Optimal Control for Epidemic Routing of Two Files with Different Priorities in Delay Tolerant Networks

Shangxing Wang, MHR. Khouzani, Bhaskar Krishnamachari and Fan Bai

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Ming Hsieh Department of Electrical Engineering – Systems
University of Southern California
Los Angeles, California 90089-2562

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Shangxing Wang[1], MHR. Khouzani[2], Bhaskar Krishnamachari[1] and Fan Bai[3]

***Abstract*—We consider the problem of joint dissemination of multiple contents with different priorities through epidemic routing in a large Delay Tolerant Network (DTN). Specifically, we consider two files a and b to be distributed in a large capacity-limited DTN through opportunistic contacts between the roaming nodes. The goal is to maximize the number of nodes that receive the files within a delay window, but with a priority for file b over file a. This preference can reflect difference in popularity or significance of a file, or offering different grades of service. The restriction is the short-lived encounter and transmission capacity of nodes, where decisions have to be made on which file to forward upon an opportunity of communication. By formulating this problem as an optimal control problem based on ordinary differential equations and analyzing it through Pontryagin's Minimum Principle, we find that the optimal routing policies follow a simple but *a priori* counter-intuitive "bang-singular-bang" structure. Through numerical evaluations, we validate our findings and provide some intuitions about how the structure of the optimal policy changes with respect to different network settings.**

I. Introduction

In Delay (or Disruption) Tolerant Networks (DTNs), permanent end-to-end connectivity is not guaranteed any more due to the intermittent connectivity between nodes. Such "challenged" networks arise in settings where nodes are sparsely distributed, highly mobile, and have limited wireless radio range. However, messages can still be delivered to their destinations thanks to the mobility of nodes that help the source to relay the messages. One of the most promising examples of DTNs is the vehicular network, in which moving vehicles contact each other through short-range wireless communications. Related applications and services provided by vehicular networking include road traffic information, automatic collision warning and global internet services.

The primary goal in the design of DTNs is to make it possible to transmit data from the source to the destination. However, routing in DTNs is still a challenging and open research area because of its inherent uncertainty about network conditions. One of the earliest works on packet distribution and routing is Epidemic Routing [1], where the authors proposed a "store-carry-forward" scheme in which each node, with unlimited bandwidth and storage, buffers copies of its received packets and moves around to transmit copies to new nodes in later encounters. Flooding without any *a priori* information on the network conditions, Epidemic Routing is probably the most simple but robust routing algorithm.

The low capacity and resource inefficiency performance of Epidemic Routing has motivated researchers to design more economic routing algorithms by using strategies such as restricting overhead [2]-[5], making use of network information [6]-[8], or network coding [9]. Though aiming at improving delivery performance and reducing resource consumption at the same time, all these studies only focus on a single file dissemination and do not consider the differences among files. However, files, transmitted in real-life networks, are different in many aspects, such as their contents and freshness. Even for the same file, it might be favored differently by different users. All these differences among files should be taken into consideration when designing routing schemes.

In this paper, we take into account the differences among files by assigning them distinct priorities. And the goal is to develop a theoretical understanding of modeling and performance optimization for dissemination of multiple files with distinct priorities through epidemic routing in a DTN. As a starting point, we consider two types of files a and b where file b has a priority over file a, and the goal is to disseminate the two files through the opportunistic encounters to as many mobile users as possible within an allowed time interval. We model the distinct priorities by assigning a higher reward for delivering of file b to each node than delivering file a. What makes the problem non-trivial is the fact that opportunities of communication are sporadic and short-lived and the transmission capacities are limited. In particular, users need to decide which file to forward during encounters where both file a and b are eligible to forward. It is best to make this decision dynamically, as the "state" of dissemination, i.e., the number of users with each type of the files is evolving over time. We model the dissemination of the two files in the DTN given the dynamic forwarding decisions using a deterministic system of non-linear ODEs as the mean-field limit of a DTN with a large number of users. We derive the structure of optimal forwarding policy as given by Pontryagin's Minimum Principle without access to closed-form solutions.

Contributions of this work are as follows: First, we model the two file dissemination through epidemic routing problem in a large scale DTN as a deterministic non-linear optimal control problem (Section II), and characterize the set of necessary conditions that the optimal solution needs to satisfy by applying the Pontryagin's Minimum Principle (Section III). Second, without access to the closed-form solution and through the investigation of conditions set by Pontryagin's Minimum Principle, we establish that the optimal control policy in the

[1]Shangxing Wang and Bhaksar Krishnamachari are with the University of Southern California, Los Angeles, CA. Their emails are {shangxiw, bkrishna}@usc.edu.

[2]MHR. Khouzani is with the Royal Holloway University of London, Egham, UK. His email is arman.khouzani@rhul.ac.uk.

[3]Fan Bai is with General Motors Global R&D, Warren, MI. His email is fan.bai@gm.com.

most general form has a simple bang-singular-bang structure (Section IV). Specifically, we prove that only one of the following four cases can occur: (1) In extreme cases, only higher-priority file b is forwarded; (2) Only file b is forwarded initially until a certain time threshold at which the fractions of users with each file become equal. After this time, there is an interval of singular control over which both files are forwarded complementarily in related encounters, and finally, only file b is forwarded again; (3) Initially, only file a is forwarded, i.e., the file with lower priority, until a time when the fractions of the two files in the network become equal. Thereafter, there will be the singular interval of forwarding both complementarily, which is followed by the interval of only forwarding file b that extends toward the end; (4) Similar to case (3) but without the singular interval: initially only the less priority file is forwarded until a threshold time, after which, only the higher priority file is forwarded. Finally, we validate our findings through numerical evaluations, and show how the optimal control policy changes according to different initial network conditions, which provides us with some intuitions in real implementations (Section V).

Related Works: Routing has always been a popular research topic ever since the earliest days of DTN research. Among all the routing schemes, epidemic related routing algorithms are relatively simple, since they make the advantage of opportunistic encounters and require minimum network information. Epidemic Routing [1] is the extreme case where a copy of a packet is forwarded whenever possible from the packet-carrying node to any other node who arrives within its transmission range and does not have the packet yet. This minimizes the delivery delay at a cost of incurring redundancies and wasting network resources. Many following works try to improve the efficient use of network resources by restricting the replications of a packet, either through setting a limit on the number of copies of a packet [5] or through using historical knowledge to make a decision on the replication [8]. In common with these prior works, we too design control policies of file dissemination through epidemic routing because of its simplicity and good delay performance. However, unlike much of the prior focus on improving average delay or per packet delivery probability in a single file dissemination network, we take the different priorities (or preferences) of files into consideration and focus on multiple file dissemination. Specifically, our focus is primarily on maximizing the number of nodes receiving files with a preference of file b over file a within a given time window.

Dissemination of multiple files in mobile wireless networks has been investigated in several papers. Ioannidis *et al.* [10] study heterogeneous services under constrained cache storages in an infrastructure-based network. But the distributed caching strategy they propose, PSEPHOS, mainly deals with buffer management to avoid overflow. Moghadam *et al.* [12] focus on disseminating multiple files through storage limited nodes in a two-tier hybrid mobile network architecture by utilizing centralized control together with distributed dissemination. However, this work mainly focuses on the assignment of helper nodes for different files instead of making decisions on which file to transmit. Recent work done by Li *et al.* [11] proposes a heuristic algorithm for the multiple contents dissemination under roadside units (RUAs) aided opportunistic networks, where vehicles obtain Internet services through the static RUAs. Our work instead studies the multiple file dissemination problem in a fully distributed mobile network.

