Efficient Mechanism Design for Competitive Uplink Carrier Selection and Rate Allocation

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I. Abstract

As cellular wireless networks see increasing amount of uplink traffic due to video applications, there is a need to explore the use of multiple carriers. In such a system, multiple operators (carriers) may compete to carry data from a customer (transmitter). In order to optimally allocate power and rate, a transmitter needs truthful channel state information from the carriers. This can be challenging as the carriers are self-interested entities. We model this as a competitive rate allocation game, in which carriers send bids about their channel quality to the transmitter, and the transmitter allocates power and rate accordingly. To ensure truthful bidding, we design an incentive mechanism where the transmitter offers a payment based on a convex piecewise linear function. We prove that as the number of bits per bid is increased, the total rate obtained by the transmitter approaches the maximum possible data rate obtained by the transmitter under perfect information about the channel statistics. To validate the performance of our proposed model, we also conduct simulations using real base station locations in London, and show that not only the customers benefit by having higher throughput, this model is also profitable to the operators. The operators gain more revenue due to more potential customers and more efficient use of the channels.

***Index Terms*—auction, mechanism design, multi-bit bids, convex piecewise liner function, truthful bidding.**

II. Introduction

Wireless networks are on the verge of a third phase of growth. The first phase was dominated by voice traffic, and the second phase, which we are currently in, is dominated by data traffic. In the third phase, it is predicted that the traffic will be dominated by videos [1].

With the development of technologies in media productions such as digital video, blogging, forums, social networking, and social media, it is now accessible and affordable for the general public to produce their own video content and publish on the internet. The development of smartphones and tablets and the continuing growth of laptops make the mobile device be able to display and capture high quality video contents. In addition, these devices are capable of supporting new interactive video applications such as video conferencing. Applications for video sharing, video blogging, and video broadcasting are also supported by these devices.

According to Cisco, in 2012, the average mobile user consumed 201 megabytes of data a month, including one hour of video, two hours of audio and downloaded one app per month. By 2017, it is predicted that the average mobile user will use 2 gigabytes of data per month, including 10 hours of video and 15 hours of audio. [2] Though most of the videos in the social media are prerecorded nowadays, in the future, the demands for live video capturing and streaming will become higher and higher. As a result, future wireless networks will need to be optimized for the delivery of mobile data services, especially for video camera based applications, which include significant uplink traffic.

The massive growth of wireless mobile traffic has led to an accelerating pace of research and development in the wireless area. New technologies are proposed and developed to provide faster data rates, higher spectrum efficiency, and larger system capacity. With technologies such as orthogonal frequency-division multiplexing (OFDM) [16], multiple-input-multiple-output (MIMO) [17], a high-rate data stream can be split into a number of lower rate streams and transmitted simultaneously over a number of subcarriers or a number of antennas. This greatly boosts the cellular radio link speed. With 4G (LTE and WiMax) systems, the peak rate can be 100 Mb/s to 1 Gb/s [3]. However, the fast rate of the technology growth still barely keeps pace with the fast growing demands from mobile traffic, especially for the video traffic [4]. To solve the bandwidth thirst problem, another trend is to increase the number of base stations (BSs) with smaller cells such as pico and femto cells. It is predicted that by 2015, there will be perhaps 50 million BSs, and some even predict that in the near future, say 10-15 years, there will be more BSs than mobile devices [5], [6], and one mobile device may connect to multiple BSs and use multiple links to transmit data simultaneously.

In the future, it will be harder and harder for a single operator to always provide high quality service to all its customers. Compared to the system capacity in the case where each mobile device must be connected to a single BS from the operator with which it has a contract, system capacity almost grows quadruply when a mobile device is allowed to connect to any nearby BSs even they are from different operators [7]. Exploring strategies of splitting data across multiple service providers attract researchers' attention and related ideas can also be found in recent literature [8]–[12].

In our opinion, the currently prevailing situation that a mobile device typically contracts with a single operator may change. This is also supported by industry trend of switching from contract model to prepaid model [13], [14]. It is anticipated that in the near future, the concept of service brokers will evolve and act as a third party in between the operators and the users. These service brokers will allow the end users more freedom to move from long-term single operator agreements to more opportunistic service models [10]. Allowing a mobile device to connect with multiple BSs from one or multiple operators is not only technically feasible, but also economically provides a win-win strategy for both cellular users and operators when the networks are close to being saturated since the cellular users will have better services, while the bandwidth burden is amortized among multiple operators.

Motivated by these trends, in this technical report, we consider a futuristic scenario in which a mobile device is able to split its data stream and connect to one or multiple BSs from different operators. Operators send bids indicating their channels' quality to the mobile device, and the mobile device selects one or multiple channels and allocates power and data rate based on its traffic, the number of its available antennas, and the bids received from the operators. If the channels are selected and used by a mobile device, that mobile device pays the corresponding operators based on a predefined payment contract. We assume that there are some monitoring software installed at the mobile device and BSs. The mobile device side software monitors the bids received from multiple operators and how the mobile device splits data stream. The BS side software monitors the bids sent and the amount of successfully received data. The bill is generated periodically (i.e., monthly) based on the predefined payment contract and the monitoring records.

In this technical report, we call such a mobile device as the transmitter and different operators as carriers who own different channels. We study in this technical report a simple yet fundamental rate allocation problem in which a transmitter does not know the state of the channels, and the corresponding carriers are self-interested.

We consider a problem that there is one transmitter who is able to transmit data simultaneously over at most K parallel channels, as shown in Fig. 1. The channels from the transmitter to each carrier are independent stochastic channels with two states: high or low. A channel in state high allows the transmitter sending data with a high data rate (sending aggressively), while a channel in state low only allows the transmitter sending data with a low data rate (sending conservatively). We further assume that the probability that a channel in state high is p, the value of p are different among different channels. When the conservative data rate is used, no matter what channel state is, the transmission is always successful. However, when the aggressive data rate is used, with probability $(1-p)$, the transmission will fail, and we assume all the data are lost. The transmitter rewards the carriers for successful transmissions (i.e., gives some positive credits to the carrier), and penalize them for failure (i.e., negative credits). As noted before, credits are converted to financial payments over a longer time period, such as monthly billing.

