

Critical Transmission Range for Channel Allocation in Multi-hop Wireless Networks

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Abstract— As in other large-scale complex systems, many global properties of interest in multihop wireless networks undergo phase transitions as the local interaction parameter – the common transmission range of nodes – exceeds a critical value. It is now well-established in the literature that there exists such a critical range for network connectivity. We show that critical transmission range thresholds are also associated with conflict free channel allocation in wireless networks. When the common communication radius of nodes exceeds this value, the probability that k channels suffice drops sharply from nearly one to nearly zero. The analysis of these phase transitions provides useful lessons about energy-efficient and bandwidth-efficient operating points as well as feasible regions for such networks. Other results presented in this paper include a polynomial algorithm for one-dimensional conflict-free channel allocation (which is an NP-hard problem in the general case), and an analysis of maximum throughput in channelized wireless networks.

I. INTRODUCTION

In large-scale complex systems consisting of many component “particles,” it is known that global effects can emerge abruptly at a critical level of interactions between these components [11]. In physical systems, for example, there are abrupt phase transitions

in macroscopic properties such as ferromagnetism or superconductivity. Recent work has shown that analogous critical thresholds occur for various properties in large scale multi-hop wireless networks [1], [2], [17], [18]. In this paper we study phase transitions in the context of channelized wireless networks.

An excellent tool to help us understand threshold effects in networks is the mathematical theory of random graphs [12]. We first review the rudiments of this theory, and make the connection to our domain by describing a random graph model that is suitable for modelling multi-hop wireless networks in section II.

The most well-known example is that of network connectivity, which we discuss in more detail in section III. There exists a critical range of radio transmission for all nodes beyond which the multi-hop network is likely to be connected with high probability (nearly one) and below which the network is likely to be connected with low probability (nearly zero).

Our main findings are presented in section IV where we focus our attention on the problem of channel allocation in wireless networks, also known as the broadcast scheduling problem. While channel allo-

cation is in general an NP-hard problem, we develop a polynomial time algorithm for one-dimensional (1D) networks. We show that there are critical transition thresholds associated with channel allocation. The probability that a conflict-free allocation is possible with k channels drops rapidly from nearly one to nearly zero at a threshold value of the transmission range. The threshold range increases monotonically with k . We will show that analysis of this phase transition is helpful in determining the feasible operating conditions for the network and the average maximum throughput that can be obtained with a channelized network.

Exact analytical expressions for the critical transition point are hard to obtain for the general channel allocation problem. In section V, we study how throughput scales with the number of nodes in channelized wireless networks and provide an analysis of the threshold transmission range for a closely related property called max- k -neighbor, which is the property that no node in the network has more than k neighbors. The critical threshold for this property is identical to conflict-free channel allocation in 1D networks, and represents a good bound for 2D networks.

Finally, we conclude with a discussion of the implications of our findings and future directions in section VI.

II. THEORY OF RANDOM GRAPHS

Phase-transition behavior has been known to mathematicians for several decades in the form of “zero-one” laws in random graphs [12], [13]. These laws show that the probability that a randomly generated graph exhibits certain monotone properties asymptotically transitions sharply from zero to one at some critical average density.

A. Random Graph Models with Independence

A random graph G is a graph that is generated by some prescribed random experiment. The description of the random experiment constitutes a *random*

graph model. These models contain a tuning parameter which varies the average graph density. The most widely used model is the Bernoulli random graph:

Bernoulli random graph $G(n, p)$: given n , and p , construct G with n vertices such that there is an edge between any two pairs of nodes with probability p . When $p = 0$, the resulting random graph has no edges, while when $p = 1$, we always get the complete graph K_n . As the parameter p increases from 0 to 1, the average density of the random graphs increases (see figure 1 (a)).

There is considerable literature on the properties of these random graph models [12]. These models are generally tractable to analysis because they show strong independence properties. In the Bernoulli model, for example, the probabilities of any two different edges being part of a generated graph are independent of each other.

B. Phase Transitions in Random Graphs

To quote Bollobás : “one of the main aims of the theory of random graphs is to determine when a given [graph] property is likely to appear... Erdős and Rényi were the first to show that most monotone properties appear rather suddenly.” [12].

In 1996, Friedgut and Kalai proved an extremely strong result: they showed that in fact *every* monotone graph property undergoes a sharp transition on Bernoulli random graphs [14]. Let $\mu_p(A)$ be the probability that a monotone property A is satisfied by $G(n, p)$, and c_1 a universal constant, then the following is their result:

Theorem 1: If $\mu_p(A) > \epsilon$, then $\mu_q(A) > 1 - \epsilon$ for all $q > p + c_1 \log(1/2\epsilon)/\log(n)$.

