Capacity Bounds For Ad-Hoc Networks Using Directional Antennas

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Abstract— **Directional antennas can be useful in significantly increasing the capacity of wireless ad hoc networks. With directional antennas, independent communications between nodes can occur in parallel, even if the nodes are within range of each other. However, mutual interference by simultaneous transmissions limits the maximum number of such concurrent communications. Furthermore, it poses bounds on the amount of capacity gain one can achieve by using directional antennas instead of omni-directional ones. These bounds depend on the specific antenna type and its parameters, as well as higher layer protocol requirements. In this paper we calculate interference-based capacity bounds for a generic antenna model as well as a real-world antenna model and analyze how these bounds are affected by important antenna parameters like gain and beam-width**

Keywords-component; — capacity; ad-hoc; directional antennas formatting;

I. INTRODUCTION

Wireless ad-hoc networks are multi-hop networks where all nodes cooperatively maintain network connectivity. The ability to be set up fast and operate without the need of any wired infrastructure (e.g. base stations, routers, etc.) makes them a promising candidate for military, disaster relief, and law enforcement applications. Furthermore, the growing interest in sensor network applications has created a need for protocols and algorithms for large-scale self-organizing ad-hoc networks, consisting of hundreds or thousands of nodes.

Until recently, most papers on wireless ad-hoc networks were assuming that all nodes are equipped with omni-directional antennas. However, during the past couple of years there has been a rapidly growing interest in the use of directional antennas in ad-hoc networks. The reason for that is the potential benefits one could have from the ability of directional antennas to concentrate the radiated power towards a specific direction. Those benefits are in terms of higher throughput [1] [2] [3] [4] [11], better energy-efficiency [5], lower interference, and more secure communications.

An important characteristic of a wireless ad-hoc network, or as a matter of fact any network in general, is its capacity. There have been quite a few papers exploring the capacity of ad-hoc networks where nodes have omni-directional antennas [7] [8], and the most well-known result is by Kumar and Gupta [6]. Although many papers following that work expanded this analysis or used simulation to confirm the results, none yet (to the best of our knowledge) has conducted any extensive capacity analysis for wireless ad-hoc networks using directional antennas.

In this paper we perform a capacity analysis for ad-hoc networks consisting of nodes that are equipped with directional antennas. We show that interference from concurrent transmissions limits the maximum achievable capacity. We provide bounds for an abstract and real-world linear array directional antenna models and show how these bounds are affected by important antenna parameters like gain and beam-width. In the next section, we briefly describe the models for the antennas we're going to assume in our subsequent analysis. The capacity analysis for these antenna models follows in section III. In section IV, we illustrate how the calculated capacity bounds are related to antenna type and parameters and provide results for a range of values for those parameters. Finally, in section V we conclude the paper and discuss open issues and future work.

II. DIRECTIONAL ANTENNA MODELS

First, we're going to consider a simple generic directional antenna model, which is usually referred to as the *flat-topped* antenna model in the literature. In this model, a specific constant gain G_1 is assumed[1] within a specific angle θ, representing the main antenna beam, and a lower constant gain G_2 is assumed for all other directions. For our analysis, we're going to assume that G_1 is equal to one and G_2 takes values between zero and one, in order to have the same transmission range as an omni-directional antenna. This could be achieved by reducing the transmitting power by a factor of $1/G_1$. We're going to call G_2 the *suppression ratio* of the antenna. This model albeit quite simplistic, can provide valuable insight on how the directional antenna characteristics affect the capacity of an ad-hoc network consisting of nodes utilizing directional antennas.

The nature of ad-hoc networks poses certain limitations on the types of antennas that could be used in that context as well as their respective parameters (i.e. gain and main beam angle). The size of the terminal (e.g. PDA, sensor, laptop) is a major restricting factor and so is the need to be able to quickly re-direct the antenna. For these reasons, we choose a simple linear

[1] The gain G of an antenna in a given direction is the ratio of the input power needed by an omni-directional antenna and that of a directional one, in order to reach the same distance.