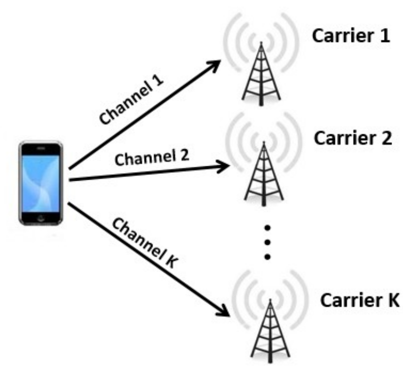

Figure 1: Illustration of Problem

We model the problem as an auction, in which the transmitter is an auctioneer, and the carriers are the bidders. The objective of the transmitter is to efficiently use the channels to meet its traffic requirement. When the traffic is heavy, this is equivalent to maximizing the total data rate over multiple channels under a power constraint. The objective of the carriers is to maximize the expected payment from the transmitter. The main focus of this technical report is to design the payment mechanism in such a way that the carriers will reveal their channel qualities truthfully, and as result, the transmitter is able to efficiently use its power and maximize the throughput.

A. Our Results

• We propose a payment mechanism using a convex piecewise linear function of channel probabilities, and prove that bidding truthfully is always a preferable action for a carrier.

•We prove that the throughput obtained by the transmitter approaches the maximum possible throughput with perfect information about the channel statistics as the number of bits per bid increases.

• Since bidding truthfully is always preferable, comparing with many existing literatures on competitive rate allocation, which typically runs multiple iterations to converge, our proposed mechanism is one-shot and does not require iterative convergence.

• The system overhead is little since the bids communication requires a few control message exchanges and happens only at the beginning of an auction cycle, and the computation of the payment is light.

• Our proposed mechanism is win-win for both customers and operators since the customers have better service (and/or lower payment) and the operators may potentially obtain larger revenues due to more customers and more efficient use of the channels. This claim is supported by simulations based on a dataset of real BS locations over a 2km× 2km area in London.

The technical report is organized as follows: section III talks about the related work; section IV introduces how the system works; section V introduces the power-rate model and investigates the optimal strategies for a transmitter under perfect information about the channel statistics; section VI discusses a payment mechanism design which ensures the truthfulness; section VII addresses implementation issues; section VIII conducts simulations and evaluates the performance; and section IX concludes the technical report.

III. Related Work

Wireless network technologies are continuously evolving to meet the increasing demands for data rate and high quality of services. Thanks to the development of technologies such as software-defined radio technique [15], Orthogonal Frequency Division Multiplexing (OFDM) [16], and Multiple-Input and Multiple-Output (MIMO) [17], a transmitter has the ability to learn and change the transmission parameters according to the radio environment and dynamically allocate rate and power.

One well-known technique of dynamic rate and power allocation and scheduling across multiple carriers is to use water-filling, which provides the maximum throughput under a power constraint [18]. A number of works are based on the idea of water-filling. Bingham proposes a finite granularity multicarrier loading algorithm which assigns bits successively to the subcarrier until the target rate is reached in [19]. Chow *et al*. improve the water-filling computation complexity by iteratively adjust the system performance margin until convergence [20]. Yu and Cioffi solve a simple two-band channel partition and power allocation problem using a water-filling algorithm in [21]. Kim *et al.* propose a joint subcarrier and power allocation algorithm in [22] and optimal power is calculated by water-filling fashion. Our work also uses the water-filling mechanism, as the transmitter does water-filling across channels. However, prior works assume truthful feedback regarding the channel states. In our problem setting, with autonomous selfish carriers, truthfulness is no longer a trivial thing. To ensure truthfulness of carriers is one of the main goals of this paper.

In communication networks, economic theories turn out to be powerful tools to deal with problems where interacting decision makers have conflicting objectives. In a competitive environment where the resources are shared, a user's utility is typically affected by other users actions. Most traditional auctions are about single unit auction, where there is only one winner. In such auctions, mechanisms such as second price auction yields truthful bidding. However, in spectrum markets, single unit auctions are generally fail to address the issues such as bidding for multiple units or multiple winners in one auction.

Pricing and auction mechanisms in dynamic spectrum access are studied in [23]–[25], where the primary users of the spectrum are the channel sellers and the secondary users are the channel buyers. In [23], Niyato and Hossain propose an equilibrium pricing scheme where the QoS performance degradation of the primary users was considered as the cost in offering the spectrum access to the secondary users. The authors analyze the problem as a Bertrand game and obtain the Nash equilibrium which provides the optimal pricing. In [24], Ghosh and Sarkar model the problem as a competitive game where the primary needs to select the price of its channel with the knowledge of its own channel state but not its competitors. Secondary users select the channels based on the states and the prices. The authors prove that there exists a unique symmetric Nash equilibrium strategy in this game setting. In [25], Gao *et al.* study the spectrum auction with multiple auctioneers and multiple bidders, and propose a mechanism in which auctioneer systematically raises the trading price and bidders subsequently choose one auctioneer for bidding. The authors analytically show that the proposed algorithm converges to the equilibrium with maximum spectrum utilization of the whole system. Our problem is also about using auction mechanisms to find the optimal. However, different from previous studies, in which the bids are about price and dynamically changed, in our mechanism, the pricing mechanism is predefined, thus, the bids are about their channel state parameters, rather than price. The price is dynamic with respect to different bids and performance. This is one major difference of our work from prior work on pricing and auction mechanism in dynamic spectrum access. Moreover, in previous studies, it typically takes some iterative learning process for the algorithm to converge to the Nash equilibrium or the optimal. However, since our mechanism ensures the truthful biddings from the carriers, there is no iterative learning algorithm whose convergence is affected.