Proof: See [14]. □

C. Random Graphs in Wireless Networks

We now describe a random graph model that is useful in studying multi-hop wireless networks – the

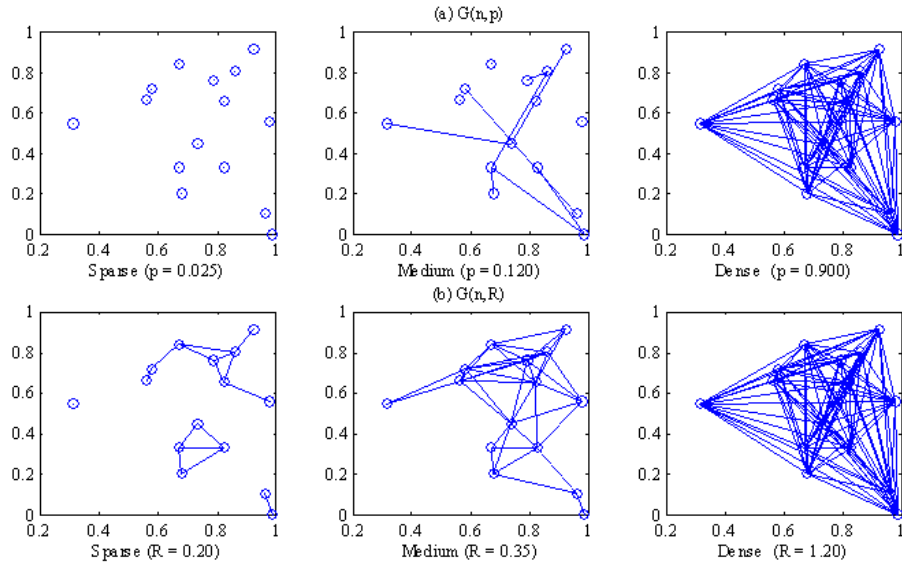


Fig. 1. Random graphs generated by (a) the Bernoulli model and (b) the fixed radius model

fixed radius model:

Fixed radius random graph: Consider an ad-hoc wireless network with n nodes, located randomly in a square area of unit size, each of which is assumed to transmit with a fixed radio power in an idealized environment. Two nodes can communicate directly with each other if and only if they are no more than a distance R apart. We can now talk about the underlying communication graph $G(n, R)$ in this fixed radius model. Let each node in the network correspond to a unique vertex in $G(n, R)$, with an edge between a pair of vertices if the corresponding nodes can communicate with each other at a particular instant. Figure 1 (b) illustrates sample random graphs generated according to this model. As the parameter R increases from 0 to $\sqrt{2}$, the average density of the random graphs increases.

It is important to note that a random graph generated using the fixed radius model has some different characteristics than the Bernoulli random graph. Consider three nodes i, j , and k . In the fixed radius model, “the event that there are links between i and j and j and k is not independent of the event that there is a link between i and k ... as the former is true given the latter only if j lies in the intersection of two discs of radius $[R]$ centered at i and k ” [1]. Still, there are clear similarities between the two

models. The density parameter p in the Bernoulli random graph is analogous to the parameter R in the fixed radius model. While there hasn’t been much work in this direction to our knowledge, we believe it may be possible to extend results from the literature on Bernoulli random graphs to the fixed radius random graphs that are of interest to us in the context of wireless ad-hoc networks. In particular, we formulate the following conjecture as a challenging open problem with great importance to the area of wireless networks:

Conjecture: ALL monotone graph properties undergo sharp phase transitions in the fixed-radius random graphs $G(n, R)$ in the following sense. Let $\mu_A(R)$ be the probability that $G(n, R)$ has the property A , and $R_\epsilon, R_{1-\epsilon}, R_c$ be radii such that $\mu_A(R_\epsilon) = \epsilon, \mu_A(R_{1-\epsilon}) = 1 - \epsilon, \mu_A(R_c) = 0.5$; then $\lim_{n \rightarrow \infty} \frac{|R_{1-\epsilon} - R_\epsilon|}{R_c} = 0$.

III. CONNECTIVITY

An important global property for a multi-hop wireless network is connectivity - does there exist a multi-hop path between every pair of nodes in the network?

Gupta and Kumar show in [1] that if n nodes are

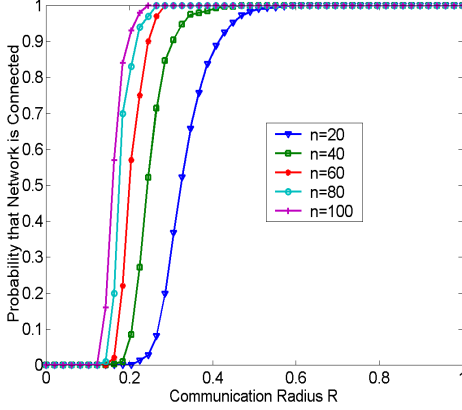


Fig. 2. Phase transitions for network connectivity in 2D fixed-radius model

placed uniformly and independently in a disc of unit area in $\mathbb{R}^2$, and each node transmits at a power level so as to cover an area of $\pi R^2 = (\log(n) + c(n))/n$, then the network is connected with probability asymptotically tending to one if and only if $c(n) \rightarrow \infty$. Xue and Kumar show in [2] that for n nodes placed uniformly iid in a unit square, the network is connected with high probability when each node is connected to $\Theta(\log(n))$ neighbors.

