is aimed at maximizing system throughput rather than fairness, in their formulation, the cost of the communication is not embedded in the users' utilities.

In the cognitive radio context, previous research has mostly focused on single user's optimal sensing policy. Zhao *et al.* [7], [8], consider a partially observable markov decision problem where the primary users' behavior can be characterized as providing stochastically independent and identical Markovian channels. In this setting, it is proved that a myopic policy that picks at each step the channel with highest probability of being free, has a simple semi-universal structure and moreover is provably optimal for the case of 2-channels. Further results by Javidi *et al.* [9] and Ahmad *et al.* [10] have considerably generalized the conditions under which a myopic scheme is optimal. Chang and Liu [3] consider a setting where a single secondary user must probe multiple channels to decide on which one to access. They consider the tradeoff between the time spent in determining which channel to sense, and using the channel (similar in spirit to the negotiation/use tradeoff that we consider in this work).

When there exist multiple secondary users in the system, we have to consider the probability of collision among secondary users since this becomes a significant source for loss of throughput. However, the research on multiple-channel multiple-user scenarios in cognitive networks is still in its infancy. Liu and Krishnamachari [5] give a solution for the static case sensing decision. They conclude that in the asymmetric case (i.e., secondary users have different valuations on contending channels), with global information, the optimal system throughput can be obtained by applying Hungarian algorithm [12]. We use this conclusion in this paper. Rather than getting a static solution on the case with full information, however, we investigate the optimal information exchange rounds when exchanging information has cost in this paper.

Liu *et.al* [6] compare the global throughput for four different sensing policies via 2-user 2-channel simulations when the channel status is formulated as a partially observed Markov decision process (POMDP) [16], [17]. The simulation results show that for the 2-user case, a distributed myopic policy (i.e. users make myopic decision without considering the other user's existence) performs worse than the policies with user cooperation in most cases. Similar results are observed in a work by Liu *et al.* [11] where a randomized policy is proposed within the POMDP framework. These results convince us that coordination is needed in the multi-user scenario.

Fu and van der Schaar [4] have recently modeled the secondary users in cognitive radio networks as self-interested autonomous agents that strategically interact in order to acquire the dynamically available spectrum opportunities. In their formulation, a virtual central spectrum manager auctions the available resources and the secondary users bid for the resources. Based on the observed resource allocations and corresponding rewards, a best response learning algorithm for wireless users to improve their bidding policy at each stage is proposed. Unlike their work, we focus on optimizing the expected throughput in a distributed manner in a given time period. The objective of this work is to maximize the coordination among secondary users with minimum information needed, taking into account the cost of information gathering.

III. The Negotiation Process

In cognitive radio networks, when the primary users do not use the communication channels, the secondary users have the opportunity to access the corresponding channels. In this paper, we consider that the time is slotted. In each time slot, a channel can be in one of two states for a particular secondary user i: busy (i.e., this channel's corresponding primary user occupies the channel in this time slot) or free (i.e., user i can use this channel in this time slot without conflicting with the primary user). A secondary user chooses a channel to sense at the beginning of the time slot. The user can access this channel if the sensing result shows that the channel is in a free state.

In the section below, we consider two kinds of channel availability conditions - if "high", a channel is available with probability p_{high}, if "low", it is available with probability p_{low}, such that $p_{low} < p_{high}$. For each channel, the probability of being in high state or low state is independent and identically distributed (i.i.d). We consider n secondary users in range of each other that are trying to coordinate access. We assume that the valuation for both channels is not necessarily identical for the different users. Further, before negotiations commence, each user is aware of only its own evaluation, not that of the other users.

If two or more than two neighboring secondary users happen to choose the same channel to transmit data in one time slot, a collision occurs among these secondary users. Upon collision, none of the secondary users will gain any throughput from this channel. In order to reduce the collisions among secondary users, a secondary user might want to know other secondary users' channel valuation [1]. However, exchanging messengers among conflicting neighbors occupies the users' actual data transmitting time. Thus, there is a tradeoff between exchanging more information with other secondary users to reduce the chance of collision and quitting the information exchange process to gain more data transmitting time.

Suppose the length of the investigated time slot is T time units. The negotiation process happens before the user decides which channel to sense. An extra communication channel exists for negotiation. Before negotiation, each user has his/her own channel valuation on hand and decides the optimal negotiation rounds *a-priori*.