Our work is in the category of mechanism design, and the main idea here is to provide incentive schemes such that the self interested entities will play a game in the favor of the mechanism designer. This idea has something in common with the mechanism design using the intervention framework. Schaar *et al*. have several interesting studies on non-cooperative resource sharing among self-interested users based on the idea of intervention [31]–[33]. To provide incentive schemes for the self-interested users to cooperate and improve the system performance, Schaar *et al*. propose to use an intervention device which is able to monitor the actions of users and affect the resource usage. An intervention manager strategically chooses an intervention device to maximize the system performance such as the sum of the utility of all users. Our proposed mechanism design is also using intervention framework to manipulate the users actions. However, different from these works, in which an optimal intervention device is selected among a set of intervention devices, and the algorithms to find the optimal or a good enough intervention device typically requires several iterations to converge, we have only a fixed intervention device/rule, which is predefined in the contract.

Our work uses the payment to control the carrier's behavior, which consequently affects the resource allocation. The payment is dynamic based on the quality of service rather than byte-counting. This idea has some similarity with the Smart Data Pricing (SDP), which uses price as a way to manage and control congestion [26]. SDP charges the end-user based on the quality of experience (QoE). It tries to match the cost of delivering application-specific desired QoE requirements of the user to the operator's congestion cost at the time of delivery. End user can control and manage the physical layer resources using some application layer software, either manually or automatedly. For example, since different applications may have different bandwidth and delay requirement, end users can specify different QoE requirements for different applications, and SDP software adjusts the physical layer resource allocation and media selection (i.e., WiFi offloading versus 3G). Since different QoE's are charged differently, this provides an incentive for end users to adjust their behavior to mitigate the network congestion [27]–[30]. Though our work also uses price to manage the interested party's behavior, different from SDP, which manipulates end users' (mobile users) behavior, our work is to manipulate the operators. Another major difference is that our work does not explicitly specify different QoE requirement at the application level.

Our work models a futuristic cellular network which a mobile has freedom to connect to any service provider based on its traffic requirement and quality of services provided by different service providers. Similar ideas are also proposed in [8]–[12], where a wireless user is able to split jobs among multiple service/network providers, and the service providers compete to get the jobs from the wireless users by sending bids. In these works, either the bids dynamically converge to the optimal [8], or the users evaluation is known [9]. In our work, the bids are about the channel state parameter, and they are fixed when the channel statistic parameter does not change. Moreover, we assume the traffic requirement of the mobile user is private information and not revealed publicly.

This work is a significant extension of our previous work [34]. In [34], we consider a simple single transmitter two carriers problem, and in each time slot, both carriers send a binary bid to the transmitter and the transmitter either allocates full power to the higher bidder or splits power equally and allocate half to each carrier. The transmitter gives a reward proportional to the amount of data transmitted for a successful transmission and gives a penalty for an unsuccessful transmission. We have proved that there exists a penalty setting such that the total rate obtained from the game is at least half of the optimal, and this is the best bound for the worst case considering all possible parameter settings. This technical report extends the single bit bid to a multiple-bit bid. The game is redesigned from a different point of view. The key idea of [34] is to find out suitable penalty settings such that PoA from the transmitter's point of view will be bounded, while in this technical report, the key aim is to guarantee truthfulness. A convex piecewise linear function is proposed to ensure truthfulness of the carriers when power is allocated. We show here that the optimal is achieved when the length of quantization interval approaches 0 with light weight computation complexity. Moreover, this work is more general, which the number of carrier can be arbitrary, and power/rate allocation is more flexible based on any given concave power-rate model, while in [34], we only focus on two carrier, equally-split power case.

IV. System Model

We consider the transmission problem of a single mobile device which can simultaneously transmit data over multiple channels provided by multiple operators. In this technical report, we call such a mobile device as the transmitter and different operators who own different channels as carriers. We study in this technical report a simple yet fundamental rate allocation problem in which a transmitter does not know the states of the channels, and the corresponding carriers are self-interested.

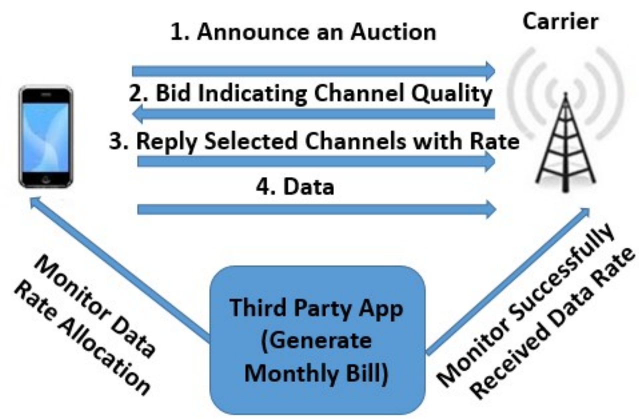

Figure 2: System Model

The system works as shown in Fig. 2: when the transmitter has some data to sent, it announces an auction to request channel resources. The nearby carriers reply to the request with a bid indicating the quality of their respective channels. After receiving the bids, the transmitter ranks the bids, estimates the channel quality, selects a set of channels, determines power and data rate allocation strategies, replies to the selected channel with allocated data rate, and then transmits data. Whenever data is successfully sent, the transmitter gives credits to the carrier. However, we assume that there is a failure and nothing gets sent if the transmission channel is bad but the transmitter uses a high data rate to send data. In this case, the carrier incurs a penalty, and returns credits to the transmitter. The transmitter stays with the same set of channels and use the same power and data allocation strategy until it finishes transmitting or announces another auction. We call this one request-reply cycle one auction cycle.

The key elements to determine the transmitter's allocation strategy is the bids from the carriers, and its own traffic requirement. The objective of the transmitter is to efficiently allocate power and data rate to meet the traffic requirement. When the traffic is heavy, the transmitter has to maximize the total data rate under a given power constraint to best meet its traffic requirement. However, if the traffic is light, a few or even a single channel can satisfy the transmitter's traffic requirement, in which case, the transmitter may only use a small set of channels.

In our mechanism, each auction cycle can be considered as a single shot game. A carrier who gets some data from the transmitter during one auction cycle may not necessarily get data from another auction cycle. We assume that the transmitter's total traffic is the transmitter's private information, and is not revealed to the carriers. Moreover, A carrier is not able to estimate such information based on the amount of data allocated to it (0 if its channel is not selected). In practice, especially in a relatively stable environment, the transmitter may select the same/similar sets of channels in different auction cycles.

