

**On the Relation between<sup>1</sup>  
Wire Length Distributions and  
Placement of Logic on  
of Master Slice ICS**

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## **Abstract**

In this paper we present a model that characterizes the relation between wire length distributions and placement of logic on master slice ICs. In particular, the model provides a firm mathematical basis for the well known empirical law known as Rent's Rule. It is shown that Rent's Rule is a manifestation of a more fundamental underlying process characterized by a function from which the distribution of wire lengths can be recovered. That is, Rent's Rule contains all the information about the distribution of wire lengths. Based on this, estimates for the average wire length are derived. Finally, experimental results from both simulated and actual chips are presented.

# Table of Contents

1 Introduction	0
2 Outline of the paper	1
3 The Model	1
4 Choices for $Z(x)$	4
4.1 Constant $Z(x) : \beta = 1$	4
4.2 Decreasing $Z(x) : 0 < \beta < 1$	4
4.3 Increasing $Z(x) : \beta > 1$	5
5 Average Wire Length	5
5.1 Estimating $\alpha$ and $\beta$	5
5.2 Graphical Method of Estimating $\alpha$ and $\beta$	6
5.3 Method of Moments	7
6 Simulation Results	8
7 Experimental Results	9
8 Conclusions	10
<b>References</b>	<b>14</b>

## List of Figures

<b>Figure 1:</b>	Wire Lengths of Graph A fitted to a Weibull	11
<b>Figure 2:</b>	Wire Lengths of Graph B fitted to a Weibull	12
<b>Figure 3:</b>	Wire Lengths of Graph C fitted to a Weibull	13

## List of Tables

<b>Table 1:</b>	Simulation Results for the Weibull Model	8
<b>Table 2:</b>	Experimental Results on Wire Length Estimation	9

## 1 Introduction

Master Slice (MS) or Gate Array is a very popular method of designing LSI/VLSI circuits, especially when fast turn-around time is the primary objective. This fast turn-around time is achieved by having a two-dimensional array of uniformly spaced logic cells of identical size that have been pre-fabricated up to the metalization layer. Placement of the logic consists of assigning the circuits to one or more of the basic cells on the array. The space between the logic cells, called channels, are used to route the wires connecting the cells. The routing of wires is usually carried out in two steps. The first step, called Global Wiring, simply assigns wires to channels without allocating the individual tracks to the wires. The second step, called Exact Embedding, assigns wires to specific tracks within a channel.

The quality of the placement and routing is measured by various parameters, the most common one being average wire length. With regard to wire length, placement and routing are mutually competing tasks and the solution space for both is exponential in nature. It seems natural then to ask how to estimate measures of placement such as average wire length, total number of wiring tracks etc, prior to routing the connections. These estimates will give some indication about the routability of the placement and, if too large, can be used to select another placement and the process repeated.

The nature of placement and routing problems necessitate a probabilistic approach to the wirability analysis of integrated circuits. Stochastic models for wiring space estimation and the relation between wire length distribution and placement optimization have received extensive attention recently [1], [6], [2], [5], [7], [3] and [4]. Much of the reported work on wire length distributions and placement of logic rests on empirical evidence that indicates that "well placed" chips exhibit a power law relation, known as Rent's Rule, between the number of components and the number of corresponding external connections. Rent's Rule has been the basis of much of the heuristic arguments used to derive upper bounds on the average wire length and the form of the wire length distribution. Rent's Rule has the form

$$T = KC^P \quad (1)$$

where  $T$  is the average number of external connections,  $C$  is the average number of components,  $K$  is number of connections per component and  $p$  is a positive constant. In [1], [2] and [4] the effect of placement on wire length distribution was introduced by assuming that a hierarchical partitioning scheme aimed at minimizing the average wire length results in a configuration that exhibits Rent's Rule. In [2] an upper bound  $\bar{r}_k$  for the average wire length between elements of different subsets of components of size  $k$  was derived, and using Rent's Rule to obtain the number of connections between such subsets, an upper bound on the average wire length was derived. In [3] the Pareto distribution is proposed for the distribution of wire lengths. Similar results were presented in [4].