For a group of pairwise conflicting secondary users, each slotted time period T contains two stages: the negotiation stage and the sensing/transmission stage. The negotiation stage may contain several rounds and each round occupies t time units. In each negotiation round, all the secondary users reveal one channel valuation with all the neighbors. Specifically, the users sort their channel valuation in descending order and in negotiation round k, every user in the negotiation process shares his/her k^{th} best channel's valuation together with the channel identity with all his/her neighbors. If there are more than one channels ranked the same to a user, the user will uniformly randomly pick one channel among these channels to share the information. Figure 1 illustrates a typical frame containing 3 negotiation rounds.

Without loss of generality, we also assume that T is large

[1] For tractability, we assume that secondary users can negotiate with each other through a low-rate control channel that is accessible to both. We set aside the much-harder problem of negotiating without a common control channel to future work.

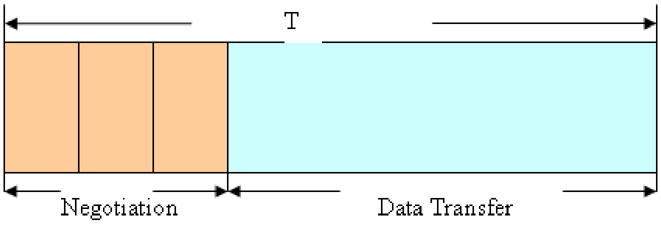

Fig. 1. An illustration of the frame containing negotiation rounds

		$C1$	$C2$
Case 1 $(1-q)q$	$P1$ $P2$	p_{high} p_{low}	p_{low} p_{high}
Case 2 q^2	$P1$ $P2$	p_{high} p_{high}	p_{low} p_{high}
Case 3 $q(1-q)$	$P1$ $P2$	p_{high} p_{high}	p_{low} p_{low}
Case 4 $(1-q)^2$	$P1$ $P2$	p_{high} p_{low}	p_{low} p_{low}

TABLE I
FOUR POSSIBLE OUTCOME CASES

enough for the secondary users to exchange all the information. That is, $T \geq tm$ where m is the number of channels. The actual number of the negotiation rounds is decided before the negotiation begins by the conflicting secondary users in a distributed manner. The negotiation process ends when at least one user quits.

A. The Utility Function

The secondary users' objectives are to maximize their own expected throughput in T time units. Mathematically, the utility function for user i can be expressed in the following:

$$U_i = (T - at)E[R] \tag{1}$$

where a is the number of negotiation rounds and $E[R] = \sum_{k=1}^{m} b_{i,k} p_{i,k} g_{-i,k}$ is the expected throughput in unit time (R denotes the throughput rate). In this equation, $b_{i,k}$ is the transmitting rate for user i on channel k in unit time and $p_{i,k}$ is the available probability of channel k for user i. $g_{-i,k}$ denotes the probability that user i will not collide with any other secondary users in channel k (*i.e.*, the probability that other users will not choose channel k to sense).

In this paper, we consider a one-shot game in the given time slot (*i.e.*, T is a given constant). Instead of considering that each user maximizes his/her throughput in a given time period T, we consider an equivalent formulation that each user maximizes the average throughput in unit time. That is, we consider $U_i' = \frac{U_i}{T}$ as follows:

$$U_i' = (1 - a\beta)E[R] \tag{2}$$

where $\beta = \frac{t}{T}$. Without loss of generality, we assume that $b_{i,k} = 1$ for all $1 \leq i \leq n$ and $1 \leq k \leq m$ where n is the number of neighboring secondary users. In the discussion below, we will use this assumption and the unit time utility function in equation (2) for simplicity.

IV. A Two-user with Two-channel Case

In this section, we discuss an example involving two secondary users ($P1$ and $P2$) and two communication channels ($C1$ and $C2$). We investigate the case where the channel valuation is limited to be one of the two values: "high" or "low". We illustrate that even in this case, deciding the optimal number of negotiation rounds is not trivial.

A. Two-value Availability Probabilities

In this section, we consider the simplest scenario of channel negotiation in cognitive radio networks. For each secondary user i, channel k's valuation $p_{i,k}$ can be either "high" or "low". If the channel state is valuated as "high" for the secondary user, the channel will in free state in this time slot for the user with probability p_{high}. Similarly, "low" channel will be available with probability p_{low} for the user. Obviously, $1 \geq p_{high} > p_{low} \geq 0$. At each time, each channel has an i.i.d. probability q of being in the high state and probability $1-q$ of being in the low state.

Without loss of generality, we discuss from secondary user $P1$'s point of view. We also assume that channel $C1$ is the "high" channel for user $P1$ and channel $C2$ is the "low" channel for him/her (*i.e.*, $p_{1,1} = p_{high}$ and $p_{1,2} = p_{low}$). Other cases can be derived using similar method as described below.