To monitor the carriers' performance and generate the payments, a third party (i.e., service broker) software is installed in the transmitter side and the carrier side. The transmitter side software monitors the bids received from multiple operators and how the mobile device splits data stream. The carrier side software monitors the bids sent and the successfully received data. As noted before, credits are converted to financial payments over a longer time period, such as monthly billing.

To emulate the real system, we have the following assumptions:

•**A1:** A transmitter is able to transmit data simultaneously over at most K parallel channels.

•**A2:** There are N carriers (channels) competing to get data from the transmitter. Typically, $N > K$. N is a variable, which changes with locations and time. Different areas may have different number of carriers nearby. Even if in the same location, a carrier may still choose not to participate in the competition for some auction cycles depending on its available resources.

•**A3:** Channel quality dynamically changes over time due to the transmitter's movement, environment changes, and many other reasons. However, in one auction period, we assume that the channel quality stays the same.

•**A4:** Transmitter's traffic requirement dynamically changes. Based on the traffic requirement and channels' quality, the transmitter may simultaneously be able to connect k channels, $k < K$, and k is a variable. In one auction period, we assume that k stays the same.

•**A5:** It is difficult for a carrier to monitor other carriers' channels' quality, or estimate the transmitter's traffic requirement. A channel which is not selected by the transmitter may not necessarily be due to bad channel quality, it is also possible that the transmitter does not need that many channels. Thus, the history helps neither in estimating other carrier's channels, nor in estimating the transmitter's selections. If a carriers gets some data from the transmitter, all it knows is that it is selected during this auction cycle; what is the percentage of the allocated data among the total traffic is unknown.

•**A6:** The carriers are risk averse, which in our context means the carriers tend to choose actions which may give a possibly lower, but a more quantifiable expected payoff rather than choose actions which give unquantifiable payoffs. Although the latter actions may sometimes give high returns, there also exists the risk to get a lower or even negative expected reward.

V. Power-Rate Model and Optimal Strategy for a Transmitter

Before we get to describe the auction and mechanism design, we first take a look at what the optimal power and data allocation strategies are for a transmitter given perfect information about multiple parallel channels.

A. Power-Rate Model

The expected data rate is a concave function of the power. We assume that there are two sets of rate allocation strategies depending on the channel state, shown in Fig. 3. A channel in high state (i.e., the channel noise level is low) allows the transmitter sending data aggressively, shown in the upper curve f_h, while a channel in low state (i.e., the channel noise level is high) only allows the transmitter sending data conservatively, shown in the lower curve f_l. We assume that if data rate allocated based on the curve f_h, the transmission will fail if the channel condition turns out to be in low state and nothing gets sent (i.e., the noise level is too high so that $SNR < SNR_{outage}$, the data got corrupted). However, if data rate is allocated based on curve f_l, the transmission will be always successful no matter what the channel states are. Whenever data are successfully sent, the transmitter rewards the carrier some credits, and the carrier incurs a penalty and returns credits to the transmitter for a failed transmission.

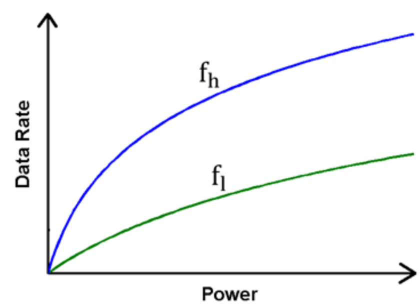

Figure 3: Data vs Power allocation

B. Optimal Power and Data Allocation

We assume the transmitter is able to simultaneously transmit data over at most K channels, as shown in Fig. 1. The indices of the carriers represent their bid rankings. For example, carrier 1 is the one who sends the highest bid.

The channels from the transmitter to each carrier are independent stochastic channels with two states: high or low. We assume that the channel states are independent and identically distributed (i.i.d.) Bernoulli random variables. Let $p_i\,(i = 1, 2, ..., N)$ be the probability that channel i is in state high during this auction cycle.

The transmitter's maximum power is P_{max}, and in each auction cycle, a transmitter ranks the carriers' bids, and selects $k\,(\leq K)$ best of them, and allocates power P_1,P_2,...P_k to their corresponding channels.

$$\sum_{i=1}^{k} P_i \leq P_{max}$$

As we know, if the transmitter has the perfect information about the channels' statistics, it can allocates power and data in an optimized way. As an example, let us assume that the transmitter is able to simultaneously transmit data over at most 2 channels, and the best two channels' probabilities in high state are p_1 and p_2, and such information is known by the transmitter. We further assume that the total power used by the transmitter is 1, and let P denote the portion of power allocated to carrier 1; the power allocated to carrier 2 is $1-P$.

Depending on the traffic τ, we can always scale up/down the total power.

The transmitter can select allocation strategies from the following: both channels' allocated data rates are based on the curve f_h; channel 1's data rate allocation is based on the curve f_h, while channel 2 data rate allocation is based on f_l; both channels' allocated data rates are based on the curve f_l.

Let $V_{opt}(P)$ denote the maximum expected data rate given that power allocated to carrier 1 is P, and let V_{opt} denote the maximum expected data rate of optimal strategy.

$$\begin{aligned} V_{opt}(P) &= max\{f_l(P) + f_l(1-P),\, p_1 f_h(P) + f_l(1-P), \\ &\quad p_1 f_h(P) + p_2 f_h(1-P)\}. \end{aligned}$$

$$V_{opt} = \max_{P\in[0,1]} \{V_{opt}(P)\}.$$

Let π_{opt} denote the allocation strategy which obtains V_{opt}.

As an example, we assume that $f_h = 10log(1+100P)$, $f_l = 10log(1+2P)$, the optimal expected data rate V_{opt} is as Fig. 4 shown; the optimal power-rate curve selection is as Fig. 5 shown; and the optimal power allocated to channel 1 is as Fig. 6 shown.