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array antenna consisting of *N* dipoles as our real-world antenna model. We will only consider *end-fire* arrays that have a single main beam [9]. The normalized gain for an *end-fire* linear array antenna as a function of θ, the angle from the direction where the antenna is pointing, is given by

$$G_N(\theta) = \frac{\sin\left(\frac{N\pi}{4}(\cos\theta - 1)\right)}{N \sin\left(\frac{\pi}{4}(\cos\theta - 1)\right)} \quad (1)$$

III. Capacity Analysis

A. Problem Formulation

Assume we have wireless ad-hoc nodes randomly placed in an unbounded flat area, which for the sake of our analysis we consider as extending to infinity. Let ρ be the uniform node density in this area and let P_{act} denote the probability that any node is engaged in communication at any point in time. The quantity $\rho * P_{act}$ denotes the active node density, which we're going to be referring to as ρ_{act} hereafter. We assume a general, parameterized ground propagation model. If P_t is the transmitting power, G_t and G_r are the transmitter and receiver antenna gains respectively, and *d* the distance between sender and receiver then the signal power P_r at the receiver's radio is given by

$$P_r = \frac{P_t * G_t * G_r * c_\alpha}{d^\alpha} \quad (2)$$

When specific results for the two-ray ground propagation model are given, c_a is replaced by h^2, where h is the antenna elevation assumed to be equal to *1.5m* for all antennas, and the attenuation factor α is equal to *4*.

We're going to assume that a directional version of the 802.11 Media Access Control protocol (DMAC) is being implemented by all ad-hoc nodes, in order to access the shared channel using their directional antennas. This directional 802.11 protocol was independently proposed in [7] and [8]. It is essentially an adaptation of the collision avoidance and virtual carrier sensing mechanisms used in the original 802.11[10] for ad-hoc networks utilizing directional antennas to communicate. For this paper to be self-contained, we will briefly summarize the protocol features that are needed for our analysis:

- All nodes have two modes of operation, directional and omni-directional.
- When nodes are idle, they're listening to the media omni-directionally. All non-broadcast packets (i.e. RTS, CTS, DATA and ACK) are transmitted directionally.
- On reception of an RTS packet a node switches to directional mode and points its antenna back to the transmitting node, based on the direction-of-arrival of the RTS packet or knowledge of the location of that sender.
- Directional virtual carrier sensing is implemented as follows. Each node keeps a directional *NAV table* with a similar use to the NAV value in 802.11. When it overhears an RTS or CTS packet, not destined to itself, it marks the direction-of-arrival in its NAV table as "busy" for the time duration contained inside the packet.
- When a node has a packet to transmit, it checks the direction of the intended recipient in its NAV table to see whether there is any ongoing transmission in that direction. If there is, it backs off and tries again later.

B. A First Level Capacity Comparison

One important effect of using 802.11 with omni-directional antennas is the following. When two nodes, say A and B, communicate then all nodes inside a circle region of radius *R* (radio range) around each of nodes A and B, are rendered unable to communicate themselves with any other node. We shall call this region the *silence region* of nodes A and B. Consequently, if every node has a packet to send to some other node, then all nodes within range of each other will have to take turns sending their traffic. Assume there is some higher layer scheduling protocol that divides these nodes into *K* independent pairs that need to communicate with each other. Then, the available channel capacity, say *C*, is going to be effectively divided into *K* equal chunks of size at most *C/K*. If node density increases and accordingly the number of nodes in the *silence region*, the per-node allocated capacity can become infinitesimally small.

On the other hand, by using very narrow beam directional antennas, one could argue that we could isolate the *K* independent pairs from the previous example and have them all communicate at the same time. This way, each pair could utilize all the available channel capacity. Furthermore, by increasing the node density of the area, one could make the relative capacity gain from the use of directional antennas arbitrary large compared to using omni-directional antennas. This is, in general, not true, because it neglects the effects of interference. Although most power is radiated (received) through the main lobe there is still some power radiated (received) from the side lobes as well. This power is perceived as interference to (from) other communicating nodes. If the *signal-to-interference ratio* (SIR) is high enough, then the receiving node may be able to *capture* the intended signal. Nevertheless, interference is an additive (and random phenomenon). A single interfering node may not be strong enough to prevent successful communication, but the sum of all interfering signals from concurrently communicating nodes could easily garble each others' useful data. Consequently, there is a bound on how many nodes one can "pack" inside a specific area, without compromising the ability of independent pairs of nodes to communicate with each other at the same time. Alternatively, there is a bound on the capacity improvement one can achieve by using directional antennas in ad-hoc networks in place of omni-directional ones. Those capacity bounds are the focus of this paper and are going to be explored in the rest of this section.