Ordinary differential equations have been used in modeling the dynamics of wireless and mobile networks, especially in the large scale networks thanks to mean-field convergence results. Zhang *et al.* [13] formulates the epidemic routing problem into an ODE-based framework and gives closed-form expressions for a number of different performance metrics. Even in a much more heterogenous network setting where a file is constantly updated and nodes are catergorized into different classes [14], Chaintreau *et al.* shows network states can be entirely characterized by differential equations when the number of nodes becomes large. This major simplification allow one to both obtain efficient numerical solutions and derive analytical analysis. We share the same idea of using ODEs to characterize the continuously changing network states, but we concentrate on a more general problem of multiple file dissemination.

Control policies to better deploy network resources have been addressed when the network has limited resources, such as fixed buffer size [15], limited packet lifetime [16] and constrained energy consumption [17]. Optimal control theory, especially the Pontryagin's Princinple, serves as a useful tool to solve non-linear optimal control problems when the system dynamics are non-linear and traditional linear system theories are not applicable. Such studies include designing optimal packet-transmission policies concerning limited energy [18]-[20], securing communication networks against malware attacks [21]-[24]. To the best of our knowledge, our work is the first to formulate the multiple file dissemination in DTNs as an optimal control problem and apply Pontryagin's Principle to obtain a theoretical understanding.

II. Problem Formulation

In this section, we present the model of the problem and the assumptions behind it. A list of main notations is provided in Table I.

Consider a Delay (Disruption) Tolerant Network (DTN) composed of N roaming nodes. If the area of the network is constrained and nodes move according to common mobility models such as the Random Waypoint model, then the intermeeting time between nodes can be approximated as an exponentially distributed random variable with parameter $\hat{\beta}$ ([25]), which is inversely proportional to the roaming area.

Two types of files, file a and file b, are to be disseminated through the opportunistic encounters of the mobile nodes. For simplicity, we assume that the two files have the same size. At time zero, some nodes possess a copy of file a, some possess a copy of file b, and some are *full*, i.e., possess both files. The rest of the nodes are *empty*, i.e., do not have either of the two files. Duplication and transmission happen when two

TABLE I: List of main notations in the problem model

Parameter	Definition
$F_a(t)$	Fraction of nodes at time t that only possess file a
$F_b(t)$	Fraction of nodes at time t that only possess file b
$F_{ab}(t)$	Fraction of nodes at time t that possess both files a, b
$E(t)$	Fraction of empty nodes at time t
$\mu_a(t)$	Probability at which a full node forwards file a (instead of file b) in an encounter to an empty node at time t
$\omega_a(t)$	Probability at which a node containing only file a forwards file a in an encounter to a node containing only file b at time t
$\hat{\beta}$	Rate of encounters of a node with all other nodes
$[0, T]$	Dissemination time window (T: maximum delay)
W_a, W_b, W_{ab}	Reward weights respectively for nodes with file a, b and both files at the end of dissemination time window

nodes encounter, i.e., enter the communication range of each other. We assume at most one copy of a file can be transmitted during an encounter between two nodes. This assumption is reasonable when one concerns the short duration of the encounter, the limited capacity of the radio transmitter and the non-trivial size of a file, which also reflects the limited communication capacity and high mobility of the nodes.

The dynamics of the dissemination of the two files is as follows: When a non-empty node that carries a copy of only one of the files meets an empty node, it duplicates a copy of its file and transmits it, i.e., *forwards* it, to the empty node. After receiving the copy, the empty node becomes a new non-empty node carrying that file. When a full node carrying both copies and a node that carries only one of the files encounter, the full node forwards the missing file. The recipient becomes a full node that carries both files. No file transmission decision needs to be made so far, since the sender simply forwards the file that the receiver does not yet have. However, a decision on which file (*a* or *b*) to transmit has to be made in the following two interesting scenarios. The first interesting cases, *Full-Empty Encounters*, happen whenever a full node (a node containing both files) encounters an empty node. Based on the assumption that at most one copy of a file can be forwarded at a time per each encounter, the full node has to make a decision about which file to forward. Let $\mu_a(t)$ represent the probability that a full node forwards a copy of file a to an encountered empty node at time t. Correspondingly, $1-\mu_a(t)$ is the probability that a full node forwards file b to an encountered empty node at time t. The second interesting cases, *Single-a-Single-b Encounters*, occur whenever a node that carries only file a and a node that carries only file b meet. Similarly, a decision on which file to transmit has to be made during such an encounter. Let $\omega_a(t)$ be the probability that at time t a copy of file a is forwarded from the node carrying only file a to the node carrying only file b, and $1-\omega_a(t)$ be the probability that a copy of file b is forwarded the other way around. The recipient becomes a node possessing both files after the transmission. We assume that nodes are distinguishable only based on their status with regards to the possession of the files. Hence, the two control variables are assumed to be the same across *all* nodes of the same state. Clearly, we must have $0 \leqslant \mu_a(t), \omega_a(t) \leqslant 1$. Note that in this problem, we do not consider resource consumption metrics like energy (battery usage), and our only constraint is the short-lived nature of the encounters and the limited communication capacity of the nodes. Therefore, one copy of a file is indeed forwarded upon constructive encounters, the questions at hand is to optimally choose the file to forward given their distinct priorities.

Let $E(t)$, $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ respectively represent the *fraction* of empty nodes, nodes that carry a copy of file a, nodes that carry a copy of file b, and nodes that carry both files, at time t. Let $\beta = \lim_{N\to\infty} N\hat{\beta}$ and assume it converges to a constant. This assumption is valid when it makes sense to speak of the "density" of nodes (number of nodes per unit area) and it is uniform over the whole roaming area. According to Theorem 3.1 in [26], as N increases, $E(t)$, $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ asymptotically converge (path-wise) to the solution of the following system of ordinary differential equations (ODEs):

$$\dot{E}(t) = -\beta E(t)(F_a(t) + F_b(t) + F_{ab}(t)) = -\beta E(t)(1 - E(t)) \quad (1a)$$

$$\dot{F}_a(t) = \beta E(t)F_a(t) + \beta\mu_a(t)E(t)F_{ab}(t) - \beta F_a(t)((1-\omega_a(t))F_b(t) + F_{ab}(t)) \quad (1b)$$

$$\dot{F}_b(t) = \beta E(t)F_b(t) + \beta(1-\mu_a(t))E(t)F_{ab}(t) - \beta F_b(t)(\omega_a(t)F_a(t) + F_{ab}(t)) \quad (1c)$$

$$\dot{F}_{ab}(t) = \beta F_{ab}(t)(F_a(t) + F_b(t)) + \beta F_a(t)F_b(t) \quad (1d)$$

with initial conditions E_0, F_{a0}, F_{b0}, F_{ab0} that are strictly positive and satisfy the following constraints:

$$E(t),\ F_a(t),\ F_b(t),\ F_{ab}(t) \geqslant 0, \qquad \forall t \in [0, T] \quad (2a)$$

$$E(t) + F_a(t) + F_b(t) + F_{ab}(t) = 1, \qquad \forall t \in [0, T] \quad (2b)$$

The above ODE system is illustrated in Figure 1.