## 2 Outline of the paper

In this paper we present a model that provides a firm mathematical basis for Rent's Rule and its relation to the wire length distribution. It will be shown that Rent's Rule, an observed fact, is a manifestation of a more fundamental underlying process characterized by a function which leads directly to a general class of wire length distributions known as the Weibull family. That is, Rent's Rule contains all the information about the distribution of wire lengths. Based on this, estimates for the average wire length are derived. Finally the theory presented here is substantiated by simulation results and data gathered from earlier works.

## 3 The Model

A master slice integrated circuit is an array of equal size components (gates) arranged in rows and columns on a planar surface. This situation is idealized by considering such a chip to be a continuum of gates, each being a point source whose distance measured from an arbitrary origin is denoted by  $x$ . This continuous model does not cause any serious inaccuracies in the analysis that follows, for one may view this "ideal chip" as a "real chip" with probability measures of zero assigned to points  $x$  that are not gates. Thus linear distances such as wire lengths are viewed as a continuum of "prospective" or "possible" gates.

Let  $[a,x]$  denote a "circular" region on the chip of radius  $x$  centered at  $a$ , encompassing



C gates. Consider the sample space consisting of all possible placements of a given logic graph on a master slice array. Let  $X$  be the random variable defined on this sample space that denotes the length of a wire from its source to its point of termination. By "terminal at  $x$ " we mean that a wire either actually terminated at  $x$  or it has crossed the boundary of the region of radius  $x$ . Thus  $X$  may be viewed as being the length of a wire up to the formation of a terminal or the length of the "inter-terminal" formation interval. Let  $A$ ,  $B$  and  $C$  denote the following events :

$$\begin{aligned} A &= \{ x < X < x+dx \} \\ B &= \{ X > x \} \\ C &= \{ A | B \} \end{aligned} \tag{2}$$

$A$  represents the unconditional event that a wire terminates between the regions of radius  $x$  and  $x+dx$ .  $B$  represents the event that a wire did not terminate with the region of radius  $x$ .  $C$  is the conditional event that a wire terminates within a region of radius  $x+dx$  given that it did not terminate with in the region of radius  $x$ . Let  $F(x)$  and  $f(x)$  represent the distribution and density functions of  $X$  respectively. Then

$$\begin{aligned} P(A) &= P(x \leq X \leq x+dx) = F(x+dx) - F(x) = f(x)dx \\ P(B) &= P(X > x) = 1 - F(x) \\ P(C) &= P(A | B) = P(x \leq X \leq x+dx | X > x) = Z(x)dx \end{aligned} \tag{3}$$

Now  $Z(x)dx$  represents the conditional probability that a wire having crossed the boundary of a region of radius  $x$  terminates before reaching the boundary of a region of radius  $x+dx$ .  $Z(x)$  is not a probability, rather it is the instantaneous rate of wires terminating or terminal formation at  $x$ . Note that rate of certain events here refers to the number of such events per unit length of wire. Since  $A \subset B$ , we have

$$Z(x)dx = P(A | B) = \frac{P(A)}{P(B)} = \frac{f(x)dx}{1 - F(x)} = \frac{-d(\ln(1 - F(x)))}{dx} \quad (4)$$

Integrating (4) we obtain the distribution of wire lengths as

$$F(x) = 1 - \text{EXP} \left\{ -\int_0^x Z(t)dt \right\} \quad (5)$$

We will now interpret Rent's Rule as a parametric model for  $Z(x)$ .

It is clear that the number of gates in  $[a, x]$  is the content or measure of the interval, which is  $x-a$ . In other words, the number of gates in the region of radius  $x$  centered at  $a$  is proportional to  $x-a$ . Rent's Rule states that the average number of terminals formed by a collection of  $(x-a)$  gates is given by

$$T = \alpha\beta(x-a)^\beta \quad (6)$$

Recall that  $Z(x)$  represents the number of terminals per unit length of wire formed at  $x$ . Since the relation between  $T$  and  $x$  is that of formation of terminal at  $x$ , the total number of terminals at  $x$  is simply  $T = (x-a)Z(x)$ . Rent's Rule states that  $Z(x)$  has the form

$$Z(x) = \alpha\beta(x-a)^{\beta-1} \quad (7)$$

Combining equations (5) and (7) the wire length distribution function has the form

