USC CENG Technical Report CENG-2009-9

Bargaining to Improve Channel Sharing between Selfish Cognitive Radios

Hua Liu[†], Allen B. MacKenzie[§], Bhaskar Krishnamachari[†]
[†]University of Southern California, Los Angeles, CA, USA
{hual, bkrishna}@usc.edu
[§]Virginia Polytechnic Institute and State University, Blacksburg, VA, USA
mackenab@vt.edu

***Abstract*—We consider a problem where two selfish cognitive radio users try to share two channels on which they each have potentially different valuations. We first formulate the problem as a non-cooperative simultaneous game, and identify its equilibria. For cases where the resulting Nash equilibria are not efficient, we then propose a novel coordinated channel access mechanism that can be implemented with low overhead in a decentralized fashion. This mechanism, based on the Nash bargaining solution, guarantees full utilization of the spectrum resources while improving the utility of each user compared to the non-cooperative setting. We quantify the resulting gains. Finally, we prove that risk-averse users that are willing to accept offered information at face value have no incentive to lie to each other about their valuations for the non-cooperative game. However, we find that truthfulness is not guaranteed in the bargaining process, suggesting as an open problem the design of an incentive compatible mechanism for bargaining.**

I. Introduction

We consider here a simple communication scenario in which two cognitive radios try to share spectrum resources on two channels. We assume that the two users have fixed valuations for the utility they would derive from each channel. Depending on the context, these valuations may reflect, for instance, the average rate or the probability of packet success (in a general cognitive radio network) or the probability that the channel is free of the presence of a primary user (in the particular case of an opportunistic spectrum access problem). The users wish to decide on the probability with which they should access each of the two channels. We assume that if two users access a channel simultaneously then each of them will get half of the utility they would get respectively if they were to access the channel alone. This assumption reflects a belief that the channel will ultimately be time-shared by the radios, for instance via CSMA.

In this scenario, it is reasonable to assume that each user cares about his/her own gain from obtaining channel resources. This selfish behavior of users intuitively motivates us introduce game theoretic tools to analyze the possible outcomes.

Our main contributions in this work are three-fold:

- First, we formulate the problem as a non-cooperative simultaneous game. Depending on the constitution of the payoff matrix (determined by the utilities that the two users ascribe to each channel), we decompose the game into several different cases, and derive the Nash equilibria of the game in each case. In some cases we show that there is a unique pure Nash equilibrium. In other cases there are multiple equilibria but there exists a unique mixed Nash equilibrium that is focal in a distributed protocol setting.
- Second, for some of the cases, where the Nash equilibrium does not provide Pareto efficiency, we propose a Nash Bargaining Solution. In this solution, which intrinsically provides a notion of fairness, there is a distributed coordination signal (that can be implemented in practice using a pseudo-random number generator) that allows the two users to each utilize both channels without overlapping, to obtain Pareto-optimal performance. We numerically characterize the utility improvement obtained via bargaining.
- Finally, we consider whether rational users may have an incentive to lie on their channel valuations in either the original non-cooperative game or in the bargaining enhancement. For risk-averse users that take offered information at face value, we show that there is no incentive to lie in the non-cooperative game scenario. However, truthfulness is not guaranteed in the bargaining process. We leave as an open problem the design of mechanisms to enforce truthfulness in the bargaining solution.

The rest of this paper is organized as follows. In section II, we discuss some related work. In section III, we formulate the two-user two-channel scenario as a non-cooperative game and identify several cases. In section IV, we analyze the Nash equilibria for each case. In section V, we introduce the Nash Bargaining Solution and show the utility improvements obtained through bargaining. Finally, in section VI, we consider the problem of truthfulness for both the non-cooperative game and for bargaining. We present concluding comments in section VII.

II. Related Work

Game theoretic tools are valuable for wireless networks in which selfish behavior of nodes belonging to different agents

This work has been supported in part by NSF through a CAREER grant (CNS-0347621).

	$C1$	$C2$
$P1$	a	b
$P2$	c	d

TABLE I
UTILITIES WITHOUT CONFLICT

$P2 \diagdown P1$	$C1$	$C2$
$C1$	$(\frac{a}{2}, \frac{c}{2})$	(b, c)
$C2$	(a, d)	$(\frac{b}{2}, \frac{d}{2})$

TABLE II
COMPLETE PAYOFF TABLE

$P2 \diagdown P1$	$C1$	$C2$
$C1$	$(\frac{a'}{2}, \frac{c'}{2})$	$(1, c')$
$C2$	$(a', 1)$	$(\frac{1}{2}, \frac{1}{2})$

TABLE III
NORMALIZED PAYOFF TABLE

may be observed. This tool-set has therefore been extensively applied in the wireless context [1], [2], [3].