There is, in fact, a phase transition that takes place around this critical transmission power level. This behavior can be seen in figure 2 which depicts simulated results from the 2D fixed-radius model. The plot on the left shows curves for the probability that the network is connected, as the communication radius R is increased. The probabilities are determined in a frequentist manner, each point evaluated by testing 100 random graphs. It is desirable to minimize the energy consumption of the wireless nodes, but if the node transmission powers are decreased below the critical level, there is a precipitous drop in the probability that the network is connected. Hence, from a power-conservation perspective, the point just to the right of the phase transition region is the ideal operating point for the system.

Sanchez *et al.* showed in [16] that this critical transition behavior for connectivity is robust with respect to different mobility models.

The proofs given for the result in [1] and [2] are

somewhat laborious, involving techniques from continuous percolation theory [15]. To facilitate an understanding of the underlying behavior, we can consider a simpler 1-dimensional model which yields an analogous result. Let the nodes be distributed with a Poisson arrival rate λ on $\mathbb{R}^+$, with the first node at the origin. We may ask the question “what is the probability $P_n(R)$ that the first n nodes form a connected network if nodes within a range R can communicate with each other?” This is equivalent to the probability that the first $n - 1$ inter-node distances (which are exponential distributed and hence satisfy the memoryless property) are less than R :

$$\begin{aligned} P_n(R) &= \prod_{i=1}^{n-1} (1 - e^{-\lambda R}) \\ &= (1 - e^{-\lambda R})^{(n-1)} \end{aligned} \quad (1)$$

We verify below that asymptotically when $R = R_c = \left(\frac{\log(n)}{\lambda}\right)$, P_n is a non-zero constant. In other words R_c is the threshold value for connectivity in this simplified model:

$$\begin{aligned} \lim_{n \rightarrow \infty} P_n \left(\frac{\log(n)}{\lambda} \right) &= \lim_{n \rightarrow \infty} (1 - e^{-\log(n)})^{n-1} \\ &= \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n} \right)^{(n-1)} \\ &= e^{-1} \end{aligned} \quad (2)$$

Note that the condition $\lambda R_c = \log(n)$ is a condition in which each node has $\log(n)$ “neighbors” to its right, hence $2\log(n)$ neighbors on average. This is analogous to the condition for the threshold function derived for the 2-dimensional case in [1]. The curves obtained by plotting equation (1) with respect to R for a fixed n also show the characteristic phase transition behavior seen in figure 2, becoming sharper with n^1 .

¹One difference that should be noted between this 1-D model and the 2-D results depicted in figure 2 is that in this simple 1-D model, since we have not restricted the spatial extent within which to place all the nodes, the critical radius R_c increases with n .

Empirical results in [17] and [18] suggest that phase transitions occur not only for basic connectivity but for many other global properties in wireless networks: k -connectivity, partition into triangles, Hamiltonian cycles, and reachability with flooding. It is shown in [18] that for constraint satisfaction problems associated with such properties, the phase transition region also corresponds to a peak in average computational complexity.

We now focus on the problem of channel allocation.

IV. CHANNEL ALLOCATION

Network connectivity is a property that is satisfied when the network's communication graph is sufficiently dense. However, while increasing transmission ranges provides gains in connectivity, it has a negative effect as well – the corresponding increase in interference. As we show in this section, there are critical thresholds of interference beyond which it is nearly impossible to allocate a fixed number of channels to nodes in a wireless network. Phase transition analysis of channel allocation reveals the trade-off between connectivity and interference, and tells us about the maximum possible throughput in multi-hop wireless networks.

While most work in medium access for multi-hop networks has been devoted to random access protocols, there is sufficient motivation for studying and developing channelized medium access schemes. Random access protocols for use multi-hop wireless networks (e.g. ALOHA [3] and Busy Tone Multiple Access (BTMA) [4]) are useful for applications with bursty traffic conditions. Channelized access techniques (such as those presented in [5], [7], [8]) are better suited for non-bursty, heavy-load traffic conditions. In a typical channelized medium access protocol, the available bandwidth is divided into multiple logical channels defined on the basis of differing time slots or frequency slots. Transmit power is limited so that a given logical channel can be simultaneously used by several nodes in the network that are far enough apart to not interfere with each other. Given a network graph showing the communication (and interference) pattern between nodes, the

goal of conflict-free channel allocation is to provide time/frequency channels to each node in such a way that no nodes within two hops of each other share the same channel. This problem is also referred to in the literature as broadcast scheduling and is known to be NP-complete [7]. The two-hop constraint in this problem ensures that there is no hidden-node terminal condition, whereby some node experiences colliding transmissions on the same channel from two separate neighbors.

Figure 3 illustrates conflict-free channel allocation in example 1D and 2D networks consisting of eight nodes each. The edges between nodes represent communication links (which are also interference edges). Each node is labelled with a number corresponding to the logical channel it has been assigned. All two-hop constraints are satisfied in both examples.

A. One Dimensional Networks

As stated above, the general channel allocation problem is NP-hard. However it turns out that for 1D networks, it is possible to allocate channels with 2-hop constraints in polynomial time. 1D networks are not a trivial application since they are likely to occur in street-like environments and linearly-deployed sensor networks. To the best of our knowledge this is a new result:

Theorem 2: For a fixed radius communication graph generated from n nodes placed on a 1D line segment, the channel allocation problem can be solved in polynomial time.

