Link Scheduling in a Single Broadcast Domain Underwater Networks

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Abstract

Because radio waves decay rapidly in sea water, acoustic communication is the most popular choic for underwater sensor networks. However, since the propagation speed of acoustic waves are 3 orders slower than radio waves, scheduling techniques designed for radio-based communication systems may not be suitable for underwater use. We consider how to time schedule each link in a broadcast domain once. We show that, unlike its terrestrial RF counterpart, this problem is NP complete. We then use a complete SAT solver to investigate the relation between the schedule length and the satisfiability of a given network. In radio-based communication systems, the minimum schedule length is equal to the number of transmitters located in the same broadcast domain. However, this is not the case for underwater acoustic setting. The minimum schedule length can be smaller due to the fact of non-negligible propagation delay. Counter-intuitively, under certain circumstances, it can also be larger than the number of transmitters. We then mathematically analyze a randomized scheduler and present its performance in terms of the average successful transmissions and throughput, with considerations of scheduling length, node density, and packet length.

I. Introduction

For underwater networks, since radio waves fade rapidly in water, acoustic communications becomes an attractive alternative. Different from radio, acoustic communication has its own characteristics, such as low bandwidth, long delay, high error rate, distance-dependent bandwidth, *etc*. These features make the scheduling for underwater acoustic sensor network fundamentally different from radio-based communication systems. For example, due to the high propagation delay nature, the applicability of existing scheduling techniques designed for radio-based communication systems is questionable. Other issues, such as the high energy consumption overhead of acoustic communications, simplex communication hardware constraints, *etc*, all make the scheduling problem under this environment challenging. More detailed description about the uniqueness and existing efforts of underwater acoustic communication system can be found in [12] [6].

Several existing papers focus on the MAC protocol design for underwater acoustic sensor networks: Syed *et al.* [7] study the performance of slotted and non-slotted ALOHA performance in underwater acoustic network setting, and propose a protocol, T-Lohi [8]. Molins *et al.* use similar ideas as slotted-CSMA/CA to design a contention based MAC protocol, Slotted-FAMA [10]. Peleato *et al.* [11] propose another one aiming to improve the low utilization issue of slotted-FAMA. Nguyen *et al.* [9] and Casari *et al.* [5] compare the existing MAC protocols, and reviewe the pros and cons of various underwater medium access techniques under different network setting. Ahn *et al.* [2] analyze the performance of ALOHA, and propose a modify variant for underwater usage [2]. Preisig *et al.* [4] focus on acoustic channel property descriptions and provide propagation phenomena that can influence network performance in a quantitative way. Other than these directions, Badia *et al.* [3] propose a cross-layer optimization framework that can determine link scheduling and packet routing.

To the best of our knowledge, none of past work discusses the algorithmic hardness of doing scheduling in underwater acoustic networks. In addition, the performance of a random scheduler has not been studied is presented. In this paper, we first demonstrate the algorithmic complexity in Section II, and then use a SAT solver to study the relation between schedule length and satisfiability in Section III. We then present the performance analysis of a random scheduler in Section IV, and conclude our findings and future directions in Section V.

II. Algorithmic Complexity

We consider a slotted time system. Every transmission can only be taken place at the beginning of a time slot, and spans exactly one time slot. A transmission fails, if and only if a collision happens. None of the packets involved in a collision can be successfully delivered. By collision, we refer to the condition that more than one transmissions arrive the same receiver in a overlap period. In addition, every node in this network is located within the same broadcast domain. Take Figure 1(a) for example, if both transmitter 1 and 2 transmit in slot 1, then all their transmissions start arriving their destinations in slot 2. Thus, both packets can be successfully delivered, due to the fact that the interference arrive in slot 10. However, if transmitter 1 and 2 transmit in slot 1 and 9, respectively, then the transmission 2 can not go through because of collision.

