EXTENDED ABSTRACT
Flow Optimization in Wireless Sensor Networks

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1 Introduction

Because of the unique characteristics of wireless sensor networks (severe energy constraints, unattended operation, many-to-one flows, data-centric operation), information routing in these systems presents novel design challenges. Several protocols have been proposed for querying, routing and data-gathering in these sensor networks, that have been primarily validated via simulations and limited experimentation, for example LEACH, PEGASIS, Directed Diffusion [1, 2, 3]. Given the severe resource constraints, application-specificity, and need for robust performance in sensor networks, it is clearly crucial to complement these ongoing routing protocol development efforts by the concurrent development of a strong theoretical understanding.

We describe a useful formal approach — the flow optimization framework — that can be used to identify the fundamental performance limits on information routing for a specified
network. In particular, we focus on a data-gathering application, consisting of $n$ nodes with given locations and finite remaining energies, and explore how information should be routed in the network to maximize the total information extractable from the network. We first survey the recent literature on variants of flow optimization models that have been proposed for wireless networks by several authors in recent years. We then present two models for the flow optimization approach – the first is an optimization model with non-linear convex constraints permitting rate adaptation through transmit power control; the second is a simpler linear optimization model in which the rate is kept fixed on all links. Finally, we present numerical examples that investigate how these two models (linear and non-linear) compare, and how they can be used to benchmark existing protocols.

Network flow optimization is an established area of Operations Research and is described in detail in the book by Ahuja, Magnanti and Orlin [4]. Prior work in the use of this methodology includes the work by Toumpis and Goldsmith, who analyze capacity regions for general-purpose multi-hop wireless networks [5]; and work by Xiao, Johansson and Boyd [6, 7], who consider jointly optimizing the routing as well as rate-adaptive power control and bandwidth allocation. These efforts do not consider energy as a finite resource. In the context of energy-limited wireless networks, there has been prior work on lifetime maximization by Chang and Tassiulas [8], Bhardwaj and Chandrakasan [9] and Kalpakis et al. [10].

The key feature of flow optimization-based modeling of sensor networks is that it is a computational framework. Thus, it yields performance bounds not in the form of closed-form expressions, but rather numerically, for specified scenarios. This framework is well suited to explore the impact of different design variables such as node location, energy distribution, rate-adaptation etc. The framework is also useful as a benchmark for comparing the per-
formance of implementable protocols on a test-suite of scenarios. The basic optimization models we present are for single-sink data gathering scenarios, and incorporate energy constraints and costs for transmission, reception and sensing, channel capacity constraints, as well as information constraints including fairness. These models assume a (TDMA/FDMA like) scheduled medium access scheme with no interference. However, as we discuss in the full version of this paper, these models can be extended in principle to incorporate soft interference, data aggregation, richer energy models, and to some extent even mobility.

2 Optimization Models for Wireless Sensor Networks

We present two flow optimization models that investigate the trade-off between maximum information extraction and minimum energy requirements for a given network topology. In both models, we assume that there are $n$ source nodes, and a sink numbered $n + 1$, located with pairwise distances $d_{ij}$ in a given area. In the basic models it is assumed that there is an overall energy budget of $E_{tot}$ (Joules) to distribute among the sensor nodes (this could be easily modified to a per-node energy budget of $E_{tot}/n$ if needed). The transmit power on link $(i, j)$ is $P_{ij}$ (Joules/sec), while $\beta$ and $C$ are the sensing and reception energy costs (Joules/bit). There is a fairness constraint for the flow emanating from each node, which ensures that no node may generate more than a fraction $\alpha$ of the total flow sent to the sink.

It is assumed that the relation between the normalized flow rate $f_{ij}$ and transmission power $P_{ij}$ on a link is given by a Shannon’s capacity:

$$f_{ij} \leq \log \left( 1 + \frac{P_{ij} d_{ij}^{-2}}{\eta} \right).$$

(1)

This expression assumes that the decay factor of the medium is $d_{ij}^{-2}$, the communication channel has a noise power of $\eta$, and that all transmissions are scheduled (e.g. via TDMA/FDMA)
such that they are non-interfering.

