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The Optimism Principle: A Unified Framework for Optimal Robotic Network Deployment in An Unknown Obstructed Environment

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Abstract— **We consider the problem of deploying a team of robots in an unknown, obstructed environment to form a multi-hop communication network. As a solution, we present a unified framework, on*L*in*E* r*O*botic *N*etwork form*A*tion (LEONA), that is general enough to permit optimizing the communication network for different utility functions in non-convex environments. LEONA adopts the principle of "optimism in the face of uncertainty" to allow the team of robots to form optimal network configurations efficiently and rapidly without having to map link qualities in the entire area. We demonstrate and evaluate this framework on two specific scenarios concerning the formation of a multi-hop communication path between fixed end-points: one minimizing the total path cost, and another maximizing the bottleneck communication rate. Our simulation-based evaluation shows that the use of the optimism principle can significantly reduce resources spent in exploring and mapping the entire region prior to network optimization. We also present a mathematical modeling of how the searched area scales with various relevant parameters in each case.**

I. INTRODUCTION

We consider the problem of deploying a team of robots in an unknown, obstructed environment to form a multi-hop communication network. In obstructed environments, such as inside buildings or outdoors in forested areas, not only is there a concern with moving efficiently through the environment while avoiding obstacles and walls, the communication channels are also cluttered and highly varying due to signal attenuation (shadowing), as well as multi-path scattering (fading).

With the exception of some recent work (e.g., [1]), most research to date on robot network deployments has assumed idealized communication models such as the unit disk model. Further, the problem of network formation has typically been treated assuming convex utility functions that can be optimized through localized potentials [2] and greedy distributed gradient descent algorithms [3]. While radio propagation models such as the simple path loss model can yield such convex optimization problems in unobstructed environments, the presence of walls introduces non-convexities. We argue in this work that a more tractable perspective is to consider a graph theoretic formulation in which vertices correspond to the set of all possible (discretized) locations for the robots in the given environment, and there are labels on the edges between the vertices that indicate the RF path loss (or a monotonic function thereof) between the corresponding positions. The network formation problem becomes one of finding subgraphs of this graph (in this work we focus on path formation, but the approach could be generalized further to trees or other graph structures) that satisfy the constraint that the number of nodes in the selected subgraph must be equal to or less than the number of available robots, and maximize a desired utility function. Such a problem could then be solved using a suitable centralized or decentralized graph algorithm (e.g., Bellman-Ford algorithm to compute the minimum cost path) to yield the optimal configuration of the robotic network.

While general enough to handle many non-convex network optimization problems such as minimum cost path formation in environments with arbitrary link qualities (which is not possible using a purely distributed potential-based approach in obstructed environments), the implementation of this graph-theoretic approach in practice faces one significant hurdle: it requires prior mapping of the area to determine the link qualities for every pair of locations, which could be prohibitively time-consuming. We address this challenge with an innovative online, iterative, approach that is based on the principle of "Optimism in the Face of Uncertainty" inspired by similar ideas in the domain of online learning and multi-armed bandits [4].

Contributions. We first propose a unified framework, called LEONA (for on**L**in**E** r**O**botic **N**etwork form**A**tion), that is general enough to allow optimizations for different utility functions in non-convex environments. The crux of the approach is the following. At each stage, an optimistic prediction of the graph edge weights (link qualities) is maintained, i.e., it is ensured that the predicted link quality is no worse than the true link quality. And at that iteration, the robots move through the environment to the network configuration computed to be optimal for that predicted graph. As they move through the environment, the robots collaborate to take additional measurements of the link qualities. These measurements, and potentially, additional inferences derived from these measurements[1], are used to update the predicted graph to a new set of values, that are still ensured to be optimistic (though now a bit "closer" to the true graph because of the updates). The iterations continue until the robots are at a configuration whose measured utility is as good as the best possible configuration in the current

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[1]For example, it may be reasonable to assume that if a particular pair of locations has a certain path loss indicative of significant attenuation due to a wall, then links corresponding to all locations that fall on or even near the same line as those locations must experience at least that much loss due to attenuation as well.

predicted graph, which can then be shown to be provably optimal because of its optimistic bias.

Second, we demonstrate and evaluate how this general framework, i.e., LEONA, works in unknown environments for two specific scenarios concerning the formation of a multi-hop robotic relay path between two fixed end-points in an obstructed environment, that differ in the path utility functions: in one we seek to minimize the total path cost, and in the other, we seek to maximize the bottleneck rate (i.e., the end-to-end data rate). Our simulation-based evaluation shows that the use of the optimism principle can significantly reduce the time spent in exploring and mapping the entire region *a priori* before the optimal network configuration is constructed. We also present a mathematical modeling of how the searched area scales with various relevant parameters in each case.