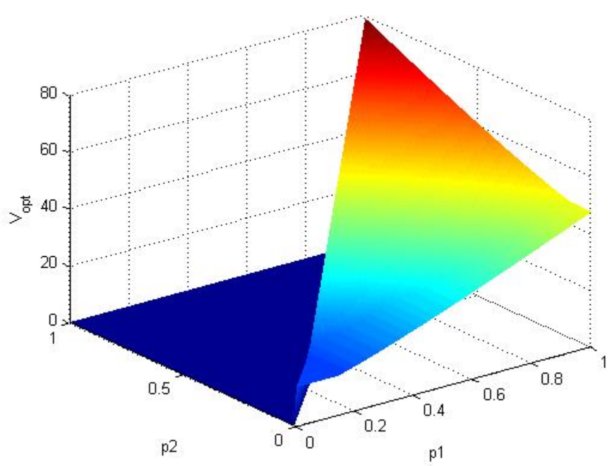

Figure 4: Optimal power and data allocation

VI. A Incentive Mechanism Design Which Ensures Truthfulness

As discussed before, to optimally allocate power and data, the key is to know the channel statistic parameters. However, only the receivers (carriers) are able to accurately estimate such parameters. How to ensure truthful bidding is the main focus in this section. We present below the details of our proposed incentive mechanism which guarantees truthfulness.

As introduced before, after the transmitter announces an auction, the nearby carriers will respond to the auction with a bid. We assume that the number of bits per bid is l; and the

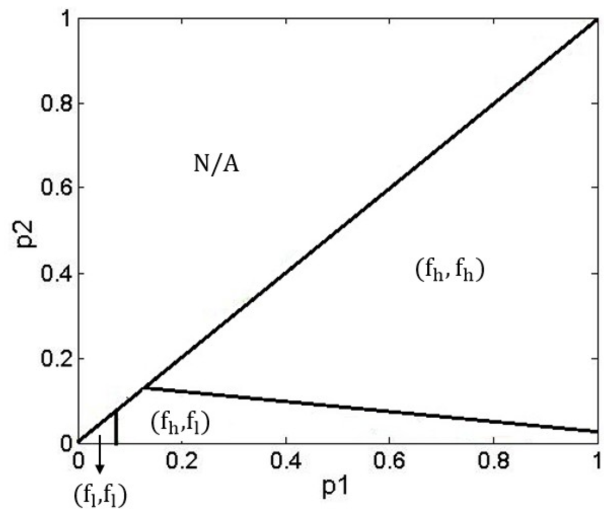

Figure 5: Optimal Power-Rate Curve Selection

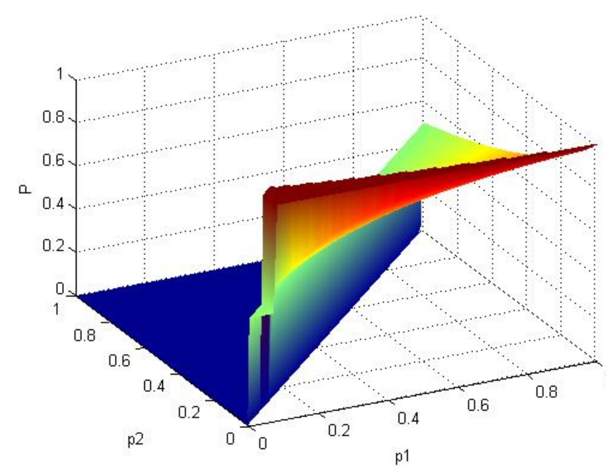

Figure 6: Optimal Power Allocated to Channel 1

whole probability interval then are divided into $n = 2^l$ smaller intervals, denoted by $[0, \alpha_1], [\alpha_1, \alpha_2]...[\alpha_{n-1}, \alpha_n]$. [1]

Each small interval $[\alpha_i, \alpha_{i+1}]$ is assigned a different reward and penalty value. The carriers' bids are based on these values and their true channel statistics.

We design the rewards and penalties based on a convex function

$$f(p) = R_x(1+\beta)^{p-1}\ (\beta > 0;\ p \in [0,1]),$$

where R_x is the allocated data rate. Given $(\alpha_i,\ f(\alpha_i))$ and$(\alpha_{i+1},\ f(\alpha_{i+1}))$, a straight line is determined. We assume that the slope of this line is k_i, and the intercept on the y axis is l_i, then we have

[1]These α_is are predefined in the contract, they can be evenly or unevenly distributed between $[0, 1]$ depending on system requirements or conditions. For example, in an area in which there are usually many good channels available, the system designer may make α_1 a little larger, i.e., 0.5, and then divide $[\alpha_1, 1]$ in a finer manner. How to optimally select α_is is out of the scope of this technical report's discussions.

$$\begin{cases} k_i\alpha_i + l_i & = f(\alpha_i) = R_x(1+\beta)^{\alpha_i - 1} \\ k_i\alpha_{i+1} + l_i & = f(\alpha_{i+1}) = R_x(1+\beta)^{\alpha_{i+1}-1} \end{cases}$$

Solving the linear equations, we get

$$\begin{cases} k_i & = \frac{R_x[(1+\beta)^{\alpha_{i+1}-1} - (1+\beta)^{\alpha_i - 1}]}{\alpha_{i+1}-\alpha_i} \\ l_i & = \frac{R_x[\alpha_{i+1}(1+\beta)^{\alpha_i - 1} - \alpha_i(1+\beta)^{\alpha_{i+1}-1}]}{\alpha_{i+1}-\alpha_i} \end{cases}$$

Since $f(p) = R_x(1+\beta)^{p-1}$ is a convex function with respect to p, the slope of the line keeps increasing when the index i increases. The set of these line segments compose a convex piecewise linear function, as shown in Fig. 7.

If a carrier bid is i, the rewards are given as follows:

$$R = \begin{cases} R_x & \text{if } R_x \text{ is allocated based on } f_l \\ k_i + l_i & \text{if } R_x \text{ is allocated based on } f_h \text{and succeed} \\ l_i & \text{if } R_x \text{ is allocated based on } f_h \text{and fail} \end{cases}$$

If the channel probability is p, and R_x is allocated based on f_h, the expected reward for bidding i is $k_i p + l_i$.