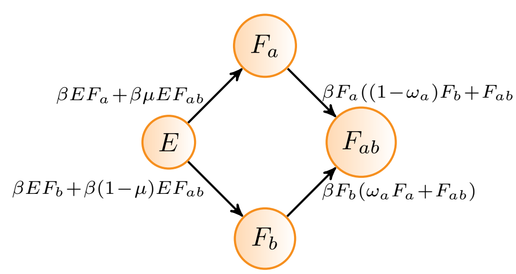


Fig. 1: System Dynamics (Fluid Model)

Note from (1a) that the evolution of the fraction of empty nodes, $E(t)$, does not depend on the control functions $\mu_a(t)$ and $\omega_a(t)$, either directly or indirectly through other variables. Indeed, the expression for $E(t)$ can be (in fact in closed-form) derived independent of the choice of the control variables from the differential equation (1a) and the initial condition $E(0) = E_0$. Moreover, the relationship $F_a(t)+F_b(t)+F_{ab}(t)+E(t) = 1$ holds for all times. Following these observations, the ODE system in (1) can be simplified by using only the following two differential equations with their initial conditions $F_a(0) = F_{a0}$ and $F_b(0) = F_{b0}$:

$$\dot{F}_a(t) = \beta F_a(t)(2E(t) + F_a(t) + \omega_a(t)F_b(t) - 1) + \beta\mu_a(t)E(t)(1 - F_a(t) - F_b(t) - E(t)) \quad (3a)$$

$$\dot{F}_b(t) = \beta F_b(t)(2E(t)+F_b(t)+ (1 - \omega_a(t))F_a(t) -1) + \beta(1 - \mu_a(t))E(t)(1-F_a(t)-F_b(t)-E(t)) \quad (3b)$$

The objective of the problem is to properly choose the control functions $\mu_a(t)$ and $\omega_a(t)$ over a time interval $[0,T]$ so that at the end time T: (a) as many nodes as possible contain both files; and (b) among nodes that contain a single file, the nodes that contain file b are favored more than those containing file a. The time interval of $[0,T]$ can be thought of as the interest window of the files, when the two files are relevant. This introduces a hard limit on the maximum allowable delay in the delivery of the two files. According to these two preferences, the system reward $\mathcal{R}$ can be designed as follows:

$$\begin{aligned}&\mathcal{R}(\mu_a)\\ &= W_aF_a(T) + W_bF_b(T) + W_{ab}F_{ab}(T)\\ &= W_aF_a(T)+W_bF_b(T)+W_{ab}(1-F_a(T)-F_b(T)-E(T))\end{aligned}$$

where $0 < W_a < W_b < W_{ab}$. A special case is when $W_{ab} = W_a+W_b$, which represents the scenario where delivering a file to a node that does not have the file brings a fixed reward that depends on the file itself but not on the state of the node with respect to possession of the other file. Specifically, delivering file b to a new node brings a reward W_b which is strictly larger than the reward for delivering a copy of file a to a new node. This reflects the priority of file b is higher than that of file a.

We seek to find optimal control $(\mu_a^*(t), \omega_a^*(t)) \in \mathcal{S}$ to achieve a maximum reward $\mathcal{R}$, where $\mathcal{S}$ is the admissible control region of any pair of piecewise continuous functions $(\mu_a, \omega_a) : [0,T] \to \mathbb{R}^2$ satisfying $\forall t \in [0,T]$, $0 \leqslant \mu_a(t), \omega_a(t) \leqslant 1$.

We start with the following technical but useful lemma, that for any admissible control function (μ_a, ω_a), the fraction of the nodes of each type is strictly positive, and hence the state constraints $(2a)$ and $(2b)$ are never active, and thus can be ignored.

***Lemma** 1:* For any $(\mu_a, \omega_a) \in \mathcal{S}$, $E(t)$, $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ are strictly positive.

Proof: From (1a), we have $\dot{E}(t) \geqslant -\beta E(t)$. Due to the initial condition $E_0 > 0$, we have $E(t) \geqslant E_0e^{-\beta t} > 0$, $\forall t \in [0,T]$. The initial conditions and continuities of $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ imply that $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ are strictly positive over some interval starting from $t = 0$. Suppose $\bar{t} \in [0,T]$ is the first time one of $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ becomes zero, and $\forall t \in [0,\bar{t})$, $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ are strictly positive. These strictly positive conditions together with (1b) and (1c) yields that $\forall t \in [0,\bar{t})$, $\dot{F}_a(t) \geqslant -2\beta F_a(t)$, $\dot{F}_b(t) \geqslant -2\beta F_b(t)$. Since $F_a(0) = F_{a0} > 0$ and $F_b(0) = F_{b0} > 0$, then $\forall t \in [0,\bar{t})$ $F_a(t) \geqslant F_{a0}e^{-2\beta t} > 0$ and $F_b(t) \geqslant F_{b0}e^{-2\beta t} > 0$. The continuity properties further imply that at time $\bar{t}$, $F_a(\bar{t}) \geqslant F_{a0}e^{-2\beta\bar{t}} > 0$ and $F_b(\bar{t}) \geqslant F_{b0}e^{-2\beta\bar{t}} > 0$. Thus, $\bar{t}$, based on our assumption, is the time at which $F_{ab}(t)$ must be 0. However, the condition $\dot{F}_{ab}(t) \geqslant \beta F_a(t)F_b(t) > 0$ $\forall t \in [0,\bar{t}]$ indicates that $F_{ab}(t)$, starting from $F_{ab0} > 0$, is increasing and thus always strictly positive in $[0,\bar{t}]$, which contradicts to the assumption $F_{ab}(\bar{t}) = 0$. Therefore, $E(t)$, $F_a(t)$, $F_b(t)$ and $F_{ab}(t)$ are strictly positive over the entire interval. ■

III. Optimal Control Policy

Since the system dynamics are non-linear, existing theories developed for linear systems cannot be applied to solve this optimal control problem. Instead, by defining a cost function as $\bar{\mathcal{R}} = -\mathcal{R}$ and converting the reward maximization problem to a cost minimization problem, we use *Pontryagin's Minimum Principle* to deal with this deterministic continuous-time optimal control problem. The corresponding *Hamiltonian function*, denoted by H, of this system characterized by equations (3) along with its objective function $\bar{\mathcal{R}}$, is defined as follows:

$$\begin{aligned}&H((\mu_a,\omega_a),(E,F_a,F_b),(\lambda_a,\lambda_b)) =\\ &\lambda_a[\beta F_a(2E + F_a + \omega_aF_b - 1) + \beta\mu_aE(1 - F_a - F_b - E)]\\ &+ \lambda_b[\beta F_b(2E+F_b+(1 - \omega_a)F_a-1)+\beta(1-\mu_a)E(1-F_a-F_b-E)]\end{aligned}$$

where $\lambda_a(t)$, $\lambda_b(t)$ are *adjoint (co-state) functions* satisfying:

$$\dot{\lambda}_a = -\frac{\partial H}{\partial F_a} = -\{\beta\lambda_a[2E+2F_a+\omega_aF_b - \mu_aE-1] - \beta\lambda_b[(1 - \mu_a)E-(1 - \omega_a)F_b]\} \quad (4a)$$

$$\dot{\lambda}_b = -\frac{\partial H}{\partial F_b} = -\{\beta\lambda_b[2E+2F_b + (1 - \omega_a)F_a-(1 - \mu_a)E-1] - \beta\lambda_a(\mu_aE - \omega_aF_a)\}. \quad (4b)$$

with final conditions $\lambda_a(T)=W_{ab}$–W_a and $\lambda_b(T)=W_{ab}$–W_b.