C. Interference-based capacity analysis: Generic Antenna Model

All nodes are assumed to be equipped with a *flat-topped* directional antenna and use *DMAC* to communicate. When a node, say A, sends a directional RTS or CTS packet, all nodes inside the main beam of that node and within range *R*, are going to successfully receive the packet, provided they're not busy themselves. Those nodes will therefore refrain from any transmission towards A's direction for the duration indicated in the RTS or CTS packet and cause no direct interference. However, those nodes within range *R* are free to engage in communication towards some other direction. Therefore, they may indirectly interfere with A through their side lobes, as long as they're outside a range *R'*, which is the range at which even side lobe interference would be high enough to corrupt A's signal on its own. All the other nodes, outside range *R*, are potential direct interference sources. For nodes lying outside A's main beam, those further away than *R'* are potential direct interference sources, while those between ranges *R'* and *R''* may interfere through their side lobes. Let P_{th} be the receiver's power threshold, P_t be the transmitting power, and α be the attenuation factor. Then *R, R'* and *R''* are given by

$$R = \left(P_t * c_\alpha / P_{th}\right)^{\frac{1}{\alpha}}, \quad R' = R * \left(G_2\right)^{\frac{1}{\alpha}}, \quad R'' = R * \left(G_2\right)^{\frac{2}{\alpha}} \qquad (3)$$

In Fig. 1 we depict the areas where nodes that can potentially interfere with node A are contained.

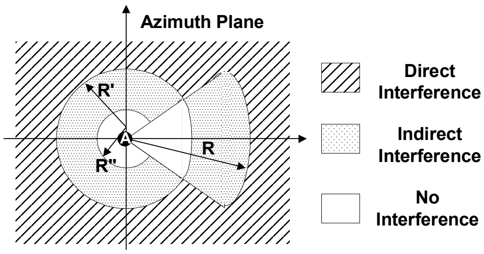

Figure 1: Area where potential interference sources lie

Those nodes that can directly interfere with A, may point their antenna to any direction with equal probability. As a result of that, the antenna gain of any interfering signal is a random variable. We choose to use the average antenna gain for the interference signal $G_I = [(2\pi - \theta) * G_2 + \theta]/2\pi$ for those nodes. On the other hand, those nodes that can only interfere indirectly have a constant interference gain $G_I' = G_2$.

We now need to calculate the total amount of interference as perceived by node A. Consider first the nodes inside the infinitesimally small arc-shaped area delimited by A's main beam and between distances *r* and *r + dr* from A. Each node in this area is going to contribute an interfering signal $I_1(r)$. Similarly, each node outside A's main beam and at distance in [*r, r + dr*] will contribute an interfering signal $I_2(r)$. $I_1(r)$ and $I_2(r)$ are given by

$$\begin{aligned} I_1(r) &= \begin{cases} (P_t * G_I * c_\alpha)/r^\alpha, & r > R \\ (P_t * G_I' * c_\alpha)/r^\alpha, & R' < r < R \end{cases} \\ I_2(r) &= \begin{cases} (P_t * G_I * G_2 * c_\alpha)/r^\alpha, & r > R' \\ (P_t * G_I' * G_2 * c_\alpha)/r^\alpha, & R'' < r < R' \end{cases} \end{aligned} \qquad (4)$$

The number of active nodes in any area of size *L* is given by $\rho_{act} * L$. Hence, the total amount of interference perceived by A, will be given by

$$I_{tot} = \theta \int_{R'}^{\infty} I_1(r) * \rho_{act} * r * dr + (2\pi - \theta) \int_{R''}^{\infty} I_2(r) * \rho_{act} * r * dr \qquad (5)$$

If the node with which A is communicating is located at distance d, then the *signal-to-interference ratio (SIR)* at A will be given by

$$SIR = \frac{P_t * c_\alpha / d^\alpha}{I_{tot}} \qquad (6)$$

We explained earlier that using directional antennas in combination with *DMAC* protocol, allows different pairs of nodes to communicate in parallel, even if they are *in range* of each other. The question is how many such transmissions like that can go on in parallel, without destroying each others' useful data.