$$F(x) = 1 - e^{-\alpha(x-a)^\beta} \quad (8)$$

Equation (8) is the well known Weibull distribution function with scale parameter  $\alpha$ , location parameter  $a$  and shape parameter  $\beta$ . With  $a$  chosen to be zero the wire length distribution and density functions are given by

$$F(x) = 1 - e^{-\alpha x^\beta} \quad (9)$$

$$f(x) = \alpha \beta x^{\beta-1} e^{-\alpha x^\beta} \quad (10)$$

#### 4 Choices for $Z(x)$

Depending on the physical situation one may choose the form of  $Z(x)$  from which the distribution function  $F(x)$  can be recovered.  $Z(x)$  can behave in one of three different ways, all parameterized by equation (7). If  $\beta = 1$  then  $Z(x) = \alpha$ , representing a constant rate of terminal formation. If  $0 \leq \beta < 1$  then  $Z(x)$  is a decreasing function of  $x$  while if  $\beta > 1$  then  $Z(x)$  is an increasing function of  $x$ .

##### 4.1 Constant $Z(x)$ : $\beta = 1$

In this case the distribution function is given by  $F(x) = 1 - e^{-\alpha x}$ , which is the exponential distribution. This represents a situation where the probability that a wire has length  $x+1$  given that it has length  $l$  is independent of  $l$ . Equivalently, the number of terminals formed at  $x$  per length of wire is constant. Except for small regions of the array this assumption may not be realistic. This is exactly the same observation made in [3] where the exponential distribution was ruled out on the basis of empirical results showing that the ratio of the number of wires of length  $k+1$  to the number of wires of length  $k$  increases with  $k$ .

##### 4.2 Decreasing $Z(x)$ : $0 < \beta < 1$

As stated earlier, this corresponds to what has been empirically observed as Rent's Rule. In this case the wire length distribution function is the Weibull distribution with shape parameter  $\beta$ . A decreasing  $Z(x)$  represents a situation where the rate of terminal formation decreases with the length of the a wire. It is this situation that is most often found in practice.

### 4.3 Increasing $Z(x) : \beta > 1$

In this case the rate at which terminals are formed at  $x$  increases with the length of the wire. Although theoretically possible, such a situation would not occur in practice as placement is usually optimized by minimizing the average wire length.

In summary, it has been shown that Rent's Rule is the exactly the function  $T = x*Z(x)$  when the corresponding  $Z(x)$  has the form  $\alpha\beta x^{\beta-1}$ , with  $0 < \beta < 1$ . Knowledge of  $Z(x)$  completely determines the wire length distribution  $F(x)$ . Thus Rent's Rule states that "well placed" chips will result in a wire length distribution from the Weibull family.

## 5 Average Wire Length

Assuming that a hierarchical placement procedure was carried out and Rent's Rule holds true, the expected value of wire length can easily be computed since the form of  $Z(x)$  is then known to be  $\alpha\beta x^{\beta-1}$  and the corresponding wire length distribution is a two parameter Weibull. Thus the expected value of wire length is given by

$$\begin{aligned} E(X) &= \int_0^{\infty} [1 - F(x)] dx = \int_0^{\infty} e^{-\alpha x^{\beta}} \\ &= \frac{1}{\beta} \left( \frac{1}{\alpha} \right)^{\frac{1}{\beta}} T \left( \frac{1}{\beta} \right) \end{aligned} \quad (11)$$

To compute  $E(x)$  from observed frequencies of wire lengths the parameters  $\alpha$  and  $\beta$  have to be estimated. In the following section methods for obtaining such estimates are presented.

### 5.1 Estimating $\alpha$ and $\beta$

There are two methods of estimating the parameters of a Weibull when the data is given in the form of quantized frequencies. The first is a graphical method and the second is known as the method of moments.

## 5.2 Graphical Method of Estimating $\alpha$ and $\beta$

This is the simplest and quickest method of obtaining estimates of Weibull parameters. The data is in the form  $(k, n_k)$ ,  $k = 1, 2, \dots, m$ , where  $n_k$  is the number of wires of length  $k$ . From this we form the pairs  $(k, F_k)$  where  $F_k$  is the cumulative frequency function defined by

$$F_k = \frac{\sum_{i=1}^k \{n_i\}}{n}$$

where  $n$  is the total number of wires.