According to *Pontryagin's Minimum Principle* ([27]), any optimal control (μ_a^*, ω_a^*) minimizes the cost $\bar{\mathcal{R}}$ also (pointwise) minimizes the Hamiltonian H as the following:

$$(\mu_a^*, \omega_a^*) \in \arg\min_{(\mu_a,\omega_a)\in\mathcal{S}} H((\mu_a, \omega_a), (E^*, F_a^*, F_b^*), (\lambda_a^*, \lambda_b^*))$$

where the state and adjoint functions $(E^*, F_a^*, F_b^*), (\lambda_a^*, \lambda_b^*)$ are absolutely continuous (and piecewise differential) functions of time satisfying (3a), (3b), (4a) and (4b) respectively. Define $\varphi \triangleq \beta E(1 - F_a - F_b - E)(\lambda_a - \lambda_b)$ and $\sigma = \beta F_aF_b(\lambda_a - \lambda_b)$. The Hamiltonian H can now be re-written as follows:

$$\begin{aligned}H = &\varphi\mu_a + \sigma\omega_a + \lambda_a[\beta F_a(2E + F_a - 1)]\\ &+\lambda_b[\beta F_b(2E+F_a+ F_b-1)+\beta E(1-F_a-F_b-E)]\end{aligned}$$

Hence, according to Pontryagin's principle, for all $(\mu_a, \omega_a) \in \mathcal{S}$, we must have:

$$\varphi^*\mu_a^* + \sigma^*\omega_a^* \leqslant \varphi^*\mu_a + \sigma^*\omega_a$$

Define the *switching function* as:

$$\phi \triangleq \lambda_a - \lambda_b \quad (5)$$

Since β, E, F_a, F_b and $F_{ab} = 1 - F_a - F_b - E$ are all strictly positive (following Lemma 1), minimizing $\varphi^*\mu_a + \sigma^*\omega_a$ is equivalent to minimizing $\phi^*\mu_a + \phi^*\omega_a = \phi^*(\mu_a + \omega_a)$. Thus, at any time t, the two-dimension control (μ_a, ω_a) depends on the same switching function $\phi^*(t)$. Therefore, in order to minimize the Hamiltonian, the optimal control policy $(\mu_a^*(t), \omega_a^*(t))$ should be chosen as follows:

$$(\mu_a^*(t), \omega_a^*(t)) = \begin{cases} (0,0) & \phi^*(t) > 0 \\ (1,1) & \phi^*(t) < 0 \end{cases}$$

Note that the Pontryagin's principle is silent on the value of the optimal control when $\phi^*(t) = 0$. If $\phi^*(t)$ is at zero over a sub-interval of non-zero length, then other means than Pontryagin's principle should be used to determine the solution, which can make the problem more challenging. The optimal control over such sub-intervals is referred to as *singular control* and the trajectory of the optimal control over such sub-intervals are often referred to as *singular subarcs*. If no singular subarc exists, the optimal control is called *non-singular*. In what follows, we observe that singular subarcs can indeed exist in our problem. From now on, we will omit the asterisks from the optimal variables (controller, state and adjoint functions), noting that, unless otherwise mentioned, all variables are according to their optimum values.

IV. Structural Results of Optimal Control

In this section, we present our main result, that the optimal dissemination policy (μ_a, ω_a) follow a simple bang-singular-bang structure. Specifically we show that there is at most one singular sub-interval that separates the whole interval into three parts: nonsingular, singular and nonsingular sub-intervals. In the first nonsingular sub-interval, μ_a and ω_a are the same and remain as either maximum (one) or minimum (zero). In the singular sub-interval, μ_a and ω_a must be adjusted to guarantee approaching to the steady status as soon as possible. And in the last nonsingular sub-interval, μ_a and ω_a become the same again and can only be the minimum. Moreover, if no singular case occurs, in the optimal control policy, μ_a and ω_a follow the same bang-bang structure and switch from the maximum to the minimum at most once with terminating in minimum towards the end of the interval.

In words, the optimal forwarding policy is bound to behave like one of the four cases: (1) In both *Full-Empty Encounter* and *Single-a-Single-b Encounter*, always forward the file with higher priority; (2) In both *Full-Empty Encounter* and *Single-a-Single-b Encounter*, initially forward the file with lower priority (somewhat counter-intuitively), and then switch to forwarding only the file with higher probability till the end; (3) In both encounters, initially, only the file with lower priority is forwarded. Then once the fractions of nodes containing different single files become equal in the network, file forwarding in *Full-Empty Encounter* and *Single-a-Single-b Encounter* become *complementary*[1] to make sure the fractions of nodes containing different single files remain equal in the network for a limited interval. But after such interval, in both two encounters, switch to forwarding only the file with higher probability that extends till the end; (4) At first in both encounters, only forward the higher priority file until the fractions of nodes with each file become equal. Then there is a sub-interval of singular control over which file forwarding in these two encounters remain complementary for some time. After that, only the higher priority file is forwarded again towards the end in both encounters.

***Theorem** 1:* The optimal control policy $(\mu_a(t), \omega_a(t))$ has one of the two structures as follows:

$$(\mu_a(t), \omega_a(t)) = \begin{cases} (1,1) & t \in [0, \tau_1) \\ (\mu_a, \omega_a) \in \mathcal{S} \text{ s.t. } (2\mu_a - 1)E(1-2F_a) + (2\omega_a - 1)F_a^2 = 0 & t \in [\tau_1, \tau_2) \\ (0,0) & t \in [\tau_2, T] \end{cases}$$

or

$$(\mu_a(t), \omega_a(t)) = \begin{cases} (0,0) & t \in [0, \tau_1) \\ (\mu_a, \omega_a) \in \mathcal{S} \text{ s.t. } (2\mu_a - 1)E(1-2F_a) + (2\omega_a - 1)F_a^2 = 0 & t \in [\tau_1, \tau_2) \\ (0,0) & t \in [\tau_2, T] \end{cases}$$

where $\tau_1 \in [0, T)$, $\tau_2 \in [0, T)$ and $\tau_1 \leqslant \tau_2$.

Proof: The proof of Theorem 1 consists of two steps. First, in the case when no singular control exists, that is, the switching function $\phi(t)$ cannot be 0 over a non-zero length sub-interval in $[0, T]$, we prove the optimal control functions $\mu_a(t)$ and $\omega_a(t)$ are the same as a bang-bang structure with at most one jump from the maximum (1) to the minimum (0) with ending in minimum. Second, when the singular control occurs, we show that there is only one singular sub-interval over which μ_a and ω_a must be chosen complementary satisfying the condition $(2\mu_a - 1)E(1-2F_a) + (2\omega_a - 1)F_a^2 = 0$ and the optimal control policy in this case has a bang-singular-bang structure.

Step 1: The structure of the optimal controls $\mu_a(t)$ and $\omega_a(t)$ in a non-singular case is given by the following lemma:

***Lemma** 2:* If $[0, T]$ is non-singular, the control functions $\mu_a(t)$ and $\omega_a(t)$ of the optimal control policy have the same bang-bang structure:

$$(\mu_a(t), \omega_a(t)) = \begin{cases} (1,1) & t \in [0, \tilde{t}) \\ (0,0) & t \in [\tilde{t}, T] \end{cases}$$

where $\tilde{t} \in [0, T]$.

Before proceeding with the proof of Lemma 2, we first present three basic real-analysis properties that will be used later.

***Property** 1:* Let $f(t)$ be a continuous and piecewise continuously differentiable function of t in the interval $[t_{min}, t_{max}]$. Assume $f(t_{max}) > 0$. Now if t_1 ($t_1 \in [t_{min}, t_{max}]$) is the last time before t_{max} such that $f(t_1) = 0$ and $\forall t \in (t_1, t_{max}]$, $f(t) > 0$, then $\dot{f}(t_1^+) \geqslant 0$.