In cognitive radio networks, bandwidth is limited and is allocated opportunistically. Many studies have been done on sharing spectrum with cognitive radios [4], [5], [6], [7], [8], [9], [10]. We here highlight a few that are most closely related. Fu and van der Schaar [10] treat the spectrum sharing problem in cognitive radio networks as a sequential auction. Cao and Zheng [5] propose a distributed channel bargaining process for a multi-hop setting; however, in their formulation users are unselfish in that they are willing to be poor in order to achieve fairness. Suris *et al.* [9] propose a cooperative game theory model to analyze a scenario where nodes in a multi-hop wireless network need to agree on a fair allocation of spectrum and investigate the fairness-efficiency tradeoff at the Nash bargaining solution. Kloeck *et. al.* [8] consider a Rubinstein-Stahl-style back-and-forth bargaining game for spectrum allocation, which too is different from the Nash bargaining solution we study in this paper.

Unlike most prior work, in which when collision happens if different users attempt to occupy the same channel, we investigate a channel "sharing" scenario in this work. Under this assumption, we propose a novel bargaining mechanism that can a) fully utilize the system resource and b) improve the utility obtained for both users with light overhead. Moreover, we model and examine the truthfulness of user reports in the interaction games in this paper.

III. PROBLEM FORMULATION

In this paper, we consider a two-user (denoted $P1$ and $P2$) two-channel (denoted $C1$ and $C2$) case. Each user's strategy is to choose which channel to use in a certain time interval. If there is no interest conflict (i.e., user can occupy the channel alone), the users' strictly positive utilities are presented in table I [1].

This table indicates that if user $P1$ picks channel $C1$ and user $P2$ chooses channel $C2$, the two users will get payoff a and d respectively. On the other hand, if user $P1$ and $P2$ choose channels $C2$ and $C1$ respectively, they will get payoff b and d respectively.

However, if the two users pick the same channel, we assume that they will share the channel in such a manner that each of them receives half of their conflict-free individual benefit for choosing the corresponding channel. Specifically, table II presents the simple non-cooperative game showing the users' payoffs in all cases.

Without loss of generality, we normalize each user's payoff in table II to get a new payoff table III where $a' = \frac{a}{b}$ and $c' = \frac{c}{d}$. We claim that the Nash equilibrium point does not change with this normalization.[2]

IV. NASH EQUILIBRIUM ANALYSIS

We discuss the Nash equilibrium outcome in this section for the non-cooperative game defined previously. For clarity, we discuss all cases while pointing out that some of them are symmetric cases.

Before getting into the Nash equilibrium solutions, we first investigate the dominant strategies for both users. It is obvious that for user $P1$, when both $\frac{a'}{2} > 1$ and $a' > \frac{1}{2}$ hold (that is, when $a' > 2$), choosing channel $C1$ strictly dominates choosing channel $C2$. When $a' = 2$, choosing channel $C1$ weakly dominates choosing channel $C2$. To be brief, we use "dominant" to represent either "strictly dominant" or "weakly dominant" in this paper. This fact implies that when $a' \geq 2$, choosing channel $C1$ is a dominant strategy for user $P1$. Similarly, when $a' \leq \frac{1}{2}$, choosing channel $C2$ is a dominant strategy for $P1$. According to symmetry, we also have the following two rules.

1) Choosing channel $C1$ is the dominant strategy for user $P2$ when $c' \geq 2$.
2) Choosing channel $C2$ is the dominant strategy for user $P2$ when $c' \leq \frac{1}{2}$.

Now we discuss the equilibrium for this problem by considering the following cases:

Case 1 $a' \geq 2$ and $c' \geq 2$

In this case, for both users, the dominant strategy is to pick channel $C1$. At the Nash equilibrium, the payoff for two users are $\frac{a'}{2}$ and $\frac{c'}{2}$, respectively.

Case 2 $a' \leq \frac{1}{2}$ and $c' \leq \frac{1}{2}$

For both users, the dominant strategy is to choose channel $C2$. Each of the users has payoff $\frac{1}{2}$ at the Nash equilibrium point. Case 2 is symmetric case with case 1. Switching the channel labels (and renormalizing the resulting payoff table) creates a one-to-one mapping between cases 1 & 2.

Case 3 $a' \geq 2$ and $c' \leq 2$

User $P1$ has dominant strategy of choosing channel $C1$. If user $P2$ chooses channel $C1$, he/she will get

[1]The utility table used here is an abstraction that can incorporate physical and link layer metrics such as SINR, power, path loss and MAC efficiency.

[2]Affine transformations of payoffs do not change the Nash equilibrium point or the Nash bargaining solution [11]. We will also show this partially by example in section IV.