Since  $F(x) = 1 - e^{-\alpha x^\beta}$ , we have

$$\ln \ln \left[ \frac{1}{1 - F(x)} \right] = \ln(\alpha) + \beta \ln(x) \quad (12)$$

Letting  $v = \ln(x)$ ,  $c = \ln(\alpha)$  and  $u = \ln \ln \left[ \frac{1}{1 - F(x)} \right]$ , equation (12) becomes  $u = \beta v + c$ . This is straight line with slope  $\beta$  and intercept  $\ln(\alpha)$ . Thus from the observed data  $(k, n_k)$  we form the data  $(v_k, u_k)$ , where  $u_k = \ln \ln \left[ \frac{1}{1 - F_k} \right]$  and  $v_k = \ln(k)$  and fit a straight line. Using the method of least squares the estimates  $\hat{\alpha}$  and  $\hat{\beta}$  of  $\alpha$  and  $\beta$  respectively are given by

$$\hat{\alpha} = e^{\bar{u} - \hat{\beta} \bar{v}} \quad (13)$$

$$\hat{\beta} = \frac{\sum_{i=1}^m (v_i - \bar{v})(u_i - \bar{u})}{\sum_{i=1}^m (v_i - \bar{v})^2} \quad (14)$$

$$\text{where } \bar{u} = \frac{\sum_{i=1}^m u_i}{m} \quad \text{and} \quad \bar{v} = \frac{\sum_{i=1}^m v_i}{m}$$

### 5.3 Method of Moments

The method of moments provides a another way of estimating  $\alpha$  and  $\beta$ . Let  $u^r$  be the  $r^{\text{th}}$  moment of zero of  $f(x; \alpha, \beta)$ . Let  $X_1, X_2, \dots, X_m$  be a sample of size  $m$  from  $f(x; \alpha, \beta)$ . Then from the data we form the sample moments

$$m_1 = \frac{\sum_{i=1}^m X_i}{m} \quad m_2 = \frac{\sum_{i=1}^m X_i^2}{m}$$

The moment estimators  $\hat{\alpha}$  and  $\hat{\beta}$  are obtained by solving the following two equations simultaneously.

$$u^1 = m_1 = \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right) \quad (15)$$

$$u^2 = m_2 = \left(\frac{1}{\alpha}\right)^{\frac{2}{\beta}} \Gamma\left(1 + \frac{2}{\beta}\right) \quad (16)$$

Solving equation (15) we obtain

$$\hat{\alpha} = \left[ \frac{\Gamma\left(1 + \frac{1}{\beta}\right)}{m_1} \right]^{\beta} \quad (17)$$

Substituting equation (17) into (16) we obtain

$$\frac{2\beta T\left(\frac{2}{\beta}\right)}{T^2\left(\frac{1}{\beta}\right)} = \frac{m_2}{m_1} \quad (18)$$

Numerical solution to equation (18) yields the estimator  $\hat{\beta}$ .

## 6 Simulation Results

To verify the quality of the estimators, simulations were carried out with sample sizes ranging from 1000 wires to 7500 wires. That is, for given values of  $\alpha$  and  $\beta$  samples from the Weibull were generated. The resulting data was quantized and estimates for  $\alpha$  and  $\beta$  were computed using both methods. Table 1 contains the results of these simulations. The method of moments provides relatively better estimates of  $\alpha$  and  $\beta$  than the graphical approach. Recall that the moment estimate for  $\beta$  involves finding the zero of equation (18). The simplicity and ease of the graphical approach enables one to obtain an initial estimate for  $\beta$  which is used in the iterative solution of equation (18).

**Table 1:** Simulation Results for the Weibull Model

n	m			Graphical		Method of Moments	
sample size	# of intervals	$\alpha$	$\beta$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
1000	38	0.5	0.8	0.557	0.715	0.484	0.787
2000	21	0.6	0.9	0.627	0.846	0.564	0.91
5000	37	0.4	1.0	0.485	0.866	0.392	0.989
7500	35	0.6	0.9	0.666	0.821	0.568	0.915

## 7 Experimental Results

The Weibull model for wire lengths was tested on data gathered from [3]. The data was gathered from three logic graphs that were placed using a hierarchical placement algorithm. Graph A consists of 2146 gates with a Rent exponent  $p = 0.75$ . Graph B, a subgraph of A, has 576 gates with an assumed Rent exponent  $p = 0.75$ . Graph C has 528 gates with a Rent exponent  $p = 0.59$ .