***Property** 2:* Let $f(t)$ be a continuous and piecewise continuously differentiable function of t in an interval $[t_{min}, t_{max}]$.

[1] By saying *Complementary* or *Complementarily*, we mean if one file is prioritized in the *Full-Empty Encounter*, the other file is prioritized in the *Single-a-Single-b Encounter*.

Assume $f(t_{min}) > 0$. Now if t_2 ($t_2 \in [t_{min}, t_{max}]$) is the first time after t_{min} such that $f(t_2) = 0$ and $\forall t \in [t_{min}, t_2)$, $f(t) > 0$, then $\dot{f}(t_2^-) \leqslant 0$.

***Property** 3:* Let $f(t)$ be a continuous and piecewise continuously differentiable function of t in an interval $[t_{min}, t_{max}]$. Assume $f(t_{min}) < 0$. Now if t_3 ($t_3 \in [t_{min}, t_{max}]$) is the first time after t_{min} such that $f(t_3) = 0$ and $\forall t \in [t_{min}, t_3)$, $f(t) < 0$, then $\dot{f}(t_3^-) \geqslant 0$.

Proof: The assumption that no singular control occurs, that is, $\phi(t)$ cannot remain zero over an interval of non-zero length in $[0, T]$, indicates that the optimal control functions $\mu_a(t)$ and $\omega_a(t)$ always follow the same structures and can only take the maximum (1) or minimum (0) over the entire interval. However, it is not *a priori* clear that how many times $\mu_a(t)$ and $\omega_a(t)$ switches between the extreme values. In the following, we show that $\mu_a(t)$ and $\omega_a(t)$ switches between 1 and 0 at most once, and they always terminates at zero.

Because of the continuity of $\phi(t)$ and its final condition $\phi(T) = \lambda_a(T) - \lambda_b(T) = W_b - W_a > 0$, there exists some interval $(t', T]$ ($0 \leqslant t' < T$) in which $\phi(t) > 0$.

Let τ be a time when $\phi(\tau) = 0$, that is, $\lambda_a(\tau) = \lambda_b(\tau)$. The time derivative of the switching function at such a point is:

$$\dot{\phi}(\tau) = \beta\lambda_a(\tau)(F_b(\tau) - F_a(\tau))$$

Suppose $\phi(t)$ could cross the t-axis at least twice. Then, let $\tau_1, \tau_2 \in [0, T]$, where $\tau_1 < \tau_2$, be the last two consecutive zero-crossing points of $\phi(t)$. Since $\phi(T) > 0$, we must have $\phi(t) > 0$ when $t \in [\tau_0, \tau_1) \bigcup (\tau_2, T]$, and $\phi(t) < 0$ when $t \in (\tau_1, \tau_2)$, for some $0 \leqslant \tau_0 < \tau_1$. This in turn would imply $\mu_a(t) = \omega_a(t) = 0$, when $t \in [\tau_0, \tau_1) \bigcup (\tau_2, T]$, and $\mu_a(t) = \omega_a(t) = 1$ when $t \in (\tau_1, \tau_2)$.

The fact that τ_1 and τ_2 are zero-crossing points together with properties 1 and 2 indicate that $\dot{\phi}(\tau_1) = \beta\lambda_a(\tau_1)(F_b(\tau_1) - F_a(\tau_1)) \leqslant 0$, and $\dot{\phi}(\tau_2) = \beta\lambda_a(\tau_2)(F_b(\tau_2) - F_a(\tau_2)) \geqslant 0$. The signs of λ_a at τ_1 and τ_2 can be determined by the following lemma.

***Lemma** 3:* $\forall t \in [0, T]$, both $\lambda_a(t)$ and $\lambda_b(t)$ are strictly positive.

Proof: See Appendix A. ■

Lemma 3 indicates λ_a and λ_b must be strictly positive at τ_1 and τ_2. Then the fact that $\dot{\phi}(\tau_1) \leqslant 0$ and $\dot{\phi}(\tau_2) \geqslant 0$ implies that at these two zero-crossing points, $F_b(\tau_1) - F_a(\tau_1) \leqslant 0$ and $F_b(\tau_2) - F_a(\tau_2) \geqslant 0$.

Consider $F_b(t) - F_a(t)$ in the interval $(\tau_1, \tau_2]$ during which $\mu_a(t) = \omega_a(t) = 1$. At time τ_1, either $F_b(\tau_1) - F_a(\tau_1) < 0$ or $F_b(\tau_1) - F_a(\tau_1) = 0$. If $F_b(\tau_1) - F_a(\tau_1) = 0$, since $\dot{F}_b(\tau_1^+) - \dot{F}_a(\tau_1^+) = -\beta F_a^2 - \beta EF_{ab} < 0$ (by Eqns. (3a), (3b) and Lemma 1), then right after τ_1, $F_b(t) - F_a(t)$ become negative. Thus, $F_b(t) - F_a(t)$ starts from a negative value in the interval $(\tau_1, \tau_2]$. Suppose $F_b(t) - F_a(t)$ may become zero in the interval. Let $\tau_3 \in (\tau_1, \tau_2]$ be the first time after τ_1 such that $F_b(\tau_3) - F_a(\tau_3) = 0$. From (3a), (3b) and Lemma 1, we have $\dot{F}_b(\tau_3^-) - \dot{F}_a(\tau_3^-) = -\beta F_a^2 - \beta EF_{ab} < 0$, which contradicts to Property 3. Thus, $F_b(t) - F_a(t)$ cannot become 0 and always remain negative in interval $(\tau_1, \tau_2]$, which contradicts to the assumption that τ_2 is another zero crossing point where $F_b(\tau_2) - F_a(\tau_2) \geqslant 0$. Hence, $\phi(t)$ cannot have two or more zero crossing points. Let $\tilde{t} \in [0, T]$ be the only one possible zero crossing point ($\tilde{t} < 0$ means there is no zero crossing point). The fact that $\phi(T) > 0$ together with its continuity property yields $\phi(t) < 0$ in $[0, \tilde{t})$ and $\phi(t) > 0$ in $(\tilde{t}, T]$. Hence, $\mu_a(t)$ and $\omega_a(t)$ must be chosen as $\mu_a(t) = \omega_a(t) = 1$ when $t \in [0, \tilde{t})$, and $\mu_a(t) = \omega_a(t) = 0$ when $t \in [\tilde{t}, T]$. ■

Step 2: This step consists of two parts: First, the singular subarc of the optimal control is computed; Second, the structure of the optimal control is discussed when the singular subarc exists.

***Lemma** 4:* If there are singular subarcs contained in the optimal control, then such singular subarcs during some time interval can only be $(\mu_a, \omega_a) \in \mathcal{S}$ satisfying $(2\mu_a - 1)E(1 - 2F_a) + (2\omega_a - 1)F_a^2 = 0$.