Proof: We sketch a constructive proof here. For a 1D network, the following greedy algorithm provides a minimal 2-hop coloring in linear time. Let C_i be the set of colors used up to the i^{th} node, labelled in order $1, 2, \dots$. Starting with the left-most node, consider each node in order. First color node 1 with the color 1. When considering node i , first examine the set of distinct colors L_i of all its neighbors to the left, and reusing any available colors from $C_i \setminus L_i$ first, color all its neighbors to the right using colors in increasing order by number, introducing additional new colors if necessary. Now repeat this process in going

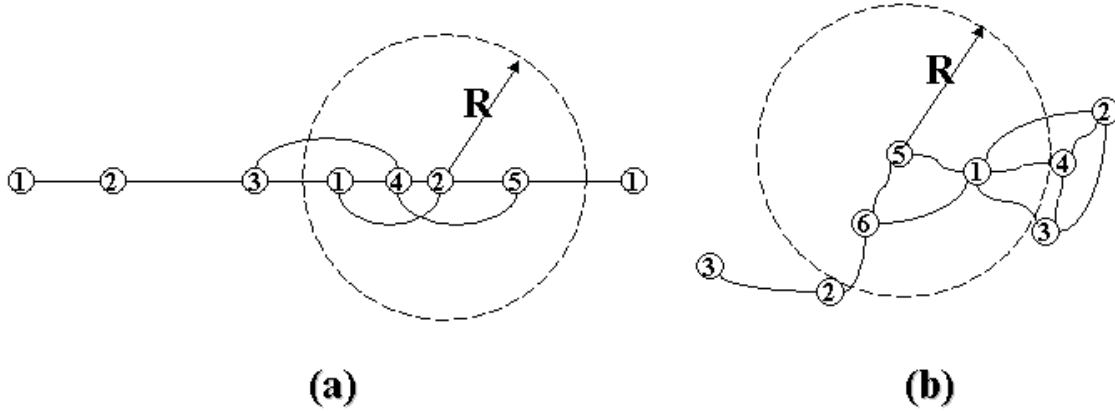


Fig. 3. Illustration of (a) 1D and (b) 2D wireless network models; nodes are labelled with sample conflict-free allocations of channels

from node i to $i + 1$. Once $i = n$, the algorithm terminates. This is a sequential greedy algorithm, and since at each step this algorithm makes the best possible decision, using no more colors than necessary, it results in the optimal answer upon termination.

This algorithm terminates in $O(n)$ time, and it is easy to see that the number of colors required by this algorithm is exactly equal to $\Delta_1 + 1$, where Δ_1 is the maximum number of one-hop neighbors of any of the nodes in the network. $\square$

In the 1D model we place n nodes randomly, independently with uniform distribution in a line segment of unit length. Figure 4 shows the phase transitions in channel allocation that occur in the 1D fixed-radius model as we vary the communication range R within which nodes can communicate. Each curve corresponds to a constant number of channels k . As the communication range is increased beyond the critical threshold, the probability that this number of channels is sufficient for allocation without violating any constraints drops sharply from nearly one to nearly zero. This happens because as the transmission range is increased, the number of edges in the network graph increases and the problem becomes over-constrained. The critical point occurs when $\Delta_1 + 1 > k$. As is to be expected, the phase transition curve moves to the right as k is increased

(i.e. when more bandwidth is added). The figure also shows the connectivity threshold for this network size ($n = 100$). Clearly only those values of k which result in phase transitions to the right of the connectivity threshold are feasible operating conditions for the network. If too few channels are available then all the constraints cannot be satisfied and there is a possibility of collisions between nodes when they transmit information.

Figure 5 shows how the average number of channels required varies with the communication radius.

If k channels are required and the available aggregate bandwidth is W , then each node can achieve a throughput of $\frac{W}{k}$. Hence the entire network would have a throughput of:

$$T = \frac{Wn}{k} = \frac{Wn}{\Delta_1 + 1} \quad (3)$$

This also gives us a way to define the maximum throughput of the network – if the average number of channels required just to the right of the 99% connectivity threshold is k_{min} , then

$$T_{max} = \frac{Wn}{k_{min}} \quad (4)$$

This expected throughput of a channelized 1D network with 100 nodes is plotted in figure 6. In this figure the throughput is normalized by the aggregate bandwidth (i.e. it shows T/W). This normalized throughput is sometimes referred to as the reuse ratio and represents how much spatial reuse is being obtained. The transmission range at which the throughput curve intersects the connectivity threshold corresponds to T_{max}/W and is thus the operating point where throughput is maximized. For a 1D model with 100 nodes, the value of this normalized throughput is between 3 and 4, indicating that the available spectrum can be reused (on average) between 3 to 4 times at most, if network connectivity is to be ensured with high probability.