$\frac{c'}{2}$. If user $P2$ picks channel $C2$, he/she will get 1. Since in this case, $1 \geq \frac{c'}{2}$, user $P2$ will use channel $C2$. The payoffs at the Nash equilibrium in this case can be expressed as tuple $(a', 1)$.

Case 4 $c' \geq 2$ and $a' \leq 2$
This is the symmetric case with case 3 with the user labels switched. Similar to case 3, the Nash equilibrium for this case is that user $P1$ picks channel $C2$ and user $P2$ chooses channel $C1$. The payoff at this Nash equilibrium is $(1, c')$.

Case 5 $\frac{1}{2} < c' < 2$ and $a' \leq \frac{1}{2}$
Applying same procedure as in previous case, the two users $P1$ and $P2$ separate their choices on channel $C2$ and $C1$, respectively. The corresponding payoff is $(1, c')$.

Case 6 $\frac{1}{2} < a' < 2$ and $c' \leq \frac{1}{2}$
This is the symmetric case with case 5. Applying the same process of eliminating dominated strategies, we know that $P1$ choosing $C1$ and $P2$ choosing $C2$ is the Nash equilibrium. The payoff is $(a', 1)$

Case 7 $\frac{1}{2} < a' < 2$ and $\frac{1}{2} < c' < 2$
This case doesn't have a dominant pure strategy for either user. Instead, two pure Nash equilibria exist for the game. In each of these two pure Nash equilibria, users are separated in two channels. However, since we assume that there is no pre-defined agreement between the two users and it is a simultaneous game, it is hard for the users to decide which channel to choose. This is the classic equilibrium selection problem. To avoid this, instead of using pure strategy Nash equilibrium, we claim that the mixed strategy Nash equilibrium is focal and that each user will employ a mixed strategy. In a mixed strategy Nash equilibrium, the users intelligently randomize their strategy selection.

Assume that user $P1$ has probability p to use channel $C1$ and user $P2$ has probability q to choose channel $C2$. If user $P1$ has employed a mixed strategy at equilibrium, then it must be the case that $P1$ is indifferent between his or her two possible pure strategies. We can use this fact to calculate user $P2$'s strategy at equilibrium point. We use the original payoff table as in II and illustrate that the mixed strategy keeps the same for payoff table II and normalized payoff table III. If user $P1$ picks channel $C1$, the expected utility is

$$\frac{aq}{2} + (1-q)a$$

If user $P1$ chooses channel $C2$, the expected utility is

$$\frac{b}{2}(1-q) + bq$$

To make user $P1$ indifferent, we need

$$\frac{aq}{2} + (1-q)a = \frac{b}{2}(1-q) + bq$$

Hence we get $q = \frac{2a-b}{a+b}$. Applying similar methods, we can obtain that $p = \frac{2c-d}{c+d}$. When using the normalized payoff table III, we get that $p = \frac{2c'-1}{c'+1}$ and $q = \frac{2a'-1}{a'+1}$; since $a' = a/b$ and $c' = c/d$, this is exactly the same equilibrium point, as expected. In the case of normalized payoffs, the corresponding expected payoff at Nash equilibrium point is $(\frac{3a'}{2(a'+1)}, \frac{3c'}{2(c'+1)})$. We point out that when $\frac{1}{2} < a' < 2$, the utility at the Nash equilibrium point is increasing with a' and $\frac{1}{2} < \frac{3a'}{2(a'+1)} < 1$. Similar result holds by c'.

Note that mixed strategy Nash equilibrium may perform worse than either pure strategy equilibrium for both users. For instance, when $1 < a' < 2$ and $1 < c' < 2$, at the two pure strategy equilibria the users gain $(1, c')$ or $(a', 1)$. Hence, the worst payoff obtained by a user in a pure strategy equilibrium is 1, which is greater than what he/she gets in the mixed Nash equilibrium. Unfortunately, as discussed previously, neither of the two pure strategy equilibria are focal, so without some coordination it is impossible for the users to preselect one of them.

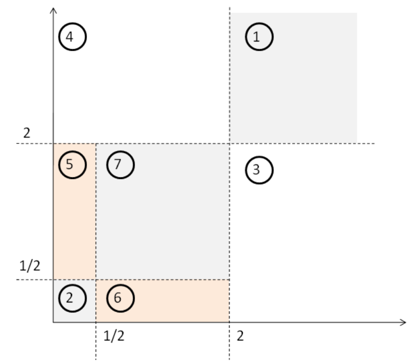

Fig. 1. Case number in corresponding regions

Figure 1 illustrates each of these cases as two-dimensional regions in a plot where the x-axis represents the value of a' and the y-axis represents the value of c'.

V. Nash Bargaining solution

In the previous section, we have analyzed the Nash equilibria of the non-cooperative game. In this section, we will discuss the Nash bargaining solution.