Proof: The singular case can only happen when the switching function $\phi(t)$ remains zero over some non-zero length time sub-interval $[t_1, t_2]$ of $[0, T]$ (in the case when multiple singular sub-intervals exist, $[t_1, t_2]$ could be any of them), which requires $\forall t \in [t_1, t_2]$

$$\phi(t) = \dot{\phi}(t) = \ddot{\phi}(t) = ... = 0 \tag{6}$$

Referring to (3a), (3b), (4a), (4b) and (5), it is straightforward to find that the candidate that satisfies (6) is $(\mu_a(t), \omega_a(t)) \in \mathcal{S}$ satisfying $(2\mu_a - 1)E(1 - 2F_a) + (2\omega_a - 1)F_a^2 = 0$ in the interval $[t_1, t_2]$. ■

Remark: Before proceeding, we give some intuitions behind the Lemma 4. During the time interval where singular case happens, the fractions of nodes containing two single files always remain the same. The condition which (μ_a, ω_a) should satisfy along the singular subarc can be re-written as $\frac{2\mu_a - 1}{1 - 2\omega_a} = \frac{F_a F_b}{EF_{ab}}$, where $F_a F_b$ indicates the happening chance of *Single-a-Single-b Encounter* and EF_{ab} indicates the happening chance of *Full-Empty Encounter*. Thus, during the singular time interval, the two control functions μ_a and ω_a are chosen complementarily to guarantee that at any time during this singular interval, if in the *Single-a-Single-b Encounter* the fraction of nodes containing a single file (say file *a*) decreases due to receiving a different file (*b*) from the sender and becoming a full node, then in the *Full-Empty Encounter*, this fraction decrease would be compensated for by full nodes forwarding this single file (*a*) to empty nodes during the *Full-Empty Encounter*. Complementarily choosing $\mu_a(t)$ and $\omega_a(t)$ in this way can make sure the fractions of nodes containing two different single files remain equal in the singular interval, and thus increases the rate of generating full nodes which have the highest priority.

Now suppose there exists a singular subarc in the optimal control, we first show that after this singular subarc, $\mu_a(t)$ and $\omega_a(t)$ must remain 0 till the end, that is, $\mu_a(t) = \omega_a(t) = 0$ $\forall t \in [\tau_2, T]$.

Consider a small interval $[\tau_2, \tau_3)$ right after the singular subarc $[\tau_1, \tau_2)$. In this small interval, there are two possibilities: either $\phi(t) < 0$ $\forall t \in [\tau_2, \tau_3)$ or $\phi(t) > 0$ $\forall t \in [\tau_2, \tau_3)$.

Suppose if $\phi(t) < 0 \ \forall t \in [\tau_2, \tau_3)$, then the optimal control $\mu_a(t)$ remains at 1 in this interval. Similarly, we have $F_b(t) - F_a(t) < 0 \ \forall t \in [\tau_2, \tau_3]$. The latter claim can be established through proof-by-contradiction: Note at τ_2, $F_b(\tau_2) = F_a(\tau_2)$ and $\dot{F}_b(\tau_2^+) - \dot{F}_a(\tau_2^+) = -\beta F_a^2 - \beta E F_{ab} < 0$, we know that right after τ_2, $F_b(t) - F_a(t) < 0$. Suppose $F_b(\hat{t}) - F_a(\hat{t}) = 0$ at some $\hat{t} \in (\tau_2, \tau_3]$, then $\dot{F}_b(\hat{t}^-) - \dot{F}_a(\hat{t}^-) = -\beta F_a^2 - \beta E F_{ab} < 0$, which contradicts to Property 3.

Now suppose $\phi(t)$ becomes 0 at $t = \tau_3$. Then $\lambda_a(\tau_3) = \lambda_b(\tau_3)$, and by Property 3, $\dot{\phi}(\tau_3^-) \geqslant 0$. However, the fact that $\lambda_a(\tau_3) = \lambda_b(\tau_3) > 0$ (by Lemma 3) and $F_b(\tau_3) - F_a(\tau_3) < 0$ imply $\dot{\phi}(\tau_3^-) = \beta \lambda_a(\tau_3)(F_b(\tau_3) - F_a(\tau_3)) < 0$, which yields a contradiction. Thus, $\phi(t)$ is still negative at τ_3. Therefore, because of the continuity of $\phi(t)$, it remains negative in a small interval after $[\tau_2, \tau_3)$. Using the same argument, we conclude that $\phi(t)$ is negative at the end of this small interval. Repeating this procedure shows that $\phi(t)$ remains negative till time T. However, this contradicts the fact that $\phi(T) > 0$. Thus, in interval $[\tau_2, \tau_3)$, we have: $\phi(t) > 0$.

The fact that $\phi(t) > 0 \ \forall t \in [\tau_2, \tau_3)$ indicates that $\mu_a(\tau_2) = \omega_a(\tau_2) = 0$ in $[\tau_2, \tau_3)$ and $F_b(t) - F_a(t) > 0$ over $[\tau_2, \tau_3]$. Using the same argument as above, one can show $\phi(t)$ remains positive from τ_2 till the end. Therefore, there is only one singular subarc (if it exists) in the optimal solution, and after such a singular subarc $\mu_a(t)$ and $\omega_a(t)$ remains 0 till the end.

Next, we show that if a singular subarc exists on some interval $[\tau_1, \tau_2)$, then $\mu_a(t)$ and $\omega_a(t)$ will be a constant as either 0 or 1 in $[0, \tau_1)$. Suppose $\phi(t)$ changes its sign once in $[0, \tau_1)$, then there are two possibilities: either $\phi(t)$ changes from negative to positive or $\phi(t)$ changes from positive to negative in $[0, \tau_1)$.

Case 1: Suppose $\phi(t)$ changes once from negative to positive over the interval $[0, \tau_1)$. Then there exists some $\tau_4 \in (0, \tau_1)$ such that $\phi(t) < 0 \ \forall t \in (0, \tau_4)$ and $\phi(t) > 0 \ \forall t \in (\tau_4, \tau_1)$. This in turn implies that $\mu_a(t) = \omega_a(t) = 1 \ \forall t \in [0, \tau_4)$ and $\mu_a(t) = \omega_a(t) = 0 \ \forall t \in [\tau_4, \tau_1)$. Moreover, at time τ_4, $\phi(\tau_4) = 0$ and $\dot{\phi}(\tau_4^-) \geqslant 0$ (by Property 3). Following Lemma 3, $\lambda_a(\tau_4) = \lambda_b(\tau_4) > 0$. Hence, $\dot{\phi}(\tau_4^-) = \beta \lambda_a(\tau_4)(F_b(\tau_4) - F_a(\tau_4)) \geqslant 0$ indicates $F_b(\tau_4) - F_a(\tau_4) \geqslant 0$. In the interval $(\tau_4, \tau_1]$, starting with $F_b(\tau_4) - F_a(\tau_4) \geqslant 0$ and choosing $\mu_a(t) = \omega_a(t) = 0$ guarantee that $F_b(t) - F_a(t) > 0 \ \forall t \in (\tau_4, \tau_1]$. Hence the corresponding result that $F_a(\tau_1) < F_b(\tau_1)$ at τ_1 cannot lead to a singular interval starting at τ_1 which requires $F_a(\tau_1) = F_b(\tau_1)$ at τ_1. Thus, case 1 cannot happen.

Case 2: Suppose $\phi(t)$ changes once from positive to negative values over the interval $[0, \tau_1)$. Then using the same argument as the that of case 1, we have $F_b(t) - F_a(t) < 0 \ \forall t \in (\tau_4, \tau_1]$. Hence the corresponding result that $F_a(\tau_1) > F_b(\tau_1)$ at τ_1 cannot lead to a singular interval starting at τ_1 which requires $F_a(\tau_1) = F_b(\tau_1)$ at τ_1. Thus, case 2 cannot happen either.

Therefore, if a singular subarc exists in some interval $[\tau_1, \tau_2)$, then $\mu_a(t)$ and $\omega_a(t)$ must be a constant as either 0 or 1 in $[0, \tau_1)$.