The users need to decide the number of negotiation rounds before the negotiation process begins. Each secondary user compares the expected throughput in three cases: a) he/she does not exchange any information with the other user; b) he/she participates in the negotiation process for one round to obtain partial system information before channel sensing; c) he/she uses two rounds of negotiation with the other user to exchange full information.

Since the inference of user $P1$ happens before the negotiation process, there are four possible outcomes of user $P2$ (listed in table I), with corresponding probability.

We assume that when the negotiation process ends, user $P1$ decides to access channel $C1$ with probability λ and to sense channel $C2$ with probability $1-\lambda$.

We now discuss user $P1$'s expected utility for the four cases in table I with the condition that user $P1$ does not participate in the negotiation process at all (i.e., the number of negotiation rounds $a = 0$).

For case 1, by symmetry, we know that user $P2$ will sense channel $C1$ with probability $1-\lambda$ and access channel $C2$ with probability λ. Therefore, user $P1$'s expected throughput in unit time for case 1 is

$$p_{high}\lambda^2 + p_{low}(1-\lambda)^2 \tag{3}$$

Similarly, for the other three cases, user $P1$'s expected throughput in unit time can be calculated as in equation (4)[2],

[2]In this case, user $P2$ would like to choose each channel to sense with probability $\frac{1}{2}$. This fact can be proved using same method as applied here to user $P1$. We omit the proof for brevity.

		$C1$	$C2$	$E[R]$
Case 1 $(1-q)q$	$P1$ $P2$	λ $1-\lambda$	$1-\lambda$ λ	$p_{high}\lambda^2 + p_{low}(1-\lambda)^2$
Case 2 q^2	$P1$ $P2$	λ $\frac{1}{2}$	$1-\lambda$ $\frac{1}{2}$	$\frac{1}{2}p_{high}\lambda + \frac{1}{2}p_{low}(1-\lambda)$
Case 3 $q(1-q)$	$P1$ $P2$	λ λ	$1-\lambda$ $1-\lambda$	$p_{high}\lambda(1-\lambda) + p_{low}\lambda(1-\lambda)$
Case 4 $(1-q)^2$	$P1$ $P2$	λ $\frac{1}{2}$	$1-\lambda$ $\frac{1}{2}$	$\frac{1}{2}p_{high}\lambda + \frac{1}{2}p_{low}(1-\lambda)$

TABLE II

CALCULATION FOR USER $P1$'S EXPECTED THROUGHPUT IN UNIT TIME WITH NO NEGOTIATION

(5) and (6) respectively.

$$\frac{1}{2}p_{high}\lambda + \frac{1}{2}p_{low}(1-\lambda) \tag{4}$$

$$p_{high}\lambda(1-\lambda) + p_{low}\lambda(1-\lambda) \tag{5}$$

$$\frac{1}{2}p_{high}\lambda + \frac{1}{2}p_{low}(1-\lambda) \tag{6}$$

Table II summaries the secondary user $P1$'s calculation for unit time throughput without any negotiation. The second and the third columns of the table are the probability of the corresponding user access channel $C1$ or $C2$ after one round negotiation, respectively. The forth column of this table illustrates the expected throughput for user $P1$ in unit time for each case.

The expected throughput for user $P1$ in unit time when he/she does not participate in the negotiation at all is $\frac{1}{2}(p_{high} - p_{low})\lambda + \frac{1}{2}p_{low}$. Notice that $p_{high} > p_{low}$, so the expected utility is maximized when $\lambda = 1$. This fact indicates that randomization on channel accessing is not helping user $P1$ to improve his/her expected utility. This fact suggests user $P1$ to sense his/her "high" channel with probability 1 if no information exchanged between the two secondary users.

Now user $P1$ considers the case where one round of negotiation happens before data transmission stage. Table III shows the process to calculate secondary user $P1$'s throughput in unit time after one round of information exchange with user $P2$.

According to the negotiation rule we defined in previous section, in the first round of negotiation, user $P1$ reveals his/her better channel to user $P2$ and *vise versa*. That is, user $P2$ gets the information that user $P1$ prefers channel one better. In case 4, the channel availability probability for user $P2$ is indifference. In order to avoid collision with secondary user $P1$, user $P2$ will access channel $C2$ with probability 1.

Case 2 in table I is more complicated here and it is split into two subcases. Since two channels are equally good for user $P2$, user $P2$ will reveal one channel condition uniformly randomly to user $P1$. Case $2a$ in table III represents the case that user $P2$ reveals channel $C1$ to user $P1$ and case $2b$ represents the other case. In case $2b$, user $P2$ will access channel $C2$ with probability 1 since user $P1$ claimed channel $C1$ as preferred and user $P2$ is indifferent to both channels.