Typically, the Nash bargaining solution is only considered for convex payoff regions. To convexify the payoff region for our game, we first introduce the notion of a coordination signal. Time is divided into slots. At the beginning of each slot, the coordinator uniformly generates a random number, $s \in [0, 1]$, which is observed by both players.

Given such a signal, we claim that all Pareto efficient outcomes for the convexified game are of the following form, for some pre-agreed value of $\alpha \in [0, 1]$: If $s \leq \alpha$, then user $P1$ picks channel $C1$ and user $P2$ picks channel $C2$ for the timeslot; otherwise, user $P1$ is assigned to channel $C2$ and user $P2$ is assigned to channel $C1$. When α is given, the expected utilities for users $P1$ and $P2$ are $u_1(\alpha) = a'\alpha + (1-\alpha)$ and $u_2(\alpha) = \alpha + c'(1-\alpha)$, respectively.

Note that these utilities are higher than those that would be obtained by the corresponding mixed strategies. In mixed strategies, users must randomize independently. Hence, there is a non-zero probability that they will land on the same channel and suffer reduced payoff. Moreover, the payoffs are higher than those that can be obtained through pure-strategy channel sharing. Namely, suppose that both users have a strong preference for $C1$. That is, $a' > 2$ and $c' > 2$. Then the maximum payoff that the users can obtain by deploying pure strategies is to share channel $C1$ and obtain payoff tuple $a'/2, c'/2$. But, if the users deploy a coordinator and set $\alpha = 1/2$, then their a priori expected payoffs are $(a'+1)/2, (c'+1)/2$. This is because the coordinator allows the user that is not selected in a given slot to use the other channel during that time slot; without a coordinator, both users will spend all their time contending for channel $C1$. In latter part of this section, we focus on how to choose α to optimize the Nash bargaining result.

The Nash bargaining outcome is dependent upon the disagreement point. This is the operating point that the users expect to prevail in the absence of bargaining. We assume the disagreement point of the Nash bargaining game is the Nash equilibrium point when the two users share a same channel or use mixed Nash strategies. When the users do not compete for the same channel, the previous pure Nash strategy equilibrium (cases 3, 4, 5, and 6) is already efficient. We discuss the Nash bargaining solution in cases 1 and 7, described in previous section. As we have pointed out in previous section, case 2 is symmetric with case 1 if the channel labels are switched. For brevity, therefore, we omit the discussion of case 2 here.

Axiomatically, the Nash bargaining solution is the only outcome that can satisfy four conditions: (1) Pareto efficiency, (2) symmetry, (3) invariance to equivalent payoff representations (affine transformations of utility), and (4) independence of irrelevant alternatives. For details on these axioms, which define reasonable expectations for the outcome of a bargaining process, see [11].

Mathematically, it can be shown that if the payoff region is convex, then the Nash bargaining solution is the point that maximizes the so-called Nash product. That is:

$$\max_{\alpha\in[0,1]} (u_1(\alpha) - u_1(ne))(u_2(\alpha) - u_2(ne))$$

where $u_i(\alpha)$ is user i's utility using the coordination signal strategy described above with parameter α and $u_i(ne)$ is user i's utility gain at the disagreement point (i.e., the Nash equilibrium point in the non-cooperative game), for $i = 1, 2$.

Case 1 Recall that in case 1, $a' \geq 2$ and $c' \geq 2$. At the disagreement point, $u_1(ne) = \frac{a'}{2}$ and $u_2(ne) = \frac{c'}{2}$. Substituting the disagreement point to the maximization problem, we need to find out the α that maximizes the following quadratic equation:

$$(a'\alpha + (1-\alpha) - \frac{a'}{2})(\alpha + c'(1-\alpha) - \frac{c'}{2}) \quad (1)$$