Our interest is, "given a set of transmitter-receivers pairs P, and pair-wise delay between every two nodes, which satisfy a metric [1], is it possible to schedule all transmissions within a target length K, such that no collision will corrupt intended communications?" We call this problem "Metric Underwater Scheduling", and claim that it is NP-complete.

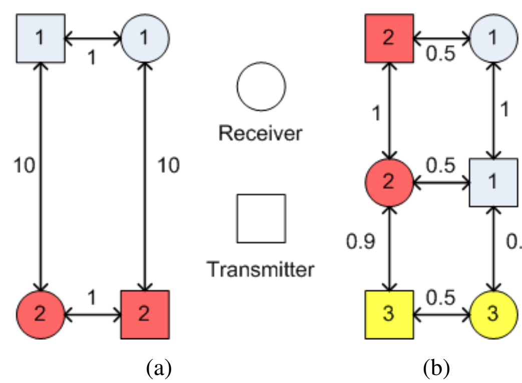


Fig. 1. Examples of transmissions.

Theorem 1: Metric Underwater Scheduling is NP-complete.

Proof: We define a length K schedule as a certificate of the Metric Underwater Scheduling. We could check this schedule, in polynomial time, whether every transmitter is scheduled within K and no intended communication is corrupted by interference. Therefore, Metric Underwater Scheduling is in NP.

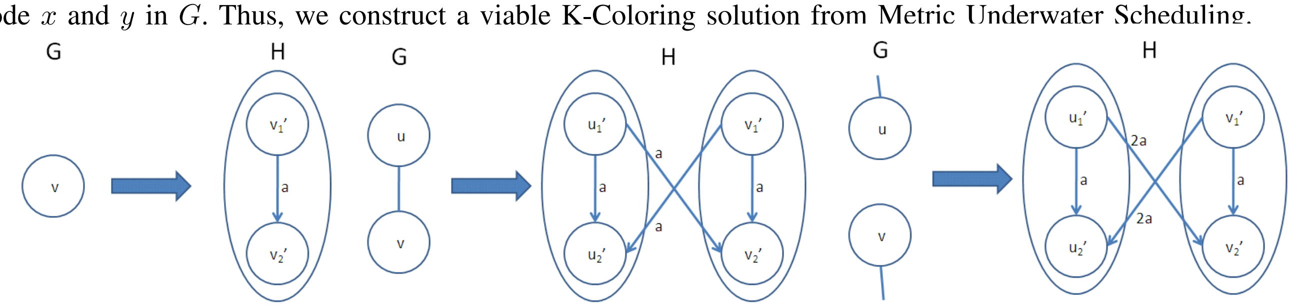
To prove the NP-hardness, we plan to reduce from K-Coloring. Consider an arbitrary instance with graph $G = (V, E)$ of K-Coloring, we build a corresponding graph $H = (V', E')$ as follows. For every node $v \in V$, we build a gadget consisting of two nodes $v'_1 \in V'$ and $v'_2 \in V'$, such that v'_1 and v'_2 can be considered as a transmitter and a receiver in Underwater Scheduling, respectively. Note that, if the highest degree of G is $\triangle$, K must be no less than $\triangle$ to be feasible. We connect every transmitter and receiver pair in H with an edge, which has length a. We set the value of a more than K. The length of an edge in H represents the propagation delay between the pair of nodes connected by this edge. For every edge $(u, v) \in E$, we add two edges, (u'_1, v'_2) and (v'_1, u'_2), with length a

[1] A valid metric satisfies the following properties: (1) non-negativity , (2) identity of indiscernible, (3) symmetry, and (4) triangle inequality.

in H. If two nodes, say x and y, are not connected by an edge in G, then we add two edges, (x'_1, y'_2) and (y'_1, x'_2), with length $2a$ in H. Because a is non-negative, the truth of symmetry, and identity of indiscernibles, by Lemma1, the construction of H satisfies a metric.