For allocating data rate based on curve f_h case, the set of all the expected rewards for bidding truthfully compose a convex piecewise linear function, shown in Fig. 7. The lines in different colors represent the expected payoffs for different probability intervals.

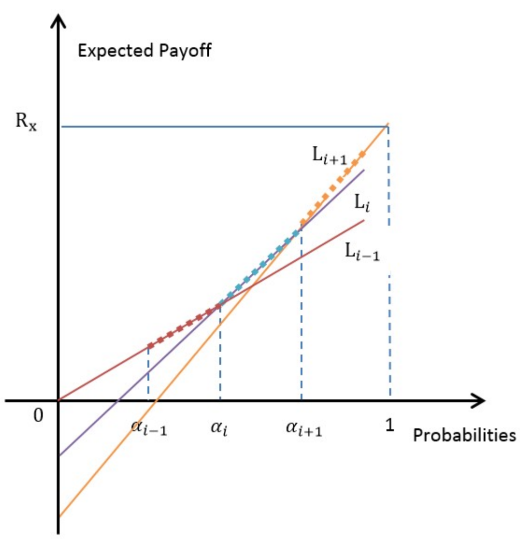

Figure 7: Payoff vs Probabilities

Theorem 1. *If the expected payoff of truthful bidding is a convex piecewise linear function with respect to channel probability p, the carriers will bid truthfully.*

Proof: If the transmitter uses the lower curve f_l to allocate power and data to a channel, no matter what the carrier bids, the allocation is the same, so is the expected payoff.

If the transmitter uses the upper curve f_h to allocate power and data, assume that a carrier's channel probability p and $p \in [\alpha_i, \alpha_{i+1}]$, with probability p, it can successfully receive the data, and with probability $(1-p)$, it will fail. Let u_i denote the payoff that it bids i, the expected payoff for bidding i is $E[u_i] = k_i p + l_i$, demonstrated by line L_i in Fig. 7.

A truthful bidding for this carrier is i. However, we assume that it bids m instead.

If $m > i$, the reward and penalty assignment will be based on line L_m. The expected reward for this carrier will be $k_m p + l_m$.

From Fig. 7, considering the line segments in the range of $[\alpha_i, \alpha_{i+1}]$, we can easily see that the expected payoff of L_i is greater than L_m. Thus, overbidding gives a lower expected reward, and a rational carrier should not overbid.

Similarly, we can prove that carrier i will not underbid.

For a channel which is selected, the above analysis shows that for the corresponding carrier, it has no incentive to lie about its channel quality.

For a channel which is not selected, overbidding may increase the possibility to be selected. However, under the assumption 4~6, overbidding may still not be a good idea for the following reasons. First, since the traffic from the transmitter is unknown, a very good channel may not be selected (The transmitter may not need that many channels). Second, without the knowledge of other channels, a channel who overbids may still not be selected, especially when there are many good channels participating the auction in that auction cycle. Thirdly, if a channel overbids and is selected, as a carrier, it does not know whether this is because of overbidding. It is still possible that it will be selected even without overbidding, in which case, overbidding yields less expected reward. Fourthly, in a real system, there are typically multiple transmitters requesting resources, a channel which is not selected by one transmitter may be selected by another, and a channel which is bad to one transmitter may turn out to be good to another transmitter. Thus, as a risk averse carrier, bidding truthfully is always a more preferable action. ■

We define the data rate efficiency, denoted by η, as the ratio of the total data rate obtained from the auction to the rate obtained by the transmitter under perfect information about the channel statistics.

Theorem 2. *For fixed rates setting, when the granularity of the probability range approaches 0, the data rate efficiency from the transmitter's point of view approaches 1.*

Proof: Refer to Fig. 4 and 5, if a grid covers no boundary of two regions, the power and data allocation is already optimal. However, if a grid covers the boundary of two regions, the optimal strategy is undetermined. For the latter case, assume the optimal data rate is V_{opt}, however, the transmitter chooses a suboptimal action which gives an expected data rate V_{subopt}.

Since the expected data rate function is continuous as Fig. 4 shown, and the expected data rate at the boundary is equal for both allocation strategies, there exists a grid length ϵ which makes $V_{opt} - V_{subopt} < \delta$. When $\epsilon \to 0, \delta \to 0, \eta = \frac{V_{subopt}}{V_{opt}} = \frac{V_{opt}-\delta}{V_{opt}} \to 1$.

Remark: *The key part of this incentive mechanism design is the convexity of the function. In general, any convex piecewise linear function can guarantee truthfulness.* ■

Taking the limit of the probability range parameters, the payoff design curve becomes a smooth convex function, shown in Fig. 8. The reward design will be based on the slope and intersection of the tangent line.

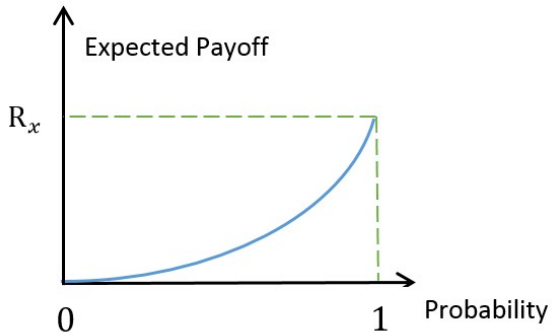

Figure 8: Expected payoff vs probabilities

VII. Implementation

In this section, we discuss some implementation issues. The following implementation is meant to be an illustrative example, it may not necessarily be the best or the unique design.

In this implementation, there is a middle man (i.e., third party software) between the transmitter and the carriers. Initially, the middle man designs the payment contract, and end-parties: the transmitter and the carriers, agree with the contract. In this example, the following items are predefined in the contract:

•The convex function which the payment functions rely on, for example

$$f(p) = R_x(1+\beta)^{p-1}\ (\beta{>}0;\ p \in [0,1]);$$

(the value of β is also predefined in the contract) .