The histograms of wire lengths for graphs A, B and C taken from [3] were fitted to the equation

$$\ln \ln \left[ \frac{1}{1 - F_i} \right] = \ln(\alpha) + \beta \ln(i)$$

Figures 1, 2 and 3 show plots of  $\ln \ln \left[ \frac{1}{1 - F_i} \right]$  versus  $\ln(i)$  for the three logic graphs. In all three cases the fit appears to be excellent. The estimates for  $\alpha$  and  $\beta$  obtained using both the graphical and the method of moments were used to compute the expected value of wire length,  $R$ , using equation (11). These values along with those reported in [3] are shown in table 2.

**Table 2:** Experimental Results on Wire Length Estimation

Graph	# of gates	# of conns	Rent expo $p$	Exptl $R$	Theor. $R$ from [3]	Theor. $R$ graphical	Theor. $R$ moments
A	2146	7302	0.75	3.53	7.54	2.99	3.02
B	576	1383	0.75	2.98	5.24	2.48	2.56
C	528	1046	0.59	2.20	4.02	1.56	1.65



## 8 Conclusions

In conclusion we have developed a mathematical formulation of Rent's Rule characterized by a function  $Z(x)$  from which the wire length distribution can be recovered. We have found that Rent's Rule is tantamount to assuming a class of wire length distributions known as the Weibull family, with the Rent exponent  $p$  corresponding to the shape parameter  $\beta$  of a Weibull distribution. The average wire length is given as a function of the Rent parameters and methods for estimating these parameters are given. Experimental results strongly indicate that the Weibull model is very accurate.