The rest of the paper is organized as follows: in section II, we present related work; in section III, we define the general utility maximization problem involved in robotic network formation in the environment; in section IV we present the general LEONA framework, while in the succeeding sections V and VI we present two case studies for specific utility functions pertaining to minimization of expected number of transmissions per successfully delivered packet (ETX) and maximization of bottleneck-rate respectively, including both theoretical analysis and simulation-based evaluations; Finally, we present a summary and discuss future work directions in section VII.

II. Related Work

Networked robots have been well investigated in recent years, especially in the field of flocking [5], formation control [6] and swarming [7]. The key idea is to allow a team of robots to cooperate and coordinate in a networked autonomous system to perform a specific task. Therefore, communication among robots plays a significant role in enabling cooperation. While the disk communication model has been used in many of these previous works to maintain connectivity among robots, such a simple model does not fit in a realistic environment, and thus degrades the performance of previous work when applied in reality.

Recently, there has been growing body of work considering more realistic communication performance in networked robots, referred to as Communication-Aware Robotics. Researchers in [8] have studied integrity problems where controllers of robots are designed while considering maintaining a desired transmission rate at the same time. A robotic router formulation problem has been studied in [1], where an optimal configuration of robots is formulated to maintain a maximized successful reception rate in realistic communication environments. ETX has been taken into consideration in [9], where researchers design a hybrid architecture to allow robots optimally configured so that each flow has a minimized ETX in a multiple flow network. But because of the convexity of utility functions each of these papers uses, they can apply potential functions or greedy gradient descent algorithms, which are implicitly based on the assumption there is only one extreme solution. This assumption no longer holds when the existence of noise sources or obstacles introduces non-convexities.

Controlling a team of robots in a network becomes more challenging when there are obstacles in the environment. Most work designs the controllers of robots under the assumption that the obstructed environment is known *a priori*. This would allow researchers to explicitly add obstacle avoidance by utilizing either linear constraints [10] or artificial potentials [11]. Moreover, when communication-oriented performance is taken into consideration, not only obstacles can block robots' movements, they can also cause signal attenuation. Few works [12], [14] consider wall attenuation when studying robots' coordination. More difficulties arise when the environment is unknown, which further requires robots to take measurements and explore the environment.

In [13], a measurement-based mapping is computed in each spatial direction between a robot's current position and the received signal strength regardless of the robots' environment and this is used to obtain a quadratic optimization yielding the best locations for a set of robotic access points to serve a set of (possibly mobile) clients. However that work does not consider the problem of forming a general utility-optimized multi-hop communication network among the robotic nodes. Another problem similar to the one we address is studied in [14] by designing a mobility control algorithm for the formation of an optimal communication chain in an environment with unknown noise sources. They use an Extremum Seeking (ES) algorithm to do estimation by injecting a perturbation signal. However, parameters have to be properly designed to maintain the stability of their algorithm. And since they use potential-based methods, there is no guarantee to find the global optimal solution.

To our knowledge, this is the first work to present a mechanism for rapid optimal multi-hop network configuration by a team of robots in an unknown realistic RF environment with obstructions, where the problem is non-convex and not amenable to solution using standard potential-based approaches.

III. Problem Formulation

We consider a team of $m \geq 3$ robots performing a task in an unknown walled environment. Robot 1 works as a source that transmits information to a destination robot m. Our goal is to find an optimal configuration of the remaining relay robots so that they can form a multi-hop communication path from the source to the destination with optimized performance.

A. Link Quality Metric

The walled environment is represented as a 2-D $L \times L$ grid and each pixel in the grid can either be a possible location for a robot or is occupied by a wall. Among all the m robots, robot 1 and robot m work as a source-destination pair, which are static and their positions are known *a priori*. The rest of the robots can move around in the space to enable and improve the communication between the source and destination. When a robot i communicates with robot j, the strength of signal decreases as it travels through air. Moreover, if there are walls between the communication pair, additional signal attenuation can occur. Taking signal attenuation caused by travelling distance and walls into consideration, the received signal (in dB) at the receiver j from transmitter i can be expressed as [15]:

$$Pr_{i,j} = Pr_0 - 10\eta \log(\frac{d_{i,j}}{d_0}) - n_{i,j}W \tag{1}$$

where $d_{i,j}$ is the distance between robots i and j; $n_{i,j}$ is the number of walls between them; Pr_0 is the received power strength at a reference point with a small distance d_0 from the transmitter; η is the path loss parameter indicating the rate at which the attenuation increases with distance; W is the attenuation effect of a single wall.