• The probability interval boundaries $(\alpha_0, \alpha_1, \alpha_{2,} ... \alpha_n)$.

With the above information, the expected reward for a carrier is purely determined by its channel probability p and the allocated data R_x.

Once the two parties agree on the contract, they install the third party software: the transmitter side software monitors the bids received from multiple operators and how the mobile device splits data stream; the carrier side software monitors the bids sent and the successfully received data rate. Periodically, a transmitter announces an auction. The carriers response to the auction by sending a bid, indicating their channel quality. Then the transmitter selects a set of channels, determines power and data rate allocation strategies, replies to the selected carriers with the data rate it will use and then starts transmitting. A dynamic reward is determined based on the successfully received data rate. The reward can be positive or negative depending on the performance. For successful transmissions, the carriers win some credits; for unsuccessful transmissions, the carriers lose some credits (return credits back). The transmitter stays with the selected channels until next auction period. The auction period length, for example, can be as long as the coherence time. Similar to current cellular network, a monthly bill is generated based on the overall performance (i.e, the credits are converted to a financial payment over a month). Different from current cellular network payment model, in our model, the price is always dynamic depending on the channel quality and traffic requirement [2].

Here are pseudo code of the transmitter side software and the carrier side software: Algorithm 1, and Algorithm 2.

Algorithm 1 Pseudo Code of the Transmitter Side Software

while There are some data to be sent **do**

Announce an Auction: The transmitter broadcasts an auction notification, requesting channel resources from nearby carriers.

Power and Rate Allocation: After receiving bids from nearby carriers, the transmitter ranks the bids, $b_1 > b_2 > ... > b_n$, converted the bids to probability values, $\hat{p_1} > \hat{p_2} > ... > \hat{p_n}$ (i.e. it randomly picks up a probablity value corresponding to each bid, for example, it randomly picks up a probability value in $[\alpha_i, \alpha_{i+1})$ if the received bid is i), selects a set of channels, and allocates power and data rate optimally based on $\hat{p_i}$ to best serve the traffic requirement τ under the total power constrain P_{max}, let (P_i, R_i) denote the power and data rate allocated to the i^{th} highest bid channel.

Reply Bids: The transmitter replies to the selected carriers with the data rate R_x and whether it is an aggressive rate or conservative rate.

Transmitting: Transmit data R_i using power P_i.

end while

Algorithm 2 Pseudo Code of the Carrier Side Software

Initialization:

• The convex function $f(p)$;

• The probability interval boundaries: $\alpha_0, \alpha_1, \alpha_2, ...\alpha_n$.

while Listening to the auction notifications **do**

if An auction notification is received, and it decides to participate in the auction **then**.

Reply the Auction: The carrier replies to the transmitter's auction with a bid, for example, i.

Receiving Data from the Transmitter: Calculate the credits based on the performance and the bids using the proposed auction model.

end if

end while

Generating Monthly Bill: A monthly bill is generated based on accumulated credits.

The duration of the auction cycle can be made dynamic, and it could depend on various issues such as overhead, protocol/standard constraints on signaling and control frequency, and also performance. For example, in a stable environment, we can use coherence time as the time length of the auction cycle. However, in a highly varying environment, we can use

[2]Note that several other researchers have previously proposed and explored various dynamic pricing schemes for cellular networks [39]–[42] .

the rate adaptation mechanism in cellular networks such as using different coding or modulation mechanisms[3] and we can make the auction cycle longer to tradeoff overhead reduction for performance reduction, and the bids are about the average channel quality.

Regarding to the overhead introduced by this system, it is small. First, the communication of the bids are small control messages, and takes very little time to exchange. Such a communication only happens at the beginning of the auction cycle. In a stable environment, after a short time to establish the channel selection, the transmission can last for a while. In a highly dynamic environment, it is possible that the channel changes to a different state after or even during the small control messages exchange. However, in such a rapidly changing environment, it would certainly be very challenging to have an optimal solution. The auction cycle duration can be optimized but this is outside of the scope of this study. Second, since the system is about a multi-carriers scenario, it requires a good design and deployment of control signaling channels for efficient data transmission control and the overall system performance, we adapt the approaches proposed in carrier aggregation [35]. For example, in LTE/LET-Advanced systems, with a minor modification of the contritol structure in LTE systems, each carrier can have its own coded control channel [35], [36].

VIII. Simulations

In this section we first present the convergence of data rate efficiency for the worst case setting, then we use a data set of real BS locations over an 2km× 2km area in London to conduct two sets of simulations: the single operator contract model (SOCM) and our proposed auction model (AM). The transmission in this simulation is time slotted: at each slot, the transmission will fail with the corresponding probability. In practice, we can also use $\frac{m_t}{m_r}$ to approximate p, where m_t is the amount of data allocated and m_r is the amount of data received. For bidding i, the expected payment will be $k_i p + l_i = k_i \frac{m_t}{m_r} + l_i$.

A. Worst Case Data Rate Efficiency

We consider the two carriers model; each carrier knows its channel parameter. To show the effectiveness of the multiple-bit bids, we use a simplified power-rate allocation model, in which either the full power is allocated to a single channel, or equally split between two channels. We use the possible data rate of LTE network and select $R_0 = 50$, $R_1 = 3500$, $R_2 = 6144$ in the unit of Kb/slot as our rate parameters [38]. R_0 is a conservative rate given half of the power is allocated to the channel; R_1 is an aggressive rate given half of the power, and R_2 is a very aggressive rate given the full power. We consider the whole probabilities range is divided into $[0, \frac{R_0}{R_1}]$, and small equally-divided intervals [4]of $[\frac{R_0}{R_1}, 1]$. We select the worst data rate efficiency η among all possible probabilities setting with increased number of bits per bid. The result is shown in Table I. We can see that the worst case data rate efficiency η converges to 1 with increased number of bits per bid.