Therefore,

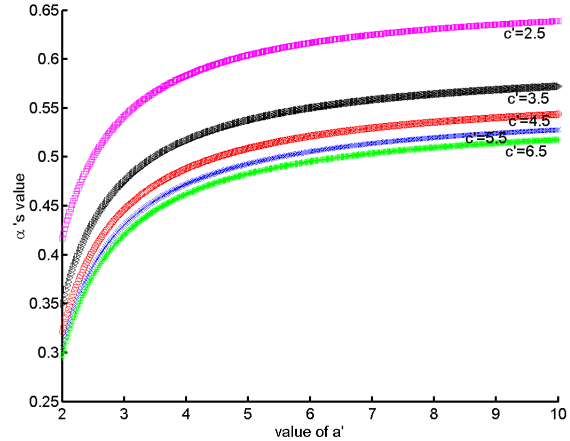

Fig. 2. Sliced plot for α's value when a' and c' vary from 2 to 10

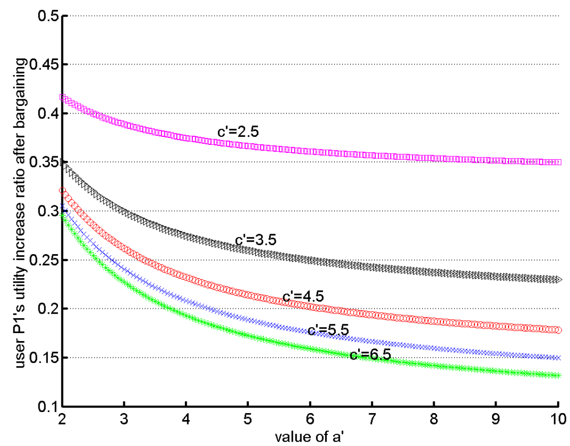

Fig. 3. User P1's utility increase ratio in case1 after Nash bargaining

$$\alpha = \frac{2a'c' - 3c' - a' + 2}{4(a'-1)(c'-1)} \quad (2)$$

Figure 2 shows the change of α's value with different a' and c' values. Compared to the disagreement point, user $P1$ increases his/her utility by $u_1(\alpha) - u_1(ne) = \frac{a'+c'-2}{4(c'-1)}$ and user $P2$ increases his/her utility by $u_2(\alpha) - u_2(ne) = \frac{a'+c'-2}{4(a'-1)}$. To illustrate the utility improvement, we define the increase ratio for user i as the ratio of utility increase after Nash bargaining to the disagreement point (Nash equilibrium) utility. Mathematically, user Pi's increase ratio R_i is

$$R_i = \frac{u_i(\alpha) - u_i(ne)}{u_i(ne)}$$

Figure 3 shows user $P1$'s utility increase ratio when $a' \geq 2$ and c' is sampled as $2.5, 3.5, 4.5, 5.5, 6.5$.

Case 7 This is the mixed strategy case. The disagreement point is set at $u_1(ne) = \frac{3a'}{2(a'+1)}$ and $u_2(ne) = \frac{3c'}{2(c'+1)}$. We focus on obtaining α that maximizes the

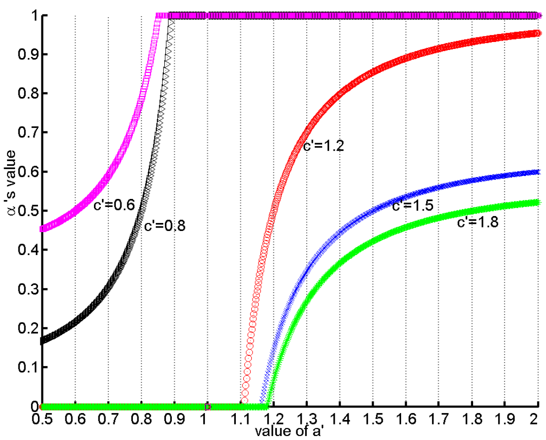

Fig. 4. Sliced plot for α's value when a' and c' vary from $\frac{1}{2}$ to 2

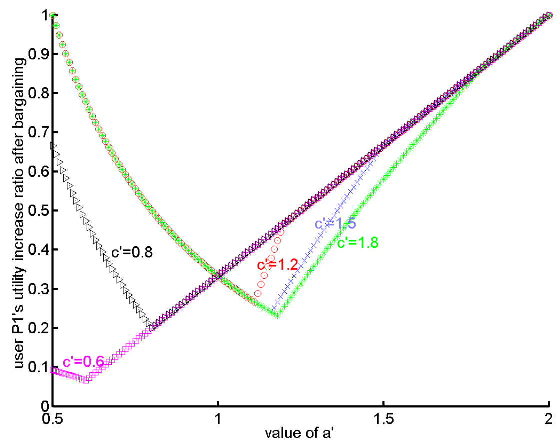

Fig. 5. User P1's utility increase ratio in case7 after Nash bargaining

following expression for the Nash product that arises in this case:

$$(a'\alpha+(1-\alpha)-\frac{3a'}{2(a'+1)})(\alpha+c'(1-\alpha)-\frac{3c'}{2(c'+1)})) \tag{3}$$

Let B denote $\frac{a'^2(2c'^2-c')+a'(c'^2-1)+c'-4c'^2+2}{4(a'+1)(c'+1)(a'-1)(c'-1)}$. The optimal value of α which maximizes the Nash product is then different for each of the following cases:

7.1 If both $a', c' \in (\frac{1}{2}, 1)$ or both $a', c' \in (1, 2)$, we have
- If $B > 1$, $\alpha = 1$ is optimal
- If $B < 0$, $\alpha = 0$ is optimal
- If $0 \leq B \leq 1$, $\alpha = B$ is optimal