Let the noise power spectral density be N_0 and spectrum bandwidth be B, then the Signal-to-Noise Ratio (SNR) at the receiver j is defined to be:

$$\gamma_{i,j} = \frac{Pr_{i,j}}{N_0 B} \tag{2}$$

We define the link quality metric $l_{i,j}$ of the communication link (i,j) as a strictly increasing function of the SNR at receiver j corresponding to transmitter i:

$$l_{i,j} = f(\gamma_{i,j}) \tag{3}$$

B. Mobility, Sensing and Environment Assumptions

Time is divided into discrete time steps of unit duration. At each time step, a mobile robot can move to one of its four neighbor positions (up, down, left, right) or stay at its current position. We assume each robot has the ability to sense and detect walls within one moving step range, which helps a robot avoid colliding with walls. The moving decision is made by each robot itself. We assume the unknown environment is connected, i.e., given any two pixels in the grid that are not occupied by walls, there always exists a path between them. This assumption ensures all the available positions in the environment could be reached by robots.

C. Objective Function

Our goal is to design an optimal configuration of mobile relay robots such that they finally form a communication path[2] connecting the source and destination with a maximized utility $\mathcal{U}$, which is a monotonic function of all link qualities:

$$\max_{(x^1,\ldots,x^m)\in\mathcal{P}} \mathcal{U}(l_{1,2},\ldots,l_{m-1,m}) \tag{4}$$

where x^k (where $1 \le k \le m$) is the position of the k^{th} robot and $\mathcal{P}$ is the set of all possible configurations.

[2]Configuration, communication path and solution are used interchangeably in the rest of the paper.

IV. ONLINE ROBOTIC NETOWRK FORMATION (LEONA)

There are three challenges inherent in the problem we address: 1) The environment is unknown so that robots need to dynamically combine exploration with configuration optimization; 2) The signal attenuation caused by walls results in a non-metric space so that prior metric-based algorithms cannot be applied to our problem; 3) The objective function in general is not convex so that potentials and convex optimization methods do not work. Therefore, one natural question to ask is: Given an unknown environment, is it possible to find an optimal configuration without fully exploring the whole space? The answer is yes, and in the following, we propose a graph-based online approach, which is guaranteed to find the optimal solution with only partial exploration.

A high-level structure of the proposed unified framework LEONA is presented in **Algorithm 1** with related steps detailed in **Algorithm 2-5**. To begin with, each mobile robot maintains a communication graph, which is represented as a complete directed graph $\mathbb{G}(\mathcal{V},\mathcal{E})$. The vertices in the graph are all pixels in the grid. And a directed edge $(i,j) \in \mathcal{E}$ represents a communication link with transmitter at pixel i and receiver at pixel j. The graph is complete in the sense that every two vertices are connected by a pair of directed edges. The weight of an edge (i,j) is set as an optimistic predication of the link quality $l_{i,j}$. We assume robots can share environment information with each other through the communication path between the source and destination, thus all of them maintain the same knowledge of the environment.

The environment information includes SNR measurements and detected wall positions. Each robot constructs its communication graph $\mathbb{G}(\mathcal{V},\mathcal{E})$ based on its current information. For each $(i,j) \in \mathcal{E}$, if its link quality has been measured, the edge weight is set as measured. Otherwise, the edge weight is predicted according to eqns. (1)-(3) based on explored walls' information. Thus, if all walls along link (i,j) are fully explored, the predicted edge weight is the same as the actual link quality; if there is still some wall information missing, the predicted edge weight is optimistic or overestimated. Based on the current communication graph $\mathbb{G}$ with optimistic prediction, robots apply $FindPath(\mathbb{G})$ to find the best possible communication path. After moving to form the best possible communication path, robots take measurements to find actual link qualities along the path, and update their communication graph $\mathbb{G}$ based on new information. They run $FindPath(\mathbb{G})$ again to find the best possible communication path. If the current communication path's measured utility is as good as the best possible path, the algorithm terminates. Otherwise, robots move to form the new best-possible communication path, and repeat above procedures until the termination condition is met.

Theorem 1: The robotic network configuration obtained from LEONA is optimal.