Table I: Worst Case Data Rate Efficiency

Bits	1	2	3	4	5	6	7	8
Worst η	0.58	0.60	0.62	0.67	0.75	0.84	0.94	0.96

B. Performance Evaluation

In this section, we evaluate the performance of our proposed aucion model by comparing it with the commonly used single operator contract model. We use the data set from Ofcom's Sitefinder [37], and obtain precise coordinates of BSs from two major operators over an 2km × 2 km area in London as shown in Fig. 9. There are 158 BSs (marked as blue) from Operator 1 and 128 BSs from Operator 2 (marked as red). We evaluate the mobile device's throughput as well as the carriers' net profit.

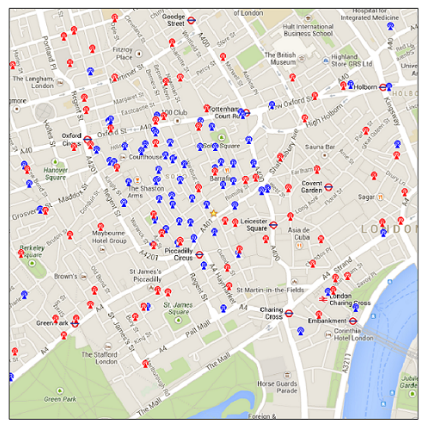

Figure 9: 2km × 2km view of the BS deployment by two major cellular operators over an area in London

1) Channel and Power-Rate Model:

We use the simple path loss model with log-normal shadowing and fading to model the channel. The received power in the unit of dB can be obtained from the following equation:

$$Pr = Pt + L - 10\eta log(\frac{d}{d_0}) + \psi,$$

where P_t is the transmission power, L is a constant, η is the path loss exponent, and ψ is the log-normal shadowing and fading, $\psi \sim N(0, \sigma^2)$ in the unit of dB . We set $P_t = 24dB$, $L = -34dB$, $\eta = 3.5$, $d_0 = 1m$, and $\sigma = 10$ in our simulation.

[3]In current cellular network, a mobile device typically uses one channel at a time, while our work supports transmitting over multiple channels simultaneously.

[4]For two carrier, equally-split power scenario, $\frac{R_0}{R_1}$ acts as a threshold. Take a single-bit bid as an example, when channel probability $p < \frac{R_0}{R_1}$, the dominant strategy for a carrier is to bid low, and when $p > \frac{R_0}{R_1}$, the dominant strategy is to bid high. More details can be found in reference [34].

We assume the noise power is $N = -100dB$, and ignore the interference from mobile devices in neighboring cells. We use signal to noise ratio (SNR) to measure the receiving signal's quality.

$$SNR = Pr - N = 90 - 35log(d) + \psi.$$

We assume SNR outage capacity $SNR_{outage} = 10dB$. When $SNR \geq SNR_{outage}$, we consider the channel is in state high, otherwise, the channel is in state low. The probability that a channel is in state high is

$$p = p(SNR > SNR_{outage}).$$

As to the power-rate model, We assume the total power is 1, and use the following equations:

$$\begin{aligned} f_l &= 10log(1 + 100P) \\ f_h &= 10log(1 + 2P). \end{aligned}$$

2) Single Operator Contract Model vs Auction Model: SOCM is the commonly used model nowadays. In this model, each transmitter contracts with a single carrier and is only allowed to connect to a single BS from the carrier it is bound to. Since the transmitter allocates all power to a single channel, plug $P = 1$ in f_l and f_h, we get rates $R_l = 11$, and $R_h = 46$ in the unit of kb/slot. When $p < \frac{11}{46}$, the transmitter transmits with rate R_l, otherwise, with rate R_h. We sample 100 customers from Operator 1 and 100 customers from Operator 2 located uniformly at random in this area. The payment to the carriers is proportional to the amount of the transmitted data. We assume that it is 10^{-4} in the unit of \$/kb (\$10 for 1 Gb data). Consequently, the payment is $u = 10^{-4}R_x$ in the unit of \$/slot. AM is our proposed model. In this evaluation, we consider $K = 2$ case. We sample 200 customers placed at the same random locations as in the single carrier contract model and use 8-bit bids: the whole probability range are evenly divided into 2^8 smaller intervals; the payment is based on $f(p) = 1.1^{p-1} \times 10^{-4}R_x$ $(p \in [0, 1])$ in the unit of \$/slot.

The simulation results are shown in Table II. We can see that our proposed model provides a win-win strategy to solve the wireless bandwidth thirst problem. The average throughput of a transmitter almost doubles while paying less per byte, and the operators make more revenue due to more potential customers and more efficient use of the spectrum by mainly serving the nearby mobile devices.

Note that the actual contract adopted in practice may depend on other market factors, but these examples show the overall benefit of carrier flexibility to both users (in terms of increased throughput and possibly reduced marginal cost) and operators (in terms of increased profit). We believe that the gain can be even more significant in future wireless system with greater carrier diversity and higher traffic.

IX. Conclusion

We have presented and investigated a competitive rate allocation game in which multiple selfish carriers compete to carry data from a transmitter in exchange for a payment. We have shown that even if the transmitter is unaware of the stochastic parameters of the channels, it can set rewards and penalties in such a way that the carriers' strategic bids yield an expected total rate that is close to the best possible expected total rate. The payment is designed according to a convex piecewise linear function; this design gives the incentive for the carriers to bid truthfully. With this design, even the worst case data rate efficiency from the transmitter's point of view converges to 1 for a large number of bits. Through simulations, we have compared our proposed model with the commonly used single carrier contract model, and have shown that our proposed model could be beneficial to both the mobile users as well as the operators.

Future work includes a more exact characterization of the data rate efficiency for a given number of bits, the optimal division of small intervals, and exploring more applications using this framework.

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Table II: Single Operator Contract model vs Proposed Auction Model

	SOCM		AM	
	Op1	Op2	Op1	Op2
Transmitter's avg throughput(kb/slot)	27.89	32.89	58.83	
Transmitter's avg payment per kb ($)	10^{-4}		7.2×10^{-5}(<10^{-4})	
Carrier's avg profit per slot ($/slot)	0.2789	0.3289	0.3649	0.4931

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