7.2 When $a' \in (\frac{1}{2}, 1)$ and $c' \in (1, 2)$, $\alpha = 0$ is optimal.

7.3 When $c' \in (\frac{1}{2}, 1)$ and $a' \in (1, 2)$, $\alpha = 1$ is optimal.

7.4 If $a' = 1$, we need to maximize $\frac{1}{4}((1-c')\alpha + c' - \frac{3c'}{2(c'+1)})$. Therefore, the optimal value of α is as follows:

$$\alpha = \begin{cases} 0 & \text{if } 1 < c' < 2 \\ 1 & \text{if } \frac{1}{2} < c' < 1 \end{cases}$$

7.5 If $c' = 1$, we need to maximize $\frac{1}{4}((a'-1)\alpha + 1 - \frac{3a'}{2(a'+1)})$. In this case, the optimal value of α is as follows:

$$\alpha = \begin{cases} 1 & \text{if } 1 < a' < 2 \\ 0 & \text{if } \frac{1}{2} < a' < 1 \end{cases}$$

Figure 4 shows α's distribution when a' varies from $\frac{1}{2}$ to 2 and c' is sampled at $0.6, 0.8, 1.2, 1.5$ and 1.8 in the interval of $(\frac{1}{2}, 2)$.

Figure 5 illustrates the utility improvement ratio for user $P1$ when $a' \in (\frac{1}{2}, 2)$ with 5 sampled c' value with the Nash bargaining solution comparing to the disagreement point (i.e. the mixed Nash strategy).

VI. Truthfulness Consideration

In this section, we consider the truthfulness of users' channel condition reports. We first present some assumptions and relevant truth-telling models and then investigate if the non-cooperative game and Nash bargaining games preserve truthfulness. Specifically, truthfulness here refers to each users' report of his or her channel condition, a' or c' for user $P1$ and $P2$, respectively.

Here, we consider players that are rational and selfish but not malicious. Such a user will not sacrifice his/her own utility in order to impact the other user's utility. We also assume that neither user has knowledge of the other user's channel condition distribution.

We need to consider two issues of individual user's behavior when analyzing truthfulness in this interaction game. The first aspect is "lying" or "truth-telling", by which we judge each user's **behavior** objectively. The second aspect is "suspicious" or "gullible," by which we identify user's subjective **beliefs** when they make rational decisions. A "suspicious" user will not trust the other user's report while a "gullible" user will take the other user's report as the truth and consider it during decision making.

We have to point that if the individual user is suspicious, the user cannot make rational decision in some cases (e.g. case 7) because it is even unclear how to compute his/her best response without knowing the other user's beliefs about the distribution of channel valuations. For this reason, we do not treat the "suspicious" case in the scope of this paper.

There are three different truthfulness models. We present them from stronger to weaker as following.

- (M1) Lying prone model: if a user will not lose anything by lying, he/she will lie.
- (M2) Neutral model: if a user can possibly gain and never lose by lying, the user will lie.
- (M3) Truth telling prone model: if a user doesn't lose by telling the truth, he/she will not lie.

We consider the neutral model (M2) in the rest of this paper. Formally, in this model, a user will lie when reporting his/her channel valuation *if and only if* the following two conditions hold:

- Incentive Condition: There exists a case in which the lie will *strictly* increase the user's utility.
- Risk Aversion Condition: In all possible cases, the user's utility is not decreased by lying.

Theorem 1: In the non-cooperative game with the gullible-user assumption, truthfulness for both users is ensured under the neutral model (M2).

Proof: Without loss of generality, we present the logic from user $P1$'s aspect of view. We consider all possible scenarios:

1 When $a' \geq 2$. We claim that there is no incentive for user $P1$ to lie about his/her channel valuation. We can infer this claim by examine the following two scenarios. Notice that the only way to lie is to under-report channel valuations in this scenario.

1.1 If the truth value of $c' > 2$, telling truth will make user $P1$ gain $\frac{a'}{2} \geq 1$. Under-reporting cannot strictly increase this value.

1.2 If the truth value of $c' < 2$, telling the truth will make user $P1$ gain a', which is the maximal possible gain for user $P1$ in the game. User $P1$ still lacks incentive to lie.

2 We claim there is no incentives for user $P1$ to lie on channel valuation when $a' \leq \frac{1}{2}$. We separate this case into three subcases.

2.1 If $P2$'s true value $c' > 2$, as we already known in previous step, $P2$ will choose channel $C1$ anyway. In this case, the best payoff $P1$ can obtain is 1. Over-reporting cannot help improve utility for user $P1$.