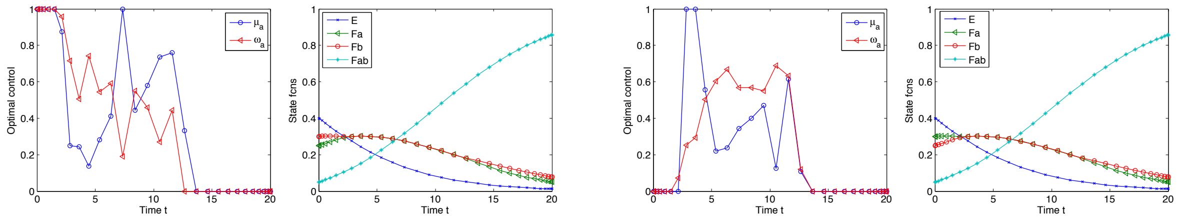
Based on the above arguments, the optimal control $(\mu_a(t), \omega_a(t))$ has a simple bang-singular-bang structure as given by Theorem 1. ■

V. Numerical Results

We first present numerical results to illustrate Theorem 1. The simulation tool we use is DIDO ([28]), which is widely used for solving optimal control problems. We use $\beta = 0.2$, $W_a = 15$, $W_b = 15.5$, $W_{ab} = 30$, $T = 20$, $E_0 = 0.4$, $F_{ab0} = 0.05$; F_{a0} and F_{b0} are varied as shown in Fig. 2(a) and 2(b). The optimal control policy is plotted together with the corresponding state functions. As can be seen from these figures, the structure of the optimal control policy has a bang-singular-bang structure, and two possible structures occur based on different initial conditions.

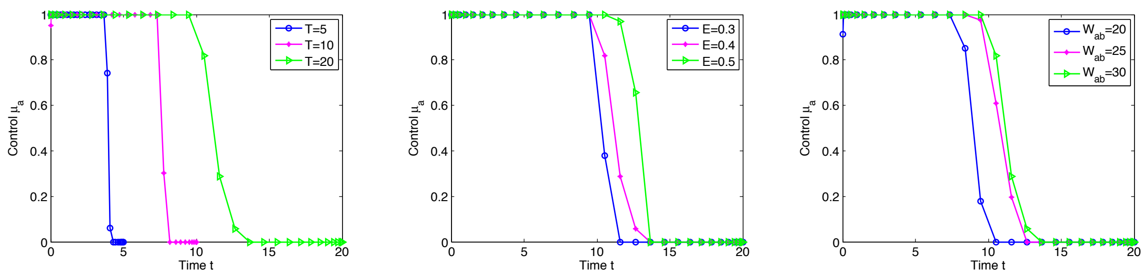
The structure of the optimal control policy reflects a phenomenon similar to the *Most Rapid Approach Path* ([29]), where the optimal control is chosen such that the system state approaches to its steady state as quickly as possible. And after reaching the steady state, the optimal control guarantee that the system stays at its steady state as long as possible before it goes to some final state to meet certain condition. The optimal control policy turns out to have the structure as it is shown in Fig. 2(a) when the lower priority file a is less than the higher priority file b in the beginning. First always transmitting file a in both *Full-Empty Encounter* and *Single-a-Single-b Encounter* increases the fraction of nodes receiving file a. This further increases the fraction of full nodes due to the packet transmission in the increased opportunistic encounters between two single file nodes. After some time when the fractions of different single file nodes become equal, optimal policy then switches to forwarding file a and b complementarily. This would keep the fractions of nodes containing two different single files equal, which maximize the encounter chance between nodes with different single files. And this in turn maximize the rate of nodes becoming a full nodes, which yields a higher system reward. However, at some later time point, the optimal control policy goes back to only transmitting file b in both encounter cases because file b has a higher priority and more nodes containing file b at the end leads to a higher system reward. Similar explanations hold for Fig. 2(b) with the only difference that the optimal control policy starts with forwarding the higher priority file b during some initial time period because of less nodes with file b in the beginning.

Second, to gain some intuitions on how the optimal control policy switches in different situations, we depict the optimal control in non-singular cases as we vary T, E_0 and W_{ab} respectively. The parameters we have used are depicted in the caption of Fig. 3. All the scenarios we consider here are non-singular. In all the three cases of Fig. 3(a), there are less nodes with the lower priority file a than those with the higher priority file b at the beginning. So according to the optimal policy, first forward the lower priority file in both *Full-Empty Encounter* and *Single-a-Single-b Encounter* until some time threshold, and then switch to only forwarding the higher priority file



(a) Control (left) and states (right) for $F_{a0} = 0.25$, $F_{b0} = 0.3$ (b) Control (left) and states (right) for $F_{a0} = 0.3$, $F_{b0} = 0.25$

Fig. 2: The two possible structures of the optimal control policy



(a) The structure of μ_a as varying T. (b) The structure of μ_a as varying E_0 (c) The structure of μ_a as varying W_{ab}

Fig. 3: The changing switching times of the optimal control policy with different initial conditions. The common parameters are $F_{a0} = 0.05$, $F_{b0} = 0.3$, $W_a = 15$, $W_b = 15.5$. For Fig.3(a), we have used $E_0 = 0.4$, $F_{ab0} = 0.25$, and $W_{ab} = 30$; For Fig.3(b), we have used $T = 20, W_{ab} = 30$ and $F_{ab0} = 0.65 - E_0$; For Fig.3(c), we have used $E_0 = 0.4$, $F_{ab0} = 0.25$ and $T = 20$.

till the end. As we increase the interest time window T, the switching time, i.e., the time when switch to forwarding the higher priority file is more postponed, which indicates the system spends more time forwarding the less priority file. This would increase the fraction of nodes with the lower priority file and make it close enough to the fraction of nodes with the higher priority file, and thus provides more chance for nodes with two different files to meet. As more such encounters happen, more nodes receive both two files due to the file transmission during these encounters, which further brings a better system reward. Similar postponement behavior of the switching time of the optimal policy is observed when we increase E_0 and W_{ab} respectively in Fig. 3(b) and 3(c). The discrepancy of $\mu_a(t)$ during switching is due to discretization in DIDO.

VI. Conclusions

In this paper, we have studied the dissemination problem of two files with distinct priorities through epidemic routing for large scale DTNs. By formulating an optimal control problem that can be dealt with by using Pontryagin's Minimum Principle, we have found that the optimal control policy for transmitting files with different priorities follows a simple bang-singular-bang structure. In addition, under some system initial settings the singular case may not exist and the optimal control policy becomes even simplified as a bang-bang structure with at most one jump from the maximum to the minimum. Though updated continuously and dynamically in time, the simple-structure optimal policy is convenient to implement in real applications. Numerical results have been presented to validate our analysis. Moreover, through numerical simulations, we have provided some intuitions on how the optimal policy changes according to different initial system settings.

There are several possible future directions. The first thing we want to point out is that though the optimal control in the singular case given by the numerical result seems disordered, it might be promising to keep both $\mu_a(t)$ and $\omega_a(t)$ be 0.5 during the singular sub-interval, since this satisfies the condition in the optimal control policy and might also yield the maximum reward. We have done a number of simulations to validate our guess, but further theoretical analysis is needed to confirm our conjecture. The second is to extend this two files dissemination problem to a general multiple files dissemination problem. The third is to introduce heterogeneity, such as heterogeneous mobility and non-uniform control functions, to the network settings. Finally, we are also interested in finding a way to theoretically characterize how the optimal control policy changes according to different initial system parameters.