Most radio receivers today are able to *capture* a useful signal between other interfering signals, as long as the *SIR* at the receiver is above a specific threshold level SIR_{th}, which depends on the coding scheme used. We can see from (6) that the SIR is inversely proportional to the active node density ρ_{act}. One could therefore claim that by decreasing the maximum distance *d* up to which two nodes can (on the average) successfully communicate it is possible to stay above SIR_{th} for any ρ_{act}. This way we could arbitrarily increase the total per-unit-of-area communication capacity. However, this is not accurate. Decreasing the transmission range indefinitely is not a good practice and could lead to opposite results from what one would expect by using the previous argument. The reason for that is the interaction of upper-layer protocols with the physical and MAC layer, as well as with one another. A short transmission range implies that a packet has to go through more hops to reach a specific destination incurring more processing and larger delays. A short effective transmission range implies also that each node will have to forward more traffic that doesn't belong to that node. This would result in a decrease of the available throughput and processing resources that each node has available to itself, in order to handle packets originating or destined to that node, as shown in [6].

For these reasons, we will assume that this *effective transmission range d* is dictated by higher layer protocols and will therefore be considered as given. In that case, the maximum active node density that allows nodes to successfully communicate at a transmission range of at least *d*, despite interference, is given by

$$\begin{aligned} \rho_{act}^{max} &= \frac{P_t * c_\alpha}{SIR_{th} * d^\alpha * I_K} \\ I_K &= \int_{R'}^{\infty} I_1(r) * \theta * r * dr + \int_{R''}^{\infty} I_2(r) * (2\pi - \theta) * r * dr \end{aligned} \qquad (7)$$

Consider now a circle area of radius *d* around any transmitting node X. We saw earlier that for the omni-directional case, 802.11 would create a *silence region* of at least $\pi * d^2$ around X. In other words, only one transmission could successfully take place in any area of size $\pi * d^2$. On the other hand, according to our previous analysis, if directional antennas were used instead, there could be up to another $\rho_{act}^{max} * \pi * d^2$ active nodes in the circle area around X, without disrupting X's communication. Assume that there exists some optimal scheduling algorithm that specifies

$\rho_{act}^{max} * \pi * d^2 / 2$ independent pairs of nodes, within the area of $\pi * d^2$, that can communicate with each other in parallel. Then, the capacity gain of using directional antennas in place of omni-directional ones is given by:

$$C_{MAX} = \frac{\rho_{act}^{max} * \pi * d^2}{2} \tag{8}$$

Finally, when the two-ray ground propagation model is assumed, the values for the maximum active node density and maximum capacity gain are given by

$$\rho_{act}^{max(4)} = \frac{h * \sqrt{P_t}}{2 * SIR_{th} * \sqrt{P_{th}}} * \frac{1}{d^4 * f(\theta, G_2)} \tag{9}$$

$$C_{MAX}^{(4)} = \frac{h * \sqrt{P_t} * \pi}{4 * SIR_{th} * \sqrt{P_{th}}} * \frac{1}{d^2 * f(\theta, G_2)} \tag{10}$$

$$f(\theta, G_2) = [(2\pi - \theta)\sqrt{G_2} + \theta] * [\theta + (2\pi - \theta)G_2 + (\sqrt{G_2} - G_2)]$$

D. Interference-based capacity analysis: Linear Array Antenna Model

The capacity analysis for the case of linear array antennas is similar to that for the generic antenna case. When a node transmits an RTS or CTS packet, the maximum range until which surrounding nodes can successfully receive it depends on the angle θ and we denote it as $R_N(\theta)$. Similarly, the distance up to which nodes can indirectly interfere is denoted by $R'_N(\theta)$. $R_N(\theta)$ and $R'_N(\theta)$ are given by

$$R_N(\theta) = \left(\frac{P_t}{P_{th}} * G_N(\theta) * c_\alpha\right)^{\frac{1}{\alpha}}, \quad R'_N(\theta) = \left(G'_I\right)^{\frac{1}{\alpha}} * R_N(\theta) \tag{11}$$