2.2 If $\frac{1}{2} \leq c' \leq 2$, telling truth will give $P1$ payoff 1, which is the highest $P1$ can obtain since $a' < 1$

2.3 If $c' < \frac{1}{2}$, telling truth will guarantee each user $\frac{1}{2}$ payoff. However, if lying on channel valuation for user $P1$ is better than telling truth, the same conclusion will be drawn by user $P2$ according to symmetry. This fact means that they either end up in mixed strategy which yields a payoff $\frac{3a'}{2(a'+1)}$ (this value is less than $\frac{1}{2}$ when $a' < \frac{1}{2}$) for user $P1$, or end up with competing in channel $C1$ which yields a payoff $\frac{a}{2} < \frac{1}{2}$ for $P1$.

3 Here, we consider the case where $\frac{1}{2} \leq a' \leq 2$. The following subcases are considered.

3.1 If the truth value of $c' > 2$, c has no incentive to lie. When $P1$ tells truth, he/she will get channel $C2$ and the corresponding payoff is 1. Since in this case, payoff 1 is dominant all other strategies for $P1$, $P1$ has no incentive to lie.

3.2 If the truth value of c' is also in $[\frac{1}{2}, 2]$, we point out a scenario that lying might hurt the user's utility which contradicts the risk averse condition in the neutral model M2. Suppose both users' true channel valuations (a' and c') are between $(1, 2)$. Assume lying is better than truth-telling in this scenario. Suppose that $P1$ reports $a' > 2$ and sees $c' > 2$ (i.e. assume telling larger channel valuations is better. [3]), according to the gullible assumption [4], the game will end up at both users using channel $C1$, which yields user $P1$ $\frac{a'}{2}$ utility. Notice that $\frac{a'}{2} < \frac{3a}{2(a+1)}$, which contradicts the risk averse condition. User $P1$ will therefore report the truth.

3.3 If $c' < \frac{1}{2}$, if $P1$ tells the truth, he/she will get payoff a' and choose channel $C1$. If lying can help forcing $P2$ choosing channel $C1$, $P1$ will be able to use channel $C2$ and gain 1. However, we can come to the conclusion that $P2$ will not choose channel $C1$ since in all cases staying at channel $C2$ is $P2$'s dominant strategy. Therefore, lying on the channel valuation will not help $P1$ in this case either.

From the above, we can conclude that truthfulness is ensured for both players. □

***Theorem 2*: Truthfulness is not ensured in current Nash bargaining mechanism under the neutral model M2.**

Proof: We provide a counterexample. Consider the case where the true value of user $P1$'s channel valuation is $1 < a' < 2$. User $P1$ considers the following cases for the true value of c'.

- $c' \geq 2$: User $P2$ might tell the truth or might report increased c'.[5] Whether or not $P2$ is truthful, if user $P1$ tells the truth, he/she will get payoff 1 since channel $C2$ is assigned to him/her after bargaining (i.e., $\alpha = 0$). If $P1$ were to (falsely) report that $a' > 2$, he/she will separate his/her time on using channel $C1$ and $C2$. The utility he/she obtains is $a'\alpha + (1 - \alpha) > 1$. Hence, lying has incentive here.
- $1 < c' < 2$: This is the tricky case. Notice the fact that since $a' > 1$, the more time user $P1$ can get on channel $C1$, the higher his/her utility. Since α is calculated based on the reported values, $\hat{a'}$ and $\hat{c'}$, if $\hat{a'} > \hat{c'}$, $\alpha > 0.5$; if $\hat{a'} < \hat{c'}$, $\alpha < 0.5$. In this case, if user $P2$ does not lie, user $P1$ can improve utility by over-reporting that $\hat{a'} > 2$. However, we assume that $P1$ and $P2$ are both rational. If we assume that the possible channel valuation values are upper bounded by a large enough value $\mu >> 2$, then we claim that both users will end up reporting $\hat{a'} = \hat{c'} =$

[3]Similar checking process can be done for other cases. We omit the details here for brevity.

[4]Notice that if the users are suspicious, he/she is not able to make action decision in this case because of insufficient knowledge.

[5]It can be proved that user $P2$ will not report a decreased c'. Intuitively, channel $C1$ is better for user $P2$ and he/she wants to claim larger channel valuation so that he/she can gain more time portion in using this channel.

μ. This is because for fixed $\hat{c'}$, α is monotonically non-decreasing with $\hat{a'}$. If a user reports a value $\hat{a'} < \mu$, then once the other users' reported value, $\hat{c'}$, is known he/she can always improve by increasing his/her reported $\hat{a'}$. The equilibrium outcome is when both report μ.