Proof: In LEONA, by construction, when updating the communication graph $\mathbb{G}$ at each step based on measurements, the predicted weight of each edge is always no worse[3] than its actual weight. Thus the communication graph is always optimistic. When LEONA terminates, the actual utility of the final configuration is at least as good as the best possible estimated configuration based on the optimistic estimates, which indicates the actual utility of the final configuration is as least as good as the actual utilities of all other possible configurations. Therefore, the final configuration is optimal. ∎

Algorithm 1 Online Robotic Network Formation (LEONA)

▷ Initialization
Robots start from current initial positions $p^* = (x^1, \ldots, x^m)$ and initialize utility of the current path p^* as $U^* = 0$. Initialize the measured SNR set and detected walls' positions set as $\mathcal{S} = \phi$ and $\mathcal{W} = \phi$ respectively
▷ Update communication graph
$\mathbb{G} \leftarrow UpdateGraph(\mathbb{G}(\mathcal{V}, \mathcal{E}), \mathcal{S}, \mathcal{W})$
▷ Find best possible path and its utility
$(p, U) \leftarrow FindPath(\mathbb{G})$
while $U^* < U$ **do**
    ▷ Robots move to form the best possible path
    $(p^*, \mathcal{W}) \leftarrow Move(p^*, p)$ [4]
    ▷ Measure SNR of each link on current path p^*
    $\mathcal{S} \leftarrow MeasureSNR(p^*)$
    ▷ Set the utility of current path p^*
    $U^* \leftarrow SetUtility(p^*, \mathcal{S})$
    ▷ Probe walls on the current path p^*
    $\mathcal{W} \leftarrow ProbeWall(p^*)$
    ▷ Update communication graph
    $\mathbb{G} \leftarrow UpdateGraph(\mathbb{G}(\mathcal{V}, \mathcal{E}), \mathcal{S}, \mathcal{W})$
    ▷ Find best possible path and its utility
    $(p, U) \leftarrow FindPath(\mathbb{G})$
end while

Remark 1: LEONA provides a unified framework to finding an optimal communication path that combines both environment exploration and configuration optimization. And it is general enough to permit optimizing for different utility functions in non-convex environments. In the following, we provide two specific case studies in which we apply LEONA with $FindPath(\mathbb{G})$ instantiated to find optimal configurations with respect to two different metrics.

[3]The predicted edge weight is the same as the actual weight if the link quality has been measured or all related wall information has been detected; overestimated if there is still some related walls' information missing.

[4]This $Move(p^*, p)$ algorithm follows the same idea of *Robotic Routing Protocol* [16], except for allowing a robot to choose using either *Righthand Traversal Rule* or *Lefthand Traversal Rule* in the *Recovery Mode*: A robot first greedily moves to its goal position in *Forwarding Mode*. When meeting a wall, it switches to move in *Recovery Mode* by following either *Righthand Traversal Rule* if its goal position is on the right side of its current position or Move in *Recovery Mode* by following *Lefthand Traversal Rule* if its goal position is on the left side of its current position to avoid walls. When arriving in a position that is closer to its goal position than the start of *Recovery Mode*, the robot switches back to *Forwarding Mode*. A robot can also record positions of walls Ω_r sensed during moving and update $\mathcal{W}$ as $\mathcal{W} \cup \Omega_r$.

[5]An element in $\mathcal{S}$ is denoted as $(i, j, \gamma_{i,j})$, where i is the transmitter's position, j is the receiver's position and $\gamma_{i,j}$ is the corresponding SNR of link (i, j). An element is $\mathcal{W}$ is a detected wall position.

Algorithm 2 UpdateGraph($\mathbb{G}(\mathcal{V}, \mathcal{E})$, $\mathcal{S}$,$\mathcal{W}$)

for each directed edge $(i, j) \in \mathcal{E}$ **do**
    if $(i, j, \gamma_{i,j}) \in \mathcal{S}$ [5] **then**
        Update the egde weight as $l_{i,j} = f(\gamma_{i,j})$
    else
        Predict $n_{i,j}$ according to $\mathcal{W}$
        Calculate $\gamma_{i,j}$ according to (1) and (2)
        Update the edge weight as $l_{i,j} = f(\gamma_{i,j})$
    end if
end for
return $\mathbb{G}$

Algorithm 3 MeasureSNR(p^*)

for each reciever robot k on p^*, where $k \in \{2, \ldots, m\}$ **do**
    Measure its SNR with transmitter robot $k-1$: $\gamma_{k-1,k}$
    $\mathcal{S} \leftarrow \mathcal{S} \cup (x^{k-1}, x^k, \gamma_{k-1,k})$
end for
return $\mathcal{S}$