We first prove that, if nodes of G are K colorable, then we can construct a solution of Metric Underwater Scheduling as follows: for each color used in K-Coloring, we map it to a unique time slot within K. If we denote the color of node $v \in V$ as $c(v)$, then we assign its corresponding time slot, $t(v)$, to the transmitter $v'_1 \in H$. Because G is K colorable, two nodes, say u and v in G, which are joint by a edge, say (u, v) in G, can not have the same color. In other words, transmitter u'_1 and v'_1 in H can not transmit at the same time slot. This assignment is feasible in H, where the transmission of u'_1 will arrive u'_2 and v'_2 at the same time $t(u) + a$. If $t(u) = t(v)$, then both transmissions will be corrupted by interference. Therefore, schedule the transmitter u'_1 and v'_1 at different slots can guarantee their transmissions will not interfere with each other. On the other hand, if two nodes, say x and y in G, are not connected by an edge, then they can be assigned either the same color , *i.e.*, $t(x) = t(y)$, or different, *i.e.*, $t(x) \neq t(y)$. Both are feasible assignments in H. Consider the case $t(x) = t(y)$. Since the packets of both x'_1 and y'_1 will arrive at their intended receivers and other's intended receivers at time $t(x) + a$ and $t(x) + 2a$, respectively, no collision occurs under this schedule. On the other hand, consider the case $t(x) \neq t(y)$. Because $t(x) \leq K < a$, transmission of x'_1 will arrive y'_2 at time $t(x) + 2a$, which is larger than $2a$. However, y'_2 already receives its packet at time $t(y) + a$, which is not larger than $2a - 1$. Similarly, we can verify that y'_1 will not cause interference with x'_2 intended communication. Thus K-colorable implies a feasible solution of Metric Metric Underwater Scheduling.

Conversely, we will prove that if Metric Underwater Scheduling has a satisfying solution, then G is also K-colorable. For a transmission schedule, say $t(u)$ for transmitter u'_1, we map it to a unique color within the K choices. If a receiver, say v'_2, is connected with another transmitter, say u'_1, with a length a edge in H, then we know u'_1 and v'_1 will interfere with each other, thus will not schedule in the same slot. By the above mapping, u and v in G are also colored in different colors. However, we know that $(u, v) \in E$. Therefore, this color assignment is feasible in G. If a receiver, say y'_2, is connected with another transmitter, say x'_1, with a length $2a$ edge in H, then we know there does not exist an edge $(x, y) \in G$. Therefore, there is no direct conflict on color choices between node x and y in G. Thus, we construct a viable K-Coloring solution from Metric Underwater Scheduling.



(a) Construction of a gadget (b) Mapping an edge of G to H (c) Mapping an edge of G to H

Fig. 2. Construction of gadgets.

■

Lemma 1: H satisfies triangular inequality.

Proof: The only combination of edges which form a triangle can be exhaustively listed as: $\{a, a, a\}, \{a, 2a, a\}, \{2a, 2a, a\}$. Obviously, they satisfy all the requirements of a valid metric. ■

III. Findings of Applying a complete SAT Solver

We plan to transform our problem into an equivalent SAT instance, and determine the satisfiability of transformed problem by using a complete SAT solver, *zchaff* [1].

A. SAT Problem Transformation of Time-Slotted Transmission Model

We first demonstrate the transformation of an arbitrary instance to a SAT problem. For example, we are given a set of transmission pairs $\{T, R\}$, K consecutive transmission slots, and a pair-wise delay matrix D. We introduce a set of indicator variables $\{t_{ij} | \forall i \in T, j \in K\}$, such that $t_{ij} = 1$ means transmitter i is allowed to transmit at slot j, or $t_{ij} = 0$ otherwise. Thus, we have $|T|$ clauses which have the following form: $(t_{i1} \vee t_{i2} \vee \ldots \vee t_{iK}), \forall i \in T$. For convenience, we call these clauses as "feasibility clauses".