Appendix A
Proof of Lemma 3

Proof: We prove Lemma 3 by first establishing the following auxiliary lemma:

***Lemma** 5:* During the interval $[0, T]$, $\lambda_a(t)$ and $\lambda_b(t)$ cannot be 0 at the same time.

Proof: Let us first rewrite equations (4a) and (4b) in a matrix form:

$$\left[\begin{array}{c}\dot{\lambda}_a\\ \dot{\lambda}_b\end{array}\right] = A\left[\begin{array}{c}\lambda_a\\ \lambda_b\end{array}\right]$$

where A is

$$\begin{bmatrix}-\beta(2E+2F_a+\omega_a F_b-\mu_a E-1) & \beta((1-\mu_a)E-(1-\omega_a)F_b)\\ \beta(\mu_a E-\omega_a F_a) & -\beta(2E+2F_b+(1-\omega_a)F_a-(1-\mu_a)E-1)\end{bmatrix}$$

Assume there exists some time $\hat{\tau} \in [0, T]$ such that $\lambda_a(\hat{\tau}) = \lambda_b(\hat{\tau}) = 0$. Since λ_a and λ_b are continuous and piecewise continuously differentiable functions of time, over some interval $[\hat{\tau}, T']$ starting at $\hat{\tau}$, there are five possible cases: 1) both λ_a and λ_b are positive and one strictly; 2) both λ_a and λ_b are negative and one strictly; 3) λ_a positive and λ_b negative and one strictly; 4) λ_b is positive and λ_a is negative and one strictly; 5) both λ_a and λ_b are strictly zero.

First, consider case 1 in which λ_a and λ_b are positive and one strictly. Then, (2b) and Lemma 1 imply that:

$$\begin{bmatrix}-\beta & -\beta\\ -\beta & -\beta\end{bmatrix}\begin{bmatrix}\lambda_a\\ \lambda_b\end{bmatrix} \leqslant \begin{bmatrix}\dot{\lambda}_a\\ \dot{\lambda}_b\end{bmatrix} \leqslant \begin{bmatrix}2\beta & \beta\\ \beta & 2\beta\end{bmatrix}\begin{bmatrix}\lambda_a\\ \lambda_b\end{bmatrix} \tag{7}$$

Define two differential equations which are the same as the left-hand side and the right-hand side of (7) respectively, and have the same initial conditions as λ_a and λ_b:

$$\begin{bmatrix}\dot{\lambda}'_a\\ \dot{\lambda}'_b\end{bmatrix} = \begin{bmatrix}-\beta & -\beta\\ -\beta & -\beta\end{bmatrix}\begin{bmatrix}\lambda'_a\\ \lambda'_b\end{bmatrix}, \tag{8}$$

and

$$\begin{bmatrix}\dot{\lambda}''_a\\ \dot{\lambda}''_b\end{bmatrix} = \begin{bmatrix}2\beta & \beta\\ \beta & 2\beta\end{bmatrix}\begin{bmatrix}\lambda''_a\\ \lambda''_b\end{bmatrix}, \tag{9}$$

with initial condition $\lambda'_a(0) = \lambda'_b(0) = \lambda''_a(0) = \lambda''_b(0) = 0$. Then for $\forall t \in [\hat{\tau}, T']$, we have:

$$\lambda'_a(t) \leqslant \lambda_a(t) \leqslant \lambda''_a(t), \qquad \lambda'_b(t) \leqslant \lambda_b(t) \leqslant \lambda''_b(t)$$

Solving (8) and (9), we have for $\forall t \in [\hat{\tau}, T']$, $\lambda'_a(t)$ $\lambda''_a(t)$ $\lambda'_b(t)$ and $\lambda''_b(t)$ are all 0. Thus, $\lambda_a(t) = 0$ and $\lambda_b(t) = 0$ over the interval $[\hat{\tau}, T']$.

Applying the same analysis method in the cases 2, 3, and 4 gives the same result that $\forall t \in [\hat{\tau}, T']$, $\lambda_a(t) = 0$ and $\lambda_b(t) = 0$. It is clear that in case 5 both λ_a and λ_b are zero in $[\hat{\tau}, T']$.

Based on the discussion of all of these five cases, λ_a and λ_b remain 0 in the interval $[\hat{\tau}, T']$. Starting from T', by using the same analysis above, we have that λ_a and λ_b remain 0 in a small interval $[T', T'']$. Then starting from T'', using the same analysis again, we have λ_a and λ_b keep 0 in a small interval right after T''. By repeating doing this, we have that λ_a and λ_b will keep being 0 till the end time T, which contracts to the final states of $\lambda_a(T) = W_{ab} - W_b > 0$ and $\lambda_b(T) = W_{ab} - W_b > 0$. Thus, no such time in $[0, T]$ exists at which λ_a and λ_b are both zero. ■

The proof of Lemma 3 now follows. We know that at time T, $\lambda_a(T) = W_{ab} - W_a > 0$, and $\lambda_b(T) = W_{ab} - W_b > 0$. Since $\lambda_a(t)$ is continuous in time, there will be some time till the end that $\lambda_a(t)$ is always non-negative. This is also true for $\lambda_b(t)$.

Going backward from T, let $\hat{t} \in [0, T]$ be the first time closest to T such that $\lambda_a(\hat{t})$ or $\lambda_b(\hat{t})$ becomes 0, and $\forall t \in (\hat{t}, T]$, $\lambda_a(t) > 0$ and $\lambda_b(t) > 0$. Now from Lemma 5, we know there are only two cases:

Case 1: At $\hat{t}$, $\lambda_a(\hat{t}) = 0$ but $\lambda_b(\hat{t}) > 0$. Because of the continuity of $\lambda_a(t)$ and $\lambda_b(t)$, there exists a small interval $[\hat{t}, \hat{t}+\delta)$ in which $\lambda_a > 0$ except for $\lambda_a(\hat{t}) = 0$, $\lambda_b > 0$ and $\lambda_a - \lambda_b < 0$. Moreover, $\mu_a(t) = \omega_a(t) = 1$ in this interval $[\hat{t}, \hat{t}+\delta]$. Thus, in the interval $[\hat{t}, \hat{t}+\delta]$, Eqn. (4a) can be expressed as

$$\dot{\lambda}_a = -\beta(E + 2F_a + F_b - 1)\lambda_a$$

Together with Eqn. 2(b) and Lemma 1, one would have

$$-\beta\lambda_a \leqslant \dot{\lambda}_a \leqslant \beta\lambda_a$$

Assume two ODEs, starting with 0 at $\hat{t}$, evolve as the lefthand side and righthand side of the above inequality in the interval $[\hat{t}, \hat{t}+\delta)$. Then, $\lambda_a(t)$ in the interval $[\hat{t}, \hat{t}+\delta)$ can be bounded by the solutions to the two ODEs. Starting with the initial condition 0 in the interval $[\hat{t}, \hat{t}+\delta)$, the solutions to the lower bound and the upper bound are both 0. Thus, $\lambda_a(t)$ remains 0 in the interval $[\hat{t}, \hat{t}+\delta)$, which contradicts to the assumption that $\hat{t}$ is the first time closet to T at which $\lambda_a(t) = 0$. Thus, $\lambda_a(t)$ is always strictly positive in $[0, T]$.

Case 2: At $\hat{t}$, $\lambda_b(\hat{t}) = 0$ but $\lambda_a(\hat{t}) > 0$. By using the same method as above, one can prove that $\lambda_b(t)$ is always strictly positive over the whole interval.

Therefore, $\lambda_a(t)$ and $\lambda_b(t)$ are always strictly positive over the interval $[0, T]$. ■

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