Algorithm 4 SetUtility(p^*,$\mathcal{S}$)

for each communication link (x^{k-1},x^k) along path p^*, where $k \in \{2, \ldots, m\}$ **do**
    Find its corresponding $\gamma_{k-1,k}$ from $\mathcal{S}$
    Update edge weight as $l_{k-1,k} = f(\gamma_{k-1,k})$
end for
$U^* \leftarrow \underset{(x^1,\ldots,x^m)=p^*}{\mathcal{U}} (l_{1,2}, \ldots, l_{m-1,m})$
return U^*

Algorithm 5 ProbeWall(p^*)

for each robot k on p^*, where $k \in \{1, \ldots, m\}$ **do**
    Send a probing signal to neighbor robots respectively
    Detect the position of closest wall w_d that reflects probing signal back to robot k: $\mathcal{W} \leftarrow \mathcal{W} \cup \{w_d\}$
end for
return $\mathcal{W}$

V. Case Study I: Finding Minimized ETX Path

One commonly used metric to measure the link quality is the expected number of transmissions per successfully delivered packet (ETX), which can be modeled as the inverse of the successful packet transmission rate $\lambda_{i,j}$ over a link (i, j). Once specifics of a communication system (modulation and coding scheme, etc.) are fixed, $\lambda_{i,j}$ can typically be expressed in terms of either a *Q function* or an exponential function of $\gamma_{i,j}$ [17]. We use *Q function* as an example here, and the corresponding successful packet transmission rate of link (i, j) is

$$\lambda_{i,j} = (1 - Q(\sqrt{c\gamma_{i,j}}))^h \qquad (5)$$

where h is the length of a packet and c is some positive constant.

And the corresponding link quality, defined as $-ETX$, is

$$l_{i,j} = \frac{-1}{(1 - Q(\sqrt{c\gamma_{i,j}}))^h} \tag{6}$$

The ETX of a path is the summation of all link ETXs, and our goal is to let robots form a communication path between the source and destination which has the minimized ETX or, equivalently, maximized utility:

$$\max_{(x^1,\ldots,x^m)\in\mathcal{P}} \sum_{k=2}^{m} l_{k-1,k} \tag{7}$$

The optimal configuration of robots can be found by applying LEONA with the $FindPath(\mathbb{G})$ implemented as running a *Shortest Path Algorithm* with a constraint that the total number of hops along a path is at most $m-1$ if the weight of each edge is set to the associated ETX on the current graph $\mathbb{G}$.

A. Analysis of the Sufficient Searched Area

As the optimality of the LEONA is guaranteed, one additional question to ask is: How much space needs to be explored in order to find an optimal configuration?

The quality of a communication path is affected by signal attenuation. In an obstructed environment, singnal attenuation is caused by two factors: the travelling distance of the signal and the number of walls affecting it. If there are no obstacles in the environment, according to [9], the optimal configuration is to place relay robots (may not use all of them) equally spaced along the straight line connecting the source and destination. This is also where LEONA starts after the first run of $FindPath(\mathbb{G})$. In an environment containing obstacles, if relay robots are still equally spaced along the straight line, there might be walls between a communication pair, which causes additional signal attenuation. The attenuation caused by walls gives incentive to robots to move away from the straight line and search for paths that may have larger distance attenuation but less wall attenuation. However, as signal attenuation caused by distance increases while robots move away, it might become so large that even though there is no wall along a path, the signal attenuation merely caused by distance makes the performance of the path worse than that of the straight-line path. Therefore, there is no need for robots to move and explore even further away, and this partial exploration is sufficient enough for robots to find the optimal configuration.

Assume the distance and the number of walls between the source robot 1 and destination robot m are denoted as d and n respectively, then the following theorem provides an upper bound of the sufficient searched area.

***Theorem** 2:* When applying LEONA, the size of the sufficient searched area $\mathcal{A}$ that guarantees robots to find the optimal configuration with minimized ETX is

$$\mathcal{A} = O(10^{n\frac{W}{5\eta}} d^2) \tag{8}$$

Further, if the distribution of n walls along the straight line connecting the source and destination allows each communication pair to have same number of walls when robots are evenly spaced along the straight line, the sufficient searched area can reduce to $O(10^{\lceil\frac{n}{m-1}\rceil\frac{W}{5\eta}} d^2)$.

***Remark** 2:* 1) The sufficient searched area scales polynomially with the distance between the source and destination and exponentially with the number of walls along the straight line connecting them. But any other wall that is not along the straight line has no effect on the size of the sufficient searched area; 2) When the distance and the number of walls along the straight line are fixed, the distribution of walls plays a crucial role in the size of searched area. The searched area becomes large when walls gather close to each other; And it becomes small when walls separately locate along the straight line; 