In order to find out which schedule may cause interference, thus corrupting intended communication, we adopt the following algorithm:

- $\forall i \in T, j \in K$, if $t_{ij} + D(i,i) < t_{kl} + D(k,i) + 1,$ for any $k \in T, k \neq i, l \in K$, or, $t_{kl} + D(k,i) < t_{ij} + D(i,i) + 1,$ for any $k \in T, k \neq i, l \in K$, then we know, if transmitter i transmits at slot j, it will cause interference, if transmitter k transmits at slot l.
- Keep iterating until all possible conflict schedules are found.

For every conflict schedule t_{ij}, t_{kl}, we define a clause $(\overline{t_{ij}} \vee \overline{t_{kl}})$. For convenience, we call these clauses as "conflict clauses".

Lemma 2: We can find a feasible schedule from the truth assignment of a transformed SAT problem.

Proof: Suppose we cannot find a viable schedule from the truth assignment. Without loss of generality, we assume the truth assignment is $\{t_{ij} | i \in T, j \in K\}$. There are only two conditions such that we cannot find a viable schedule from the truth assignment. One is that some transmitter are not assigned any slots to transmit, and the other one is the occurrence of interference.

Due to the the existence of feasibility clauses, every feasibility clause has at least one indicator variable that is one. We can always choose one indicator variable in each feasibility clause accordingly. Thus, it is not possible that a transmitter is not assigned any slot in the truth assignment.

On the other hand, the existence of conflict clauses also guarantees that, at most one of the indicator variable involved in a conflict clause can be assigned to 1. Therefore, interference will not take place in a truth assignment, thus proving our claim. ■

B. Results of Equivalent SAT Problems

We choose two variables in our simulations. One is the number of available slots, and the other one is node density. All the results are the mean of 100 randomly formed networks with given parameters.

In Figure 3, we notice two different scenarios. While for the case of $K = 5$, every instance with number of pairs less 8 can be satisfied. On the other hand, a certain percentage of 10-pair instances is not satisfiable when $K = 10$. In other words, the minimum schedule length that guarantees satisfiability can be more or less than the number of pairs located in the same broadcast domain, depending on different network density. An example of the later case is plotted in Figure 1(b). While there are only 3 transmission pairs in this network, the minimum schedule length is 4 time slots in this case (with assumption that packet length is 1 time slot).

Another observation is the existence of a transition region between feasible and infeasible instances. Take $K = 5$ for instance, it is highly likely that we can find a feasible schedule when there are no more than 7 pairs. On the contrary, it is almost impossible to satisfy more than 8 pairs under the same setting. In addition, the transition region, which located between feasible and infeasible cases, becomes wider when the schedule length K increases.

IV. Performance Analysis of a Random Scheduler

As demonstrated in the prior sections, because of the complexity of scheduling in underwater networks, we then focus on the simplest strategy: a randomized scheduler, and analyze its performance in this section.

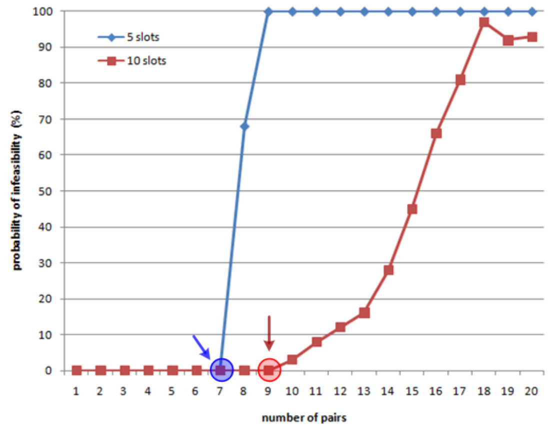


Fig. 3. Probability of infeasibility when schedule length is fixed.

A. System Modeling

In a 1×1 square area, a network is independently and randomly deployed with $2N$ nodes according to a uniform distribution. Exactly N of them are transmitters, and the remaining nodes are receivers. Each transmitter is randomly and exclusively coupled with a receiver, and has exactly one packet to transmit. The scheduling length is K, and each transmitter independently chooses an instance within this period to start transmitting a packet according to a continuous uniform distribution. More over, every packet takes exactly P unit of time to transmit, and a transmission fails if and only if interference takes place. We are interested in the probability that this randomly formed schedule becomes a feasible solution, such that every transmitter is able to successfully deliver a packet to its intended receiver within this period. In addition, we will also look at the average successful transmissions, system throughput, and per user throughput using this random scheduler.