B. Two Dimensional Networks

For the 2D fixed radius model the above algorithm does not hold and the problem is still NP-complete. We generated experimental results using a generic graph coloring solver (DSATUR with backtracking, maximum branching factor setting of 5 [9]) on random networks with $n = 30$ for various values of the communication radius. These are shown in figure 7. Just as in the 1D networks, we see that the probability that k channels suffice to provide a conflict-free channel allocation transitions from one to zero at a critical threshold. As the bandwidth k is increased this transition profile shifts to the right as well, supporting greater interference within the network.

Let Δ_2 be the maximum number of 2-hop neighbors that any node has in an arbitrary graph. We know that the minimum number of channels required k for a 2-hop coloring must satisfy the following bounds [10]:

$$\Delta_1 + 1 \leq k \leq \Delta_2 + 1 \quad (5)$$

These bounds are shown for the 2D fixed radius model, along with the average channels required by the computed solution, in figure 8 (averaged over 100 runs). One remarkable observation is that these average results are extremely close to the lower

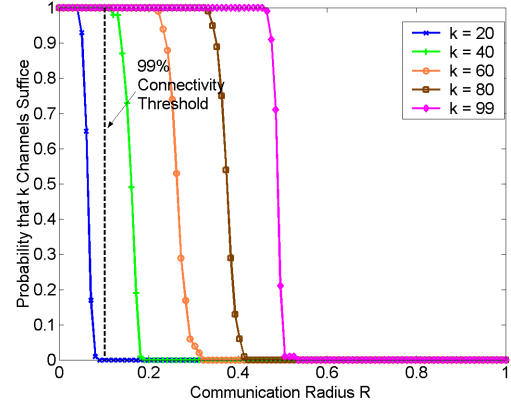


Fig. 4. Phase transitions for channel allocation in 1D model ($n = 100$)

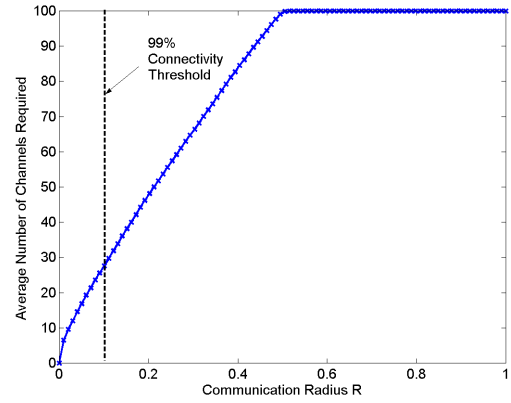


Fig. 5. Average number of channels required for conflict-free allocation in 1D model

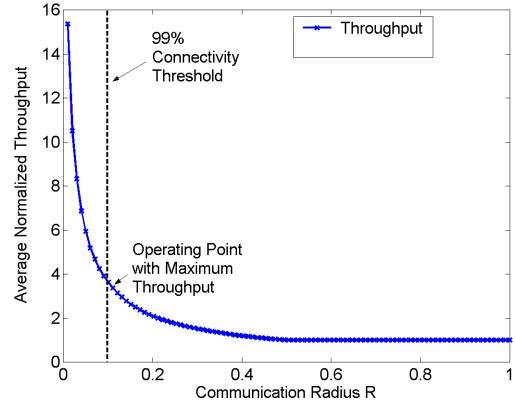


Fig. 6. Average normalized network-wide throughput (T/W) with channelization in 1D model

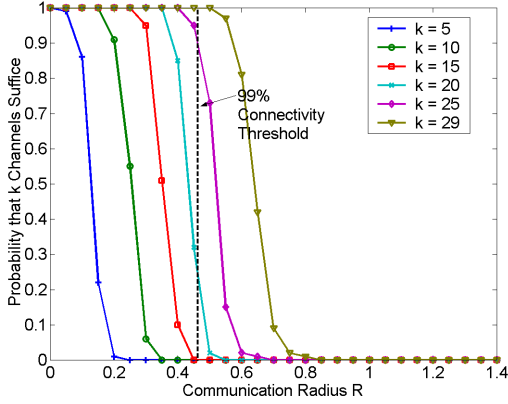


Fig. 7. Phase transitions for channel allocation in 2D model ($n = 30$)

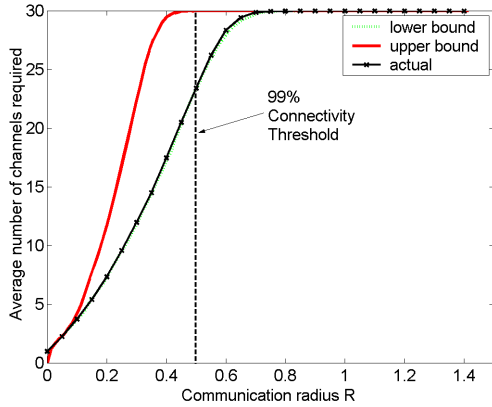


Fig. 8. Average number of channels required for conflict-free allocation in 2D model

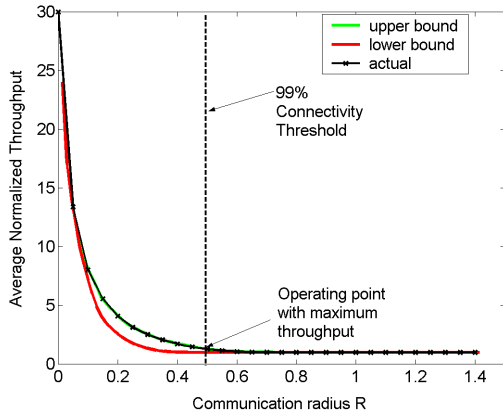


Fig. 9. Average normalized network-wide throughput (T/W) with channelization in 2D model

bound, showing that the number of channels required for channel allocation is with high probability very close to $\Delta_1 + 1$ one more than the maximum degree in the network.