The average antenna gain for a direct and an indirect interfering signal G_I and G'_I, respectively are given by

$$G_I = \frac{1}{2\pi} * \int_0^{2\pi} G(\theta) * d\theta, \quad G'_I = \frac{1}{2\pi - \theta} * \int_{\theta/2}^{2\pi - \theta/2} G(\theta) * d\theta \tag{12}$$

The total interference level and the signal-to-interference ratio when using N-element linear array antennas are given by

$$I_{tot}(N) = \int_0^{2\pi} \int_{R_N(\theta)}^{\infty} \frac{P_t * G_I * G_N(\theta) * c_\alpha}{r^\alpha} * \rho_{act} * r * d\theta * dr + \int_0^{2\pi} \int_{R'_N(\theta)}^{R_N(\theta)} \frac{P_t * G'_I * G_N(\theta) * c_\alpha}{r^\alpha} * \rho_{act} * r * d\theta * dr \tag{13}$$

$$SIR(N) = \frac{P_t * c_\alpha / d^\alpha}{I_{tot}(N)} \tag{14}$$

Finally, using similar arguments as in the case of the generic antenna model we can calculate the maximum active node density $\rho_{act}^{max}(N)$ and from that the maximum capacity gain $C_{MAX}(N)$ as follows:

$$C_{MAX}(N) = \frac{\pi * W(N)}{2 * d^{\alpha-2} * SIR_{th}} \tag{15}$$

$$W(N) = \left(\int_0^{2\pi} \int_{R_N(\theta)}^{\infty} \frac{G_I\, G_N(\theta)}{r^{\alpha-1}} d\theta\, dr + \int_0^{2\pi} \int_{R'_N(\theta)}^{R_N(\theta)} \frac{G'_I\, G_N(\theta)}{r^{\alpha-1}} d\theta\, dr\right)^{-1}$$

IV. RESULTS

A. Generic Antenna Model

In this section we're going to examine how the *suppression-ratio* G_2 and *beam-width* θ affect the maximum achievable capacity, based on (7) and (8). In Fig.2 we can see how C_{MAX} changes with decreasing *suppression-ratio* for several fixed antenna beam-widths as well as different effective transmissions range *d*.

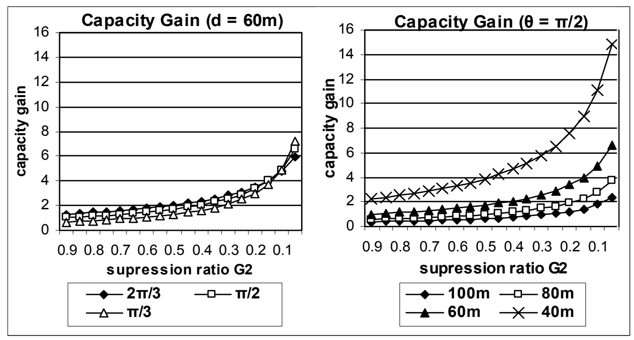

Figure 2: Capacity gain as a function of antenna suppression ratio for fixed effective transmission range 60m (left) and fixed antenna beam-width of 90° (right).

It is evident from the two graphs that the maximum capacity gain increases exponentially with lower suppression ratio for any beam-width and any effective transmission range. On the other hand, the capacity gain is not as sensitive to the antenna beam-width.

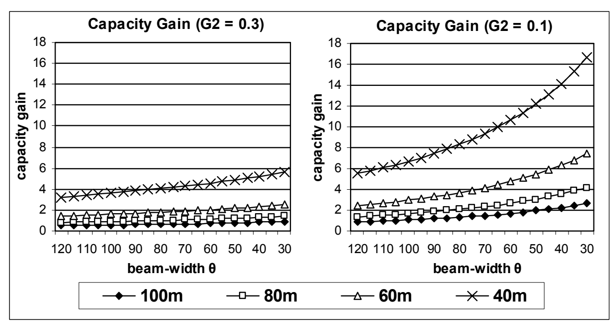

Figure 3: Capacity gain as a function of antenna beam-width θ for high suppression ratios (left) and low suppression rations (right)

Contrary to one's intuition, narrower beams result in marginally higher capacity gains for high suppression ratios (> 0.2). The reason for that is the following. Nearby nodes that lie within the main antenna beam can only interfere with their side lobes with the ongoing communication, as we saw earlier. Other nodes, however, do not learn about the upcoming communication and may directly interfere. Therefore, the narrower the beam is the more the potential nearby interference sources. This problem gets aggravated by suppression ratios, which are not low enough to cancel any potential interference, from such nearby sources.