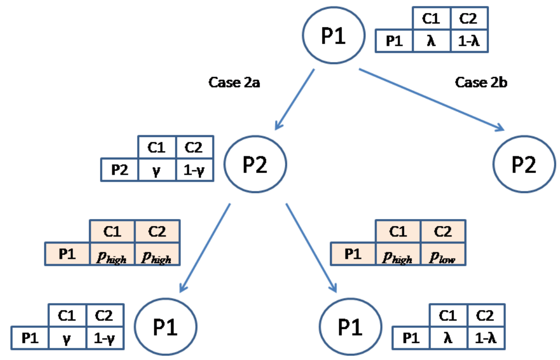

Fig. 2. Illustration of calculating case 2a

In case $2a$, user $P2$ needs to consider two cases [3]: either $p_{1,2} = p_{high}$ or $p_{1,2} = p_{low}$, with probability q and $1-q$ respectively. Let user $P2$'s optimal probability on accessing channel $C1$ be γ. If $p_{1,2} = p_{low}$ (i.e., $P1$ will sense channel $C1$ with probability λ), $P2$'s expected throughput in unit time is $p_{high}(\lambda(1-\gamma) + (1-\lambda)\gamma)$ in this case. If $p_{1,2} = p_{high}$, by symmetry, $P1$ will sense channel $C1$ with probability γ. Thus $P2$'s expected throughput in unit time is $2\gamma(1-\gamma)p_{high}$ in this case. In order to maximize $P2$'s throughput in unit time after $P1$ reveals $p_{1,1} = p_{high}$, $P2$ will set γ such that $p_{high}(\lambda(1-\gamma) + (1-\lambda)\gamma)(1-q) + 2\gamma(1-\gamma)p_{high}q$ is maximized. That is $\gamma = \frac{1+q+2q\lambda-2\lambda}{4q}$. Figure 2 illustrates the calculation process for case $2a$.

The expected throughput for user $P1$ after one round negotiation can be expressed as $\frac{1}{4}\lambda^2(p_{high} + p_{low})(q - q^2) + \lambda((\frac{7}{8}q^2 - \frac{9}{8}q + 1)p_{high} + (\frac{9}{8}q^2 - \frac{11}{8}q)p_{low})$. To maximize this value, user $P1$ needs to assign $\lambda = 1$ [4]. The maximal possible expected utility for user $P1$ after one round negotiation is $\frac{1}{8}p_{high}(-7q + 5q^2 + 8)(1-\beta)$.

When the negotiation rounds increase to 2, that is, both secondary users will have perfect information about the system, it is easy for users to make their best choice to maximize their expected throughput in unit time. In this case, a polynomial time algorithm for maximum weight bipartite matching, known as Hungarian algorithm [6], [12] can be employed to calculate the global optimal channel sensing policy in a distributed manner. Table IV shows the calculation for this case. The expected maximal utility is $p_{high}(\frac{1}{2}q^2 - \frac{1}{2}q + 1) - p_{low}(\frac{1}{2}q^2 - \frac{1}{2}q)$ with full information exchanged between two secondary users.

Users decide the number of negotiation rounds in order to maximize their expected utility U_i'. Figure 3 illustrate user

[3]Note that after one round negotiation, $P2$ only knows that $p_{1,1} = p_{high}$ according to the negotiation rule. In order to obtain his/her optimal throughput, user $P2$ needs to consider both possible cases on $P1$'s channel 2 valuation. User $P1$ needs to take user $P2$'s optimal decision into consideration when calculate his/her own maximized utility.

[4]For brevity, we omit the process for the quadratic function optimization here.

		$C1$	$C2$	$E[R]$
Case 1	$P1$	λ	$1-\lambda$	
$(1-q)q$	$P2$	$1-\lambda$	λ	$p_{high}\lambda^2 + p_{low}(1-\lambda)^2$
Case $2a$	$P1$	λ	$1-\lambda$	$(1-\frac{1+q+2q\lambda-2\lambda}{4q})p_{high}\lambda$
$\frac{1}{2}q^2$	$P2$	$\frac{1+q+2q\lambda-2\lambda}{4q}$	$1-\frac{1+q+2q\lambda-2\lambda}{4q}$	$+(\frac{1+q+2q\lambda-2\lambda}{4q})p_{low}(1-\lambda)$
Case $2b$	$P1$	λ	$1-\lambda$	
$\frac{1}{2}q^2$	$P2$	0	1	$p_{high}\lambda$
Case 3	$P1$	λ	$1-\lambda$	
$q(1-q)$	$P2$	λ	$1-\lambda$	$(p_{high}+p_{low})\lambda(1-\lambda)$
Case 4	$P1$	λ	$1-\lambda$	
$(1-q)^2$	$P2$	0	1	$p_{high}\lambda$