The bounds on the overall network throughput T on this case, are:

$$\frac{Wn}{\Delta_2 + 1} \leq T \leq \frac{Wn}{\Delta_1 + 1} \quad (6)$$

Figure 9 shows these bounds and the corresponding experimental average throughput (T/W), along with the 99% connectivity threshold which defines the maximum throughput operating point. For the 2D model with 30 nodes, the average number of channels required at the connectivity threshold is about 25, yielding a maximum normalized throughput of about 1.25. This normalized network-wide throughput increases with the size of the network, as we shall see next.

V. ANALYSIS

We can do a first-order analysis of how the maximum throughput increases with the number of nodes n in a channelized network. We know from [2] that asymptotically a network is connected when each node has $\Theta(\log(n))$ neighbors. We now know that the maximum throughput is obtained at the range where the network is just connected. Thus the minimum number of channels required $k_{min} > \Delta_1 + 1 = \Omega(\log(n))$. Hence the normalized maximum throughput for a channelized network increases with n as $O(\frac{n}{\log(n)})$. On a per-node basis, however, it decreases as $O(\frac{1}{\log(n)})$. This is in keeping with recent results in the literature ([20], [21], [22]) showing that the capacity of nodes in a wireless network tends asymptotically to zero².

²In [20], capacity is measured in bits/meter-seconds. If we organize all nodes into nearest-neighbor sender-receiver pairs, the average sender-receiver distance decreases as $\frac{1}{\sqrt{n}}$, hence the corresponding capacity for a channelized wireless network decreases with respect to n as $O(\frac{1}{\sqrt{n} \log(n)})$.

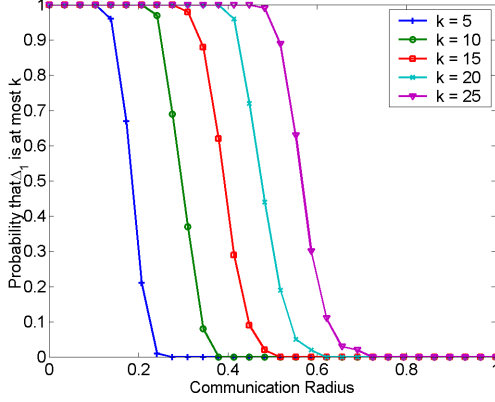


Fig. 10. Phase Transitions for max-k-neighbor

It is difficult to obtain exact analytical expressions for the critical radius at which the conflict free channel-allocation can be performed, in general. We will instead provide an analysis of the closely related property *max-k-neighbor*: the property that no node has more than k one-hop neighbors, i.e. that $\Delta_1 \leq k$. For the 1D case, as we have shown above, this is identical to the probability that $k+1$ channels suffice for channel allocation. For the 2D case, this property provides a fairly good bound, as seen in figure 8.

We first present empirical results to show the phase transition that occurs in max-k-neighbor for the 2D fixed-radius random graph model in figure 10. The figure shows the probability of this property for various k , based on simulations involving $n = 30$.

We now turn to analytical results for max-k-neighbor. Let A_i be the event that the i^{th} node has at most k neighbors. The max-k-neighbor property that all nodes have at most k neighbors is therefore the event $\{\bigcap_{i=1}^n A_i\}$. The following are simple upper and lower bounds on the probability of this event:

$$Pr(A_i) \geq Pr\left(\bigcap_{i=1}^n A_i\right) \geq \max(1 - nPr(A_i^c), 0) \quad (7)$$

We proceed to determine $Pr(A_i)$. In the case of the 2D model, once we fix the location of the i^{th} node, the other $(n-1)$ nodes are located independently with a uniform distribution in the square area. Ignor-

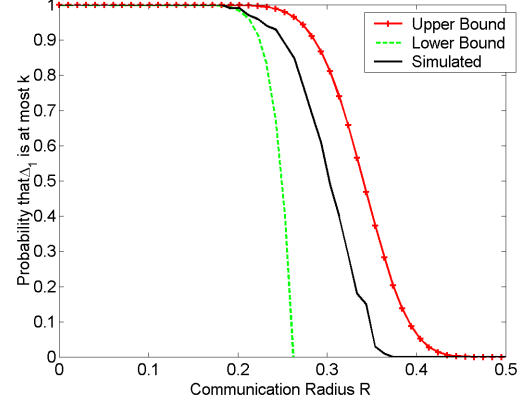


Fig. 11. Illustration of bounds for max-k-neighbor ($n = 30$, $k = 10$)

ing edge-effects (by making the boundaries of the unit area overlap), we wish to determine the probability that at most k of these $n-1$ nodes are located within distance R of the fixed node. $Pr(A_i^c)$ is therefore given by the following expression:

$$Pr(A_i^c) = \sum_{j=0}^k \binom{n-1}{j} (\pi R^2)^j (1 - \pi R^2)^{n-1-j} \quad (8)$$

The bounds in expression 7 are illustrated in figure 11 for $n = 30$ nodes, and $k = 10$.