X_i	a RV representing the X-axis coordinate of node i	$F_I(i)$	the CDF of RV I
Y_i	a RV representing the Y-axis coordinate of node i	$f_I(i)$	the PDF of RV I
T_i	a RV representing the transmission starting time of node i	K	the schedule length
N	num of transmission pairs in the same broadcast domain	P	the packet length
A_{ij}	a RV representing the $1st$ bit transmitted by node i, arriving time at node j		
Z_{ij}	a RV representing the Euclidean distance between node i and j		

B. Theoretical Analysis

By using basic probability techniques, we can have the following conclusion.

Lemma 3: S is defined as the square of horizontal distance between any two nodes, that is $S = (X_i - X_j)^2$. The CDF of S is as follows:

$$F_S(s) = Pr[S \leq s] = \begin{cases} 2\sqrt{s} - s & \text{if } 0 \leq s \leq 1 \\ 1 & \text{if } s \geq 1 \\ 0 & \text{if } s \leq 0 \end{cases}$$

Similarly, we can get S', square of the vertical distance between two nodes. Hence, the distance between two nodes $Z = \sqrt{S + S'}$. By applying Lemma 3, we can calculate the CDF of random variable Z as:

Lemma 4: The CDF of random variable Z, F_Z, is:

$$F_Z(z) = \begin{cases} \frac{1}{3} + 2\sqrt{z^2-1} - 2z^2 - \frac{z^4}{2} - \frac{1}{6}[\frac{-16z^4+20z^2-4}{\sqrt{z^2-1}} + 12z^2 tan^{-1}(\frac{z^2-2}{2\sqrt{z^2-1}})] & \text{if } z \leq 1 \\ \pi z^2 - \frac{8}{3}z^3 + \frac{z^4}{2} & \text{if } z > 1 \end{cases}$$

For convenience, we denote F_Z when $Z \leq 1$ as F_{Z1}, and the other case as F_{Z2}. Because $A = T + Z$, and A is uniformly distributed between $[0, K]$, we can calculate F_A, the CDF of A, by applying Lemma 4:

Lemma 5: When $K \leq \sqrt{2}$, the CDF of random variable A is:

$$Pr[A \leq a] = \begin{cases} \frac{1}{K}(a - \sqrt{2} + \int_{a-K}^{\sqrt{2}} F_{Z1} dz) & \text{when } a > K+1, \text{ and } a > \sqrt{2} \\ \frac{a-2}{K} + \frac{1}{K}(\int_{a-K}^{1} F_{Z1} dz + \int_{1}^{\sqrt{2}} F_{Z2} dz) & \text{when } K < a \leq K+1, \text{ and } a > \sqrt{2} \\ \frac{1}{K}\int_{a-K}^{a} F_{Z1} dz & \text{when } K < a \leq K+1, \text{ and } a \leq 1 \\ \frac{1}{K}(\int_{a-K}^{1} F_{Z1} dz + \int_{1}^{a} F_{Z2} dz) & \text{when } K < a \leq K+1, \text{ and } 1 < a \leq \sqrt{2} \\ \frac{1}{K}\int_{a-K}^{a} F_{Z2} dz & \text{when } \sqrt{2} \geq a > K+1 \\ \frac{1}{K}\int_{0}^{a} F_{Z1} dz & \text{when } a \leq min(1, K) \\ \frac{1}{K}(\int_{0}^{1} F_{Z1} dz + \int_{1}^{a} F_{Z2} dz) & \text{when } 1 < a \leq min(K, \sqrt{2}) \end{cases}$$