TABLE III

CALCULATION FOR USER $P1$'S EXPECTED THROUGHPUT IN UNIT TIME AFTER ONE ROUND NEGOTIATION

		$C1$	$C2$	$E[R]$
Case 1	$P1$	1	0	
$(1-q)q$	$P2$	0	1	p_{high}
Case 2	$P1$	1	0	
q^2	$P2$	0	1	p_{high}
Case 3	$P1$	0.5	0.5	
$q(1-q)$	$P2$	0.5	0.5	$\frac{1}{2}(p_{high}+p_{low})$
Case 4	$P1$	1	0	
$(1-q)^2$	$P2$	0	1	p_{high}

TABLE IV

CALCULATION FOR USER $P1$'S EXPECTED THROUGHPUT IN UNIT TIME WITH PERFECT INFORMATION

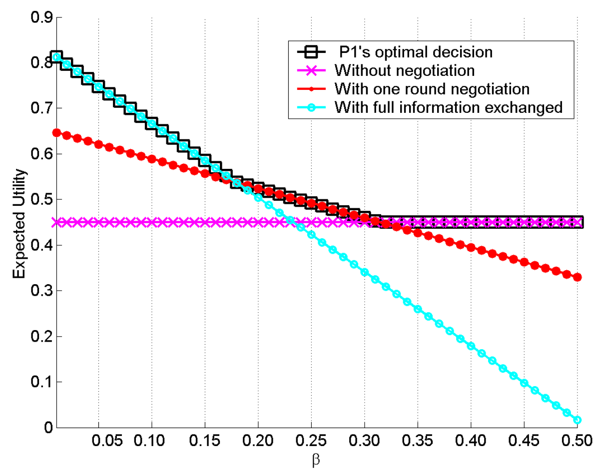

Fig. 3. Illustration of user $P1$'s optimal decision on number of negotiation rounds

$P1$'s optimal decision according to β value. In this plot, $p_{high} = 0.9$, $p_{low} = 0.2$ and $q = 0.5$. When $q = 0.5$, user $P1$'s expected utility is $\frac{1}{2}(p_{high} - p_{low})\lambda + \frac{1}{2}p_{low}$, $\frac{23}{32}p_{high}(1-\beta)$ and $(1-2\beta)(\frac{3p_{high}}{4} + \frac{p_{high}+p_{low}}{8})$ for no negotiation, one round negotiation and two rounds negotiation respectively.

From the plot, we observe that when β is small (i.e., each round negotiation is short compare to the usable time period), the user prefers to get the perfect information about the system before choosing a channel to transmit data. With the increase of β, the user is willing to gather partial information of the system to eliminate certain level of uncertainty of the system to improve expected utility. Only when β is large enough, the user prefers to begin transmit without any negotiation.

V. CONCLUSION

In this paper, we have investigated how secondary users should coordinate with each other to determine channel sensing policy in cognitive radio networks by formulating and analyzing a relevant 2-user 2-channel game. Global information gathering can help reduce collisions among secondary users, and hence, improve the utilization of the limited spectrum resource. On the other hand, exchanging information requires time and possibly other resources, which are modeled as costs. The problem we have considered is the optimal decision on how much information exchange is needed to maximize expected throughput in a given time period.

We illustrated the optimal solution in a two-value case. Even in this simplest case, the optimal decision calculation is not trivial. Our analytical results quantify how the cost of gathering the information affects the optimal number of rounds of negotiation. An interesting observation from the calculation is that randomizing on the choice of sensing channel won't help improving the expected utility in this 2-user 2-channel 2-value case. This observation motivates us to tackle also the case where the channel valuation is not limited to 2 values in the future. We plan to consider a setting where the channel valuation distribution (for example, uniformly distributed in the interval $[0, 1]$) is given as global knowledge.

There are many other interesting future directions and open problems that are suggested by this work. One is to generalize the problem to multiple-user and multiple-channel case. Solving for the optimal time-to-quit-negotiating in a dynamic fashion (instead of *a priori* is also something we are exploring at present. It is also of interest to develop incentive mechanisms for truth-telling that will ensure that selfish secondary users reveal their true valuations of the channels, something that we have taken for granted in this work. Future work could also consider other objectives for the negotiation process, such as fairness.

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