Theorem 3: The property that all nodes have at most k neighbors undergoes a zero-one transition asymptotically at the transmission radius $R_c = \sqrt{\frac{k}{\pi(n-1)}}$

Proof: Let $R = R_c = \sqrt{\frac{k}{\pi(n-1)}}$, at which setting the expected number of neighbors for the i^{th} node is k . Let M_n be the actual number of neighbors of the i^{th} node when there are n nodes in the area, then by the weak law of large numbers [19], for any $\epsilon > 0$:

$$\lim_{n \rightarrow \infty} Pr(|M_n - k| < \epsilon) = 1 \quad (9)$$

The number of neighbors increases monotonically with the radius R , hence

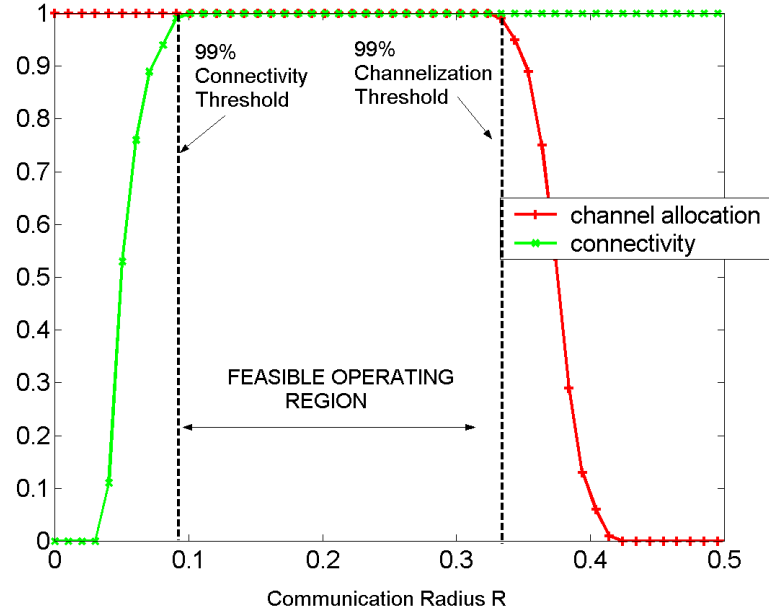


Fig. 12. Illustration of the intersection of the channel allocation and connectivity phase transitions

$$\begin{aligned} \forall R < R_c, \lim_{n \rightarrow \infty} Pr(M_n < k) &= 1 \\ &\Rightarrow \lim_{n \rightarrow \infty} Pr(A_i) = 1 \end{aligned} \quad (10)$$

$$\begin{aligned} \forall R > R_c, \lim_{n \rightarrow \infty} Pr(M_n > k) &= 1 \\ &\Rightarrow \lim_{n \rightarrow \infty} Pr(A_i) = 0 \end{aligned} \quad (11)$$

In this limit, it can be seen that the bounds in relation (7) actually coincide since $Pr(A_i) = 0$ or $1 \Rightarrow 1 - nPr(A_i^c) = 0$ or 1 . Therefore

$$\forall R < R_c, \lim_{n \rightarrow \infty} Pr\left(\bigcap_{i=1}^n A_i\right) = 1 \quad (12)$$

$$\forall R > R_c, \lim_{n \rightarrow \infty} Pr\left(\bigcap_{i=1}^n A_i\right) = 0 \quad (13)$$

□

It is straightforward to carry out the above analysis also for the 1D model, in which case the critical radius for max-k-neighbor is $R_c = \frac{k}{n-1}$. Note that since exactly $\Delta_1 + 1$ colors are needed for coloring in the 1D case, the max-k-neighbor property co-

incides with the conflict-free channel allocation of $(k+1)$ channels.

VI. DISCUSSION AND CONCLUSIONS

We have shown that there exists a critical transmission range beyond which it is rarely possible to allocate k channels in a wireless network without violating the 2-hop constraints. While the problem of scheduling channels with the 2-hop constraint is generally NP-complete, we established a polynomial algorithm for the 1D case to illustrate the phase transitions. For the 2D case we presented some simple bounds on the number of channels required. We showed that increasing the number of channels available shifts the critical threshold to the right, allowing for higher transmit powers and interference. We also showed how the throughput decreases with the transmission range in channel wireless networks. It is hard to quantify the critical threshold point in general, but we provided exact result for 1D networks and bounded it for the 2D case by analyzing the related max-k-neighbors property.