When $K > \sqrt{2}$, the CDF of random variable A is:

$$Pr[A \leq a] = \begin{cases} \frac{1}{K}(a - \sqrt{2} + \int_{a-K}^{1} F_{Z1} dz + \int_{1}^{\sqrt{2}} F_{Z2} dz), & \text{when } K+1 \geq a > K \\ \frac{1}{K}(a - \sqrt{2} + \int_{a-K}^{\sqrt{2}} F_{Z2} dz), & \text{when } \sqrt{2} + K \geq a > K+1 \\ \frac{1}{K}(a - \sqrt{2} + \int_{0}^{1} F_{Z1} dz + \int_{1}^{\sqrt{2}} F_{Z2} dz), & \text{when } K \geq a > \sqrt{2} \\ \frac{1}{K}\int_{0}^{a} F_{Z1} dz, & \text{when } a \leq 1 \\ \frac{1}{K}(\int_{0}^{1} F_{Z1} dz + \int_{1}^{a} F_{Z2} dz), & \text{when } 1 < a \leq \sqrt{2} \end{cases}$$

By applying Lemma 3, 4 and 5, we can calculate the probability that a node can successfully deliver a packet as:

$$Pr(success) = \begin{cases} \int_{0}^{P} f_A[1 - F_A(a+P)]^{N-1} da + \int_{P}^{\sqrt{2}+K-P} f_A[1 - F_A(a+P) + F_A(a-P)]^{N-1} da \\ + \int_{\sqrt{2}+K-P}^{\sqrt{2}+K} f_A[1 - F_A(a-P)]^{N-1} da, \ \text{when } 2P \leq \sqrt{2} + K \\ \int_{0}^{\sqrt{2}+K-P} f_A[1 - F_A(a+P)]^{N-1} da \\ + \int_{\sqrt{2}+K-P}^{\sqrt{2}+K} f_A[1 - F_A(a-P)]^{N-1} da, \ \text{when } 2P > \sqrt{2} + K \end{cases}$$

Therefore, the probability that a randomly formed schedule becomes a feasible schedule (i.e. every node can successfully deliver a packet) can be computed as $Pr(feasible) = Pr(success)^N$. Because of independency, the average goodput of this random scheduling policy equals to (number of transmitters) $\times$ (probability of success) $= NPr(success)$.

C. Approximation of F_{Z2}

Note that, F_A and f_A are required in the calculation of feasibility probability. While, these two values may not be able to evaluate exactly (due to the fact that the explicit integral of F_{Z2} does not exist), we found the following polynomial can provide less than 0.01% error in all our results.

$$F_{Z2} \sim 0.0146z^7 - 0.1253z^6 + 0.4614z^5 - 0.9421z^4 + 1.1522z^3 - 0.8441z^2 + 0.3430z - 0.0596 \quad (1)$$

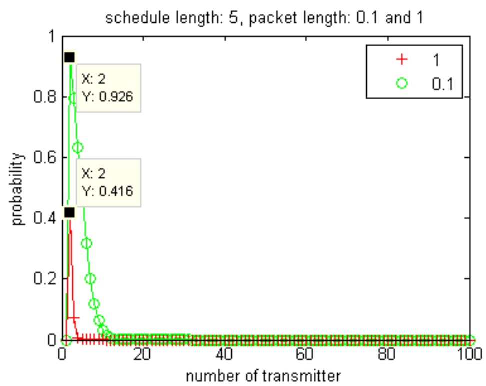
D. Numerical Evaluation

First, as plotted in Figure 4(a), we notice that while the schedule length is fixed, there exist an optimal number of transmitters, in terms of probability of feasibility. In addition, the number of transmitters increases as the schedule length goes up. While we only present the result of Figure 4(a), the optimal number of transmitters ia always 2 for all settings, the probability of feasibility always exponentially decreases as the transmitter number goes up, and this value increases with schedule length. In Figure 4(b), we can see that the probability of feasibility increases exponentially for short schedule length, and increases sub-linearly for long schedule length.