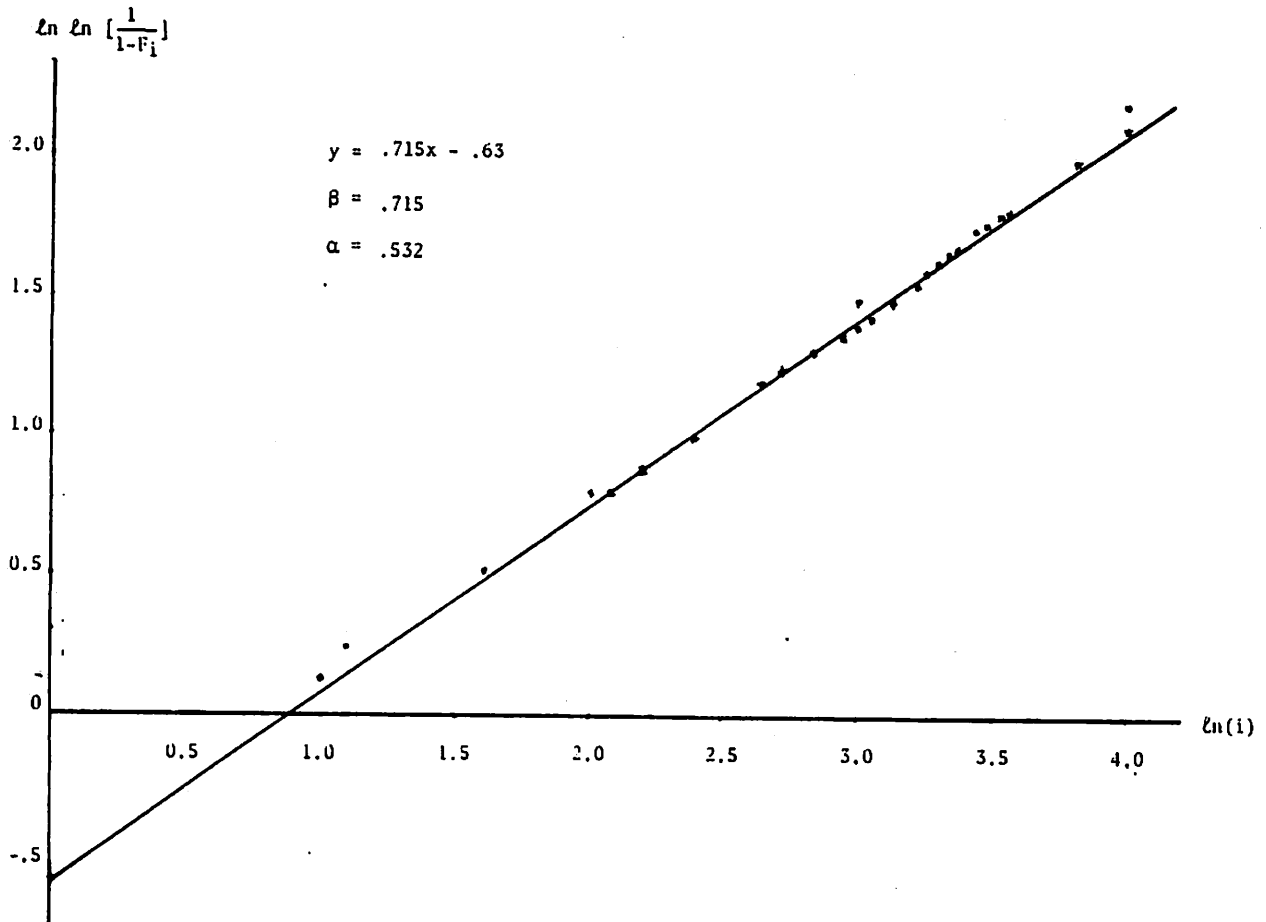
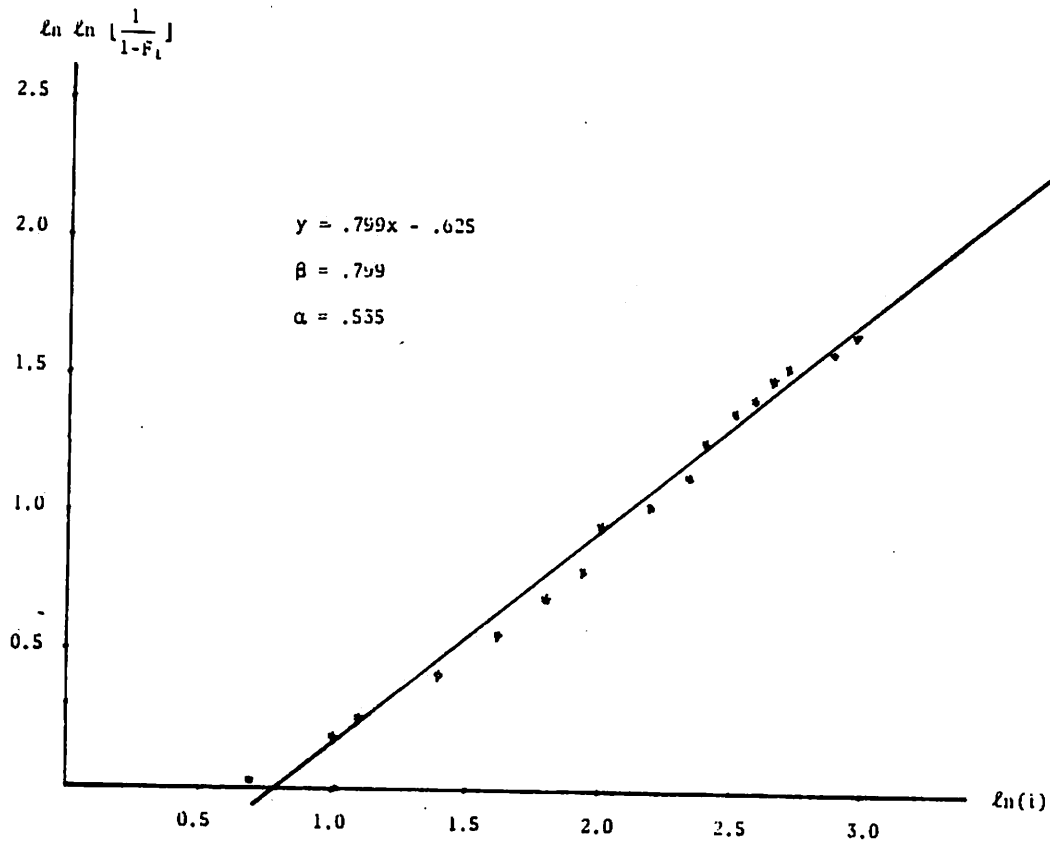
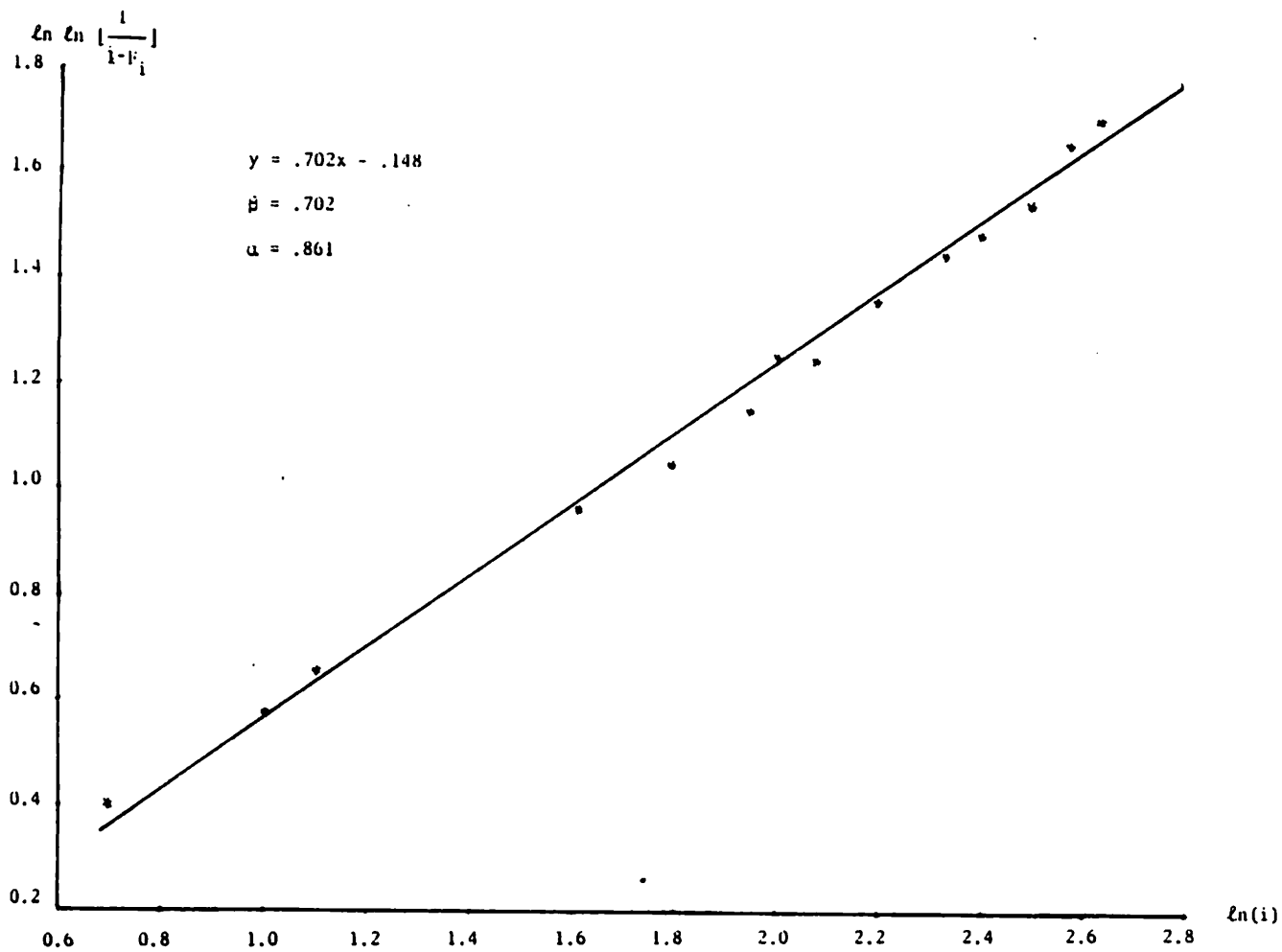


Figure 1: Wire Lengths of Graph A fitted to a Weibull



**Figure 2:** Wire Lengths of Graph B fitted to a Weibull



**Figure 3:** Wire Lengths of Graph C fitted to a Weibull