In Fig. 3 we look closer at how the antenna beam-width affects the potential capacity gains for low (< 0.2) and high suppression ratios (> 0.2), respectively. We can see that the sensitivity of the capacity gain to the antenna beam-width increases with decreasing suppression ratios. Alternatively, the respective antenna beam-width becomes an important factor only when the suppression-ratio is low enough to cancel nearby interfering transmissions

B. Linear Array Antenna Model

We will analyze how the maximum capacity gain behaves as a function of the number of elements *N* in the array. We have used the "Mathematica" tool [12] to numerically calculate all integrals involved in (12)-(15). In Fig. 4 we depict the maximum achievable capacity gain as a function of the number of elements *N* in the array. Results are given for both the two-ray ground propagation model and second propagation model, where the attenuation factor α is equal to 3 and c_a is still equal to h^2.

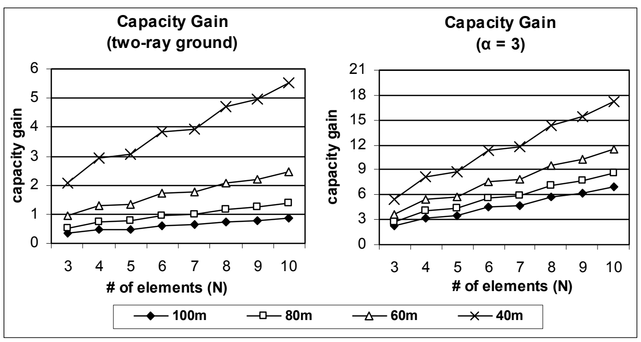

Figure 4: Capacity gain as a function of the number of dipoles *N* in linear array antenna for the two-ray ground propagation model (left) and a less harsh $\{\alpha = 3, c_a = h^2\}$ propagation model (right).

It is evident that maximum capacity gain increases with higher number of elements in the array in both cases, as expected, since both suppression ratio and antenna beam-width of a linear array antenna decrease when *N* increases. However, the more complex antenna pattern considered, as well as the counterintuitive impact of antenna beam-width for high suppression ratios, results in a less smooth capacity gain curve compared to the ideal *flat-topped* antenna model. Furthermore, we see that the propagation model assumed has an important effect on the achievable capacity. Specifically, it is evident in Fig.4 that for a less harsh environment, the maximum capacity gains are higher in every case. This behavior is justified by the larger *silence region,* which results from the higher radio range in combination with the less attenuated useful signal received.

V. DISCUSSION

It is important to note that all capacity results presented in this paper, are technology-based and not information theoretic. Specific assumptions are made about technology being used (e.g. DMAC protocol, 802.11, etc.), in order to realistically model current practice in ad-hoc networks. The implication of that is that derived capacity bounds apply to the technology and protocols being assumed. In a recent work [14], Kumar tries to derive general, information theoretic, bounds for the capacity of wireless networks. However, as the authors note, open questions still abound.

Another point to be made is that directional antennas do not change the scaling order of ad-hoc network capacity with the total number of nodes n, as shown in [6] or [14]. Only the constants in front of the order do change. However, these theoretical asymptotic capacity bounds derived in [6] are inherent to the need for multi-hop routing in ad-hoc networks.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we've analyzed how interference from simultaneous transmissions affects the capacity of ad-hoc networks utilizing directional antennas to communicate. We have calculated upper bounds for the capacity gains of using directional antennas in place of omni-directional ones, for both an ideal *flat-topped* generic antenna model as well as a real-world linear array antenna model. Furthermore, we examined how important antenna parameters like gain and beam-width affect those bounds in every case. Finally, we briefly illustrated the importance of the propagation model assumed in terms of the resulting network capacity. In future work, we're planning to incorporate smart antenna models as well as Ricean and Rayleigh propagation models [13] in our analysis.

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