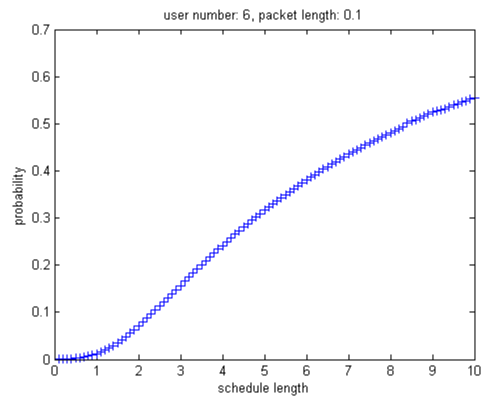
In addition to the probability of feasibility, we are also interested in the number of successful transmissions of this random scheduling policy. In Figure 4(c), we plot the average successful transmissions of this random scheduler. While the packet and schedule length are fixed, there exists an unique optimal transmitter number, and the value increases as schedule length goes up. Again, these conclusions are also valid for other settings.

While we fix the network density and schedule length, we notice the successful transmission goes down, as the packet length goes up. On the other, although the successful transmissions decreases, the amount of information that can be sent by a packet increases. To understand the amount of information that can be successfully delivered by this random scheduler, we plot the throughput, which is computed as (successful transmissions) $\times$ (packet length), in Figure 4(d) and 4(e). In general, there exists an optimal transmitter number and packet length, and these two values monotonically increase as the schedule length goes up.

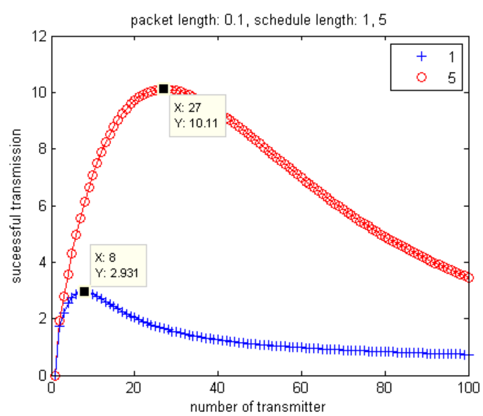
In addition to the throughput of the network, another interesting quantity is the "per user throughput", which represents the expected amount of information that each transmitter can deliver using this random scheduler. Because the calculation of per user throughput is overall throughput divided by user number, thus, it has the same trend as plotted in Figure 4(e) with respect to packet length. On the other and, we found that the per user throughput monotonically decreases as the network density goes up, but increases with schedule length, as seen in Figure 4(f).



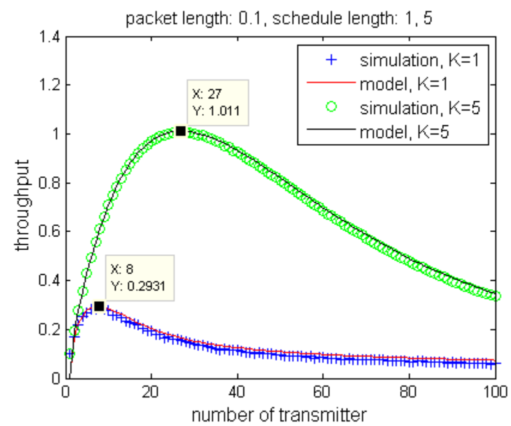
(a) Probability of feasibility vs. network density



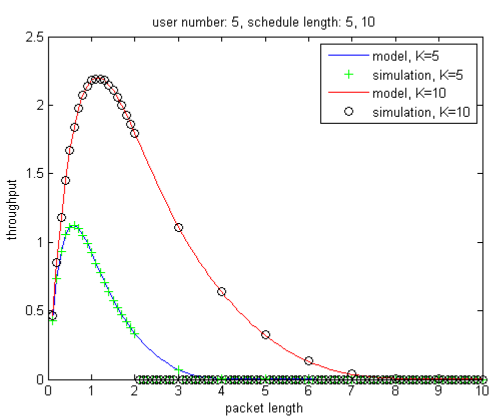
(b) Probability of feasibility vs. schedule length.