The critical thresholds for conflict-free channel allocation are best considered in conjunction with the es-

tablished results on connectivity. Our study suggests that the transmission range and bandwidth should be chosen by considering the intersection of thresholds for the two properties. This is illustrated in figure 12. We can view the results in two ways: first, for networks in which the number of channels is a variable quantity, we need to ensure that there is sufficient bandwidth to allow for conflict free allocation and connectivity. Here sufficiency is quantified in terms of the minimum value of k for which the channelization threshold is to the right of the connectivity threshold. Second, for networks in which the number of channels is a fixed quantity (e.g. certain FDMA systems), the region between the two thresholds represents the feasible region of node transmission powers that can be sustained within the network. Only within this feasible operating region will a network be both connected and have conflict-free channel allocation with high probability.

We caution the reader that the phase transition results presented here rely upon the assumption that the nodes are located at random with a uniform distribution in the area. While this is a reasonable assumption, particularly when considering static snapshots of mobile nodes with anisotropic mobility patterns or random deployment patterns for static nodes, other location distributions could be examined. It is our belief that while the details of the critical threshold point may be different, the qualitative behavior will be the same. This remains to be evaluated.

We believe that phase transition analysis is a promising approach for developing design principles and gaining a better understanding of multi-hop wireless networks, particularly in answering questions about resource-efficiency.

The simplest use of phase transition phenomena is to perform the analysis off-line to determine and set the power settings and bandwidth requirement of nodes before they are deployed in the operational environment. A more sophisticated approach would be to incorporate these results into online, adaptive mechanisms. This is an open area for research and development.

VII. ACKNOWLEDGEMENTS

We would like to acknowledge the help of several people in this work. The linear Poisson model for connectivity discussed in section III was developed jointly with Marc Pearlman. We have also benefitted greatly from conversations on the subject of phase transitions with Dr. Ramon Bejar, Dr. Bart Selman, Dr. Carla Gomes and Xi Xie.

REFERENCES

- [1] P. Gupta and P. R. Kumar, Critical power for asymptotic connectivity in wireless networks. *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming, W.M. McEneaney, G. Yin, and Q. Zhang (Eds.)*, Birkhauser, Boston, 1998.”
- [2] F. Xue and P. R. Kumar, “The number of neighbors needed for connectivity of wireless networks.” *Submitted to Wireless Networks*, 2002.
- [3] N. Abramson, “The ALOHA system – another alternative for computer communications,” *Proceedings of Fall Joint Computer Conference*, 1970.
- [4] F. A. Tobagi and L. Kleinrock, “Packet switching in radio channels, part ii: the hidden terminal in carrier sense multiple access and the busy-tone solution,” *IEEE Transactions on Communication*, Vol. 23, pp. 1417-1453, 1975.
- [5] L. Bao and J. J. Garcia-Luna-Aceves, “Collision-free topology-dependent channel access scheduling,” *MILCOM 2000. 21st Century Military Communications Conference Proceedings*, pp. 507-511, 2000.
- [6] G. Cao and M. Singhal, “Efficient distributed channel allocation for mobile cellular networks,” *7th International Conference on Computer Communications and Networks*, pp. 50-57, 1998.
- [7] R. Ramaswami and K. K. Parhi, “Distributed scheduling of broadcasts in a radio network”, *INFOCOM*, pp. 497-504, 1989.
- [8] L. C. Pong and V. O. K. Li, “A distributed time-slot assignment protocol for mobile multi-hop broadcast packet radio networks,” *MILCOM*, pp. 70-74, 1989.
- [9] J. Culberson “Iterated greedy graph coloring and the difficulty landscape ” Technical Report TR92-07, Department of Computer Science, University of Alberta, 1992.
- [10] A. Gibbons, *Algorithmic Graph Theory*, Cambridge University Press, 1985.
- [11] S. A. Kauffman, *The Origins of Order: Self-organization and Selection in Evolution*, Oxford University Press, 1993.

- [12] B. Bollobás, *Random Graphs*, Academic Press, 1985.
- [13] J. Spencer, *Ten Lectures on the Probabilistic Method*, SIAM, 1987.
- [14] E. Friedgut and G. Kalai, "Every Monotone Graph Property Has a Sharp Threshold," *Proc. Amer. Math. Soc.*, vol. 124, pp. 2993-3002, 1996.
- [15] R. Meester, and R. Roy, *Continuum Percolation*, Cambridge University Press, 1996.
- [16] M. Sanchez, P. Manzoni, and Z. J. Haas, "Determination of critical transmission range in ad-hoc networks," *Multiaccess Mobility and Teletraffic for Wireless Communications Workshop (MMT'99)*, Venice, Italy, October 1999.
- [17] B. Krishnamachari, S. B. Wicker, and R. Bejar, "Phase transitions in wireless ad-hoc networks," *IEEE GlobeCom 2001*, November 2001.
- [18] B. Krishnamachari, R. Bejar, and S. B. Wicker, "Distributed problem solving and the boundaries of self-configuration in multi-hop wireless networks," *35th Hawaii International Conference on System Sciences*, January 2002.
- [19] A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*, Second Edition, Addison-Wesley, 1994.
- [20] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.
- [21] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *INFOCOM 2001*.
- [22] J. Li *et al.*, "Capacity of Ad Hoc Wireless Networks," *MOBICOM 2001*.