It is worth noting, though, that at this liar's equilibrium where $\hat{a'} = \hat{c'} = \mu$, $\alpha = 0.5$. If the true value of $a' > c'$, then $P1$'s utility would have been higher in the truthful outcome. Nevertheless, telling the truth is not advantageous for $P1$, as $P2$ will take advantage of his/her truth-telling and suppress α's value.

- $\frac{1}{2} < c' \leq 1$: If both users tell truth in this scenario, then $C1$ is allocated to $P1$ and $C2$ to $P2$ (i.e., $\alpha = 1$). User $P2$ does not have incentive to compete with $P1$ for channel $C1$. However, even if $P1$ over-reports channel $C1$'s valuation (e.g., $P1$ reports that $\hat{a'} > 2$), α will not change.
- $c' \leq \frac{1}{2}$: In this scenario, user $P2$ prefers to use channel $C2$ alone so there is no incentive to over-report the value of c'. Moreover, it is obvious that over-reporting $\hat{a'} > 2$ does not reduce the utility gained by user $P1$.

From the analysis above, we can see that when $1 < a' < 2$, lying (reporting $\hat{a'} > 2$) is advantageous in the case where $c' \geq 2$ and $1 < c' < 2$ and does not violate the risk aversion condition in any cases. Thus, lying by over-reporting a' is beneficial for user $P1$. $\square$

We conclude that truthful channel condition reporting is not incentivized in the current Nash bargaining mechanism. Thus some mechanism is needed to enforce the truthfulness during the bargaining process. We leave the investigation of such a mechanism as an open problem for future work.

VII. Conclusion

In this work, in addition to analyzing the Nash equilibria in a non-cooperative game formulation, we have proposed a novel channel bargaining mechanism for cognitive radios that can be implemented with low overhead in a decentralized fashion. This mechanism, which uses the Nash Bargaining Solution, guarantees 100% utilization of the available spectrum resources, while providing improvements for each user compared to the non-cooperative outcome.

We have seen that even this basic problem involving just two users and two channels has surprising complexity in many dimensions: in the number of cases that arise with respect to the equilibria; in the non-trivial behavior of the Nash bargaining solution in some cases; and in the modeling involved in reasoning about truthfulness.

There are several directions for future work. One is to design a mechanism that can enforce truthfulness in the bargaining process; another is to extend this analysis to multiple users and multiple channels.

References

[1] V. Srivastava, J. Neel, A. B. Mackenzie, R. Menon, L. A. Dasilva, J. E. Hicks, J. H. Reed, R. P. Gilles, "Using game theory to analyze wireless ad hoc networks," *Communications Surveys and Tutorials, IEEE*, vol.7, no.4, pp. 46-56, Fourth Quarter 2005.

[2] A. B. MacKenzie, and L. A. DaSilva, *Game Theory for Wireless Engineers (Synthesis Lectures on Communications)*, Morgan & Claypool Publishers, 2006

[3] H. Liu, B. Krishnamachari, and S. Kapadia, "Game Theoretic Tools Applied to Wireless Networks," *Encyclopedia of Ad Hoc and Ubiquitous Computing*,Editor D. Agrawal, World Scientific Publishers, 2008.

[4] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized Cognitive MAC for Opportunistic Spectrum Access in Ad Hoc Networks: A POMDP Framework," *IEEE Journal on Selected Areas in Communications: Special Issue on Adaptive, Spectrum Agile and Cognitive Wireles Networks*, April 2007.

[5] L. Cao and H. Zheng, "Distributed Spectrum Allocation via Local Bargaining," *Proc. IEEE SECON 05*, 2005.

[6] H. Zheng and C. Peng, "Collaboration and Fairness in Opportunistic Spectrum Access," in *Proceedings of IEEE International Conference on Communications (ICC)*, 2005.

[7] M. Halldorsson, J. Halpern, L. Li, and V. Mirrokni, "On spectrum sharing games," in *Proc. of the 23rd Annual ACM Symposium on Principles of Distributed Computing*, pp. 107–114, 2004.

[8] D. Grandblaise, C. Kloeck, T. Renk, P. Bag, P. Levine, K. Moessner, J. Yang, M. Pan and K. Zhang, "Microeconomics Inspired Mechanisms to Manage Dynamic Spectrum Allocation," in *Proc. IEEE DySPAN, 2007*, pp. 452–461.

[9] J. E. Suris, L. A. DaSilva, Z. Han, and A. B. MacKenzie, "Cooperative game theory for distributed spectrum sharing," in *Proceedings of IEEE International Conference Communications (ICC)*, pp. 5282–5287, June 2007.

[10] F. Fu and M. van der Schaar, "Learning for Dynamic Bidding in Cognitive Radio Resources," CoRR abs/0709.2446 (2007).

[11] M. J. Osborne, *An Introduction to Game Theory*, Oxford University Press, 2003.