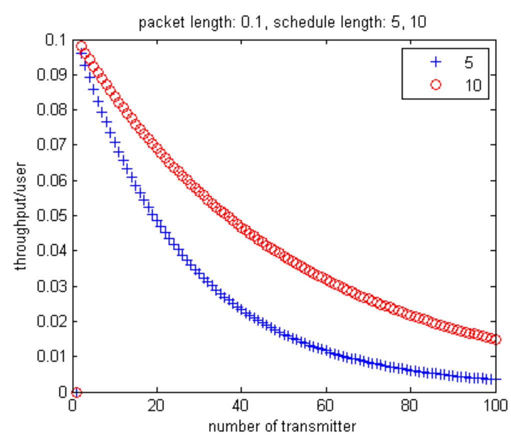
(c) Average successful transmissions vs. network density.



(d) Average throughput vs. network density.



(e) Average throughput vs. packet length.



(f) Throughput per user vs. number of users.

Fig. 4. Numerical evaluation results.

V. Conclusion

In this paper, we prove the NP-completeness of Metric Underwater Scheduling problem. By using a complete solver, we found the minimum schedule length can be larger or shorter than the number of transmitters located in the same broadcast domain, depending on the network density. We then analyze the performance, in terms of probability of feasibility, average successful transmissions and throughput, of a random scheduler. We provide both an exact and an approximate expression of above values, and present typical relations between network density, schedule length, and packet length. In the future, we plan to analyze this random scheduler under the situation, such that each transmitter has more than a packet to transmit, and study its performance. As another direction, we plan to design a repeatable scheduling policy, which aims to optimize the throughput of the network, under different arrival rates.

References

[1] A complete SAT solver, *zchaff*, “http://www.princeton.edu/ chaff/zchaff.html

[2] J. Ahn and B. Krishnamachari, “Performance of a Propagation Delay Tolerant ALOHA Protocol for Underwater Wireless Networks”, DCOSS 2008.

[3] L. Badia, M. Mastrogiovanni, C. Petrioli, S. Stefanakos, M. Zorzi, Analysis and algoritm: an optimization framework for joint sensor deployment, link scheduling and routing in underwater sensor networks, in Proc. ACM WUWNet06, Los Angeles, CA, USA, 25 Sep. 2006.

[4] J. Preisig, “Acoustic propagation considerations for underwater acoustic communications network development”, ACM WUWNet 2006.

[5] P. Casari, S. Marella and M. Zorzi, “A Comparison of Multiple Access Techniques in Clustered Underwater Acoustic Networks”, OCEANS 2007, Europe.

[6] J.Heidemann, W.Ye, J.Wills, A.Syed, and Y.Li, “Research Challenges and Applications for Underwater Sensor Networking”, In Submitted to Proc. IEEE Wireless Comm. and Networking Conf., Apr. 2006.

[7] A. Syed, B. Krishnamachari and J.Heidemann, “Understanding spatio-temporal uncertainty inmedium access with ALOHA protocols”, ACM WUWNET, 2007.

[8] A. Syed, W. Ye and J.Heidemann, “T-lohi: a new class of mac protocols for underwater acoustic sensor networks”, IEEE Infocom, 2007.

[9] H, Nguyen, S. Shin, and S. Park, “State-of-the-Art in MAC Protocols for Underwater Acoustics Sensor Networks”, EUC workshop, 2007.

[10] M. Molins and M. Stojanovic, “Slotted-FAMA: a MAC protocol for underwater acoustic networks”, IEEE, 2006.

[11] B. Peleato and M. Stojanovic, “A MAC protocol for ad-hoc underwater acoustic sensor networks”, WUWNET, 2006.

[12] I. Akyildiz, D. Pompili and T. Melodia, “Underwater acoustic sensor networks: research challenges”, Elsevier, 2005.