2.1 Non-linear Adaptive Rate Models

In this model the link rates \( f_{ij} \) and transmission powers \( P_{ij} \) are design variables, and \( T \) is a fixed total time duration (in seconds) of communication on each link. Thus \( \sum_{j=1}^{n} f_{j,n+1}T \) represents the total number of bits extracted by the sink from the network. The objective of our first model is therefore to find the coordinated operation of all nodes by setting \( P_{ij} \) and \( f_{ij} \) to maximize the amount of information that reaches the sink, which is equivalent to

\[
\max \sum_{j=1}^{n} f_{j,n+1}T \\
\text{s.t.} \quad 0 \leq Nf \leq \alpha \sum_{j=1}^{n} f_{j,n+1} \\
\sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij}T + \eta d_{ij} (e^{f_{ij}} - 1) T = E_{\text{tot}} \\
f \geq 0 .
\]

Here we simplified the notation by (a) defining \( \kappa_j = C \) if \( j = 1 : n \) and \( \kappa_{n+1} = \beta \) and (b) using the arc-incidence matrix \( N \), which for a network with \( n+1 \) nodes and \( m \) arcs, is a \( n+1 \) by \( m \) matrix, defined so that \( N_{i,(k,l)} \) is 1 if \( i = k \), -1 if \( i = l \), and 0 otherwise.

2.2 Linear Constant Rate Model

An alternative, computationally simpler, linear flow optimization model is obtained if we do not permit rate adaptation and assume that there is a fixed transmission rate \( f_{ij} = R \) (bits/sec) for each link in the network. The transmission powers are therefore also fixed and given by \( P_{ij} = \eta d_{ij} (e^{R} - 1) \) (in Joules/sec). In this model our decision variables are how
many bits to send from $i$ to $j$, $b_{ij}$. Given that the transmission rate is fixed the time taken for transmission on a given link is therefore variable and depends on the number of bits sent (recall that this was a constant $T$ for all links in the previous model).

The corresponding problem to Problem (2) in which the goal is to maximize the bits extracted for a given total energy budget is

$$
\max \sum_{j=1}^{n} b_{jn+1} \\
\text{s.t. } 0 \leq N b \leq \alpha \sum_{j=1}^{n} b_{jn+1} \\
\sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_i b_{ij} + \frac{\eta d_{ij}^2}{R} (e^R - 1) b_{ij} = E_{\text{tot}} \\
b \geq 0.
$$

We undertake a comparison of the adaptive and constant rate models. The latter model, involving a linear program is computationally more tractable, but we find that the loss of a degree of freedom (rate adaptation) can result in significant inefficiency.

### 3 A Comparison of the Non-linear and Linear models

We will compare how much information $b_{\text{out}}$ (bits) can be extracted from a given sensor network according to each model, when we are given a limited budget of overall energy $E_{\text{tot}}$ (Joules). For the comparisons we will tune the model parameter $T$ for the non-linear adaptive rate model, and the parameter $R$ for the linear constant rate model. These parameters in effect tune the throughput (bits/second) with which information is extracted from the network in each model. In Figure 1 we compare the total bits extracted for both models as a function of the total rate to the sink for a small 9 node example. Note that the non-linear
model outputs much more information for the same level of energy, at all rate levels. This shows that the computational tractability of the constant rate linear models comes at the expense of some inefficiency. Rate adaptation can provide significant additional information for the same budget (it is nearly an order of magnitude higher in this scenario).

4 Optimization as a Benchmark

We now use these models to study the performance of a practical proposed scheme for sensor networks, specifically, the LEACH protocol [1].

An important reason for the efficiency of the LEACH protocol is the fact that it aggregates data at a small number of cluster nodes before communicating the information to the sink (which is typically located far from the sensor net). To compare LEACH’s performance to optimal solutions, we introduce two main variations to Problem (3) above. The first is to represent the initial energy on each sensor node. The second variation is to consider
data aggregation, but only on transmissions to the sink. For an aggregation cost of \( \delta \) (in Joules/bits/message) and a data reduction due to aggregation of \( \lambda \in (0,1] \), this leads to changing transmission powers (to the sink) to 
\[ P_{m+1} = \lambda \frac{\eta d_{ij}^2}{R} \left( e^{R} - 1 \right) + \frac{\delta}{\lambda}. \]

In Figure 2 we compare the total bits outputed for different initial energy levels by LEACH, the modified linear model with no aggregation, and the modified linear model with aggregation. We consider an example with 100 random sensor nodes similar to the experiments presented in [1].

![Figure 2: Total bits sent to sink versus initial energy at nodes for LEACH, linear model (no aggregation), and linear model (with aggregation)](image)

We observe that the amount of information that can be extracted by LEACH is only slightly (about 30%) higher than the optimal to the linear model with no aggregation. The linear model with aggregation obtains a solution that aggregates all the data sensed within each node and sends it directly to the sink. This provides a (potentially loose) upper bound on the efficiency of any protocol that performs data aggregation when transmitting to the sink.
The LEACH solution is quite far from this upper bound (less than 5% of the value), but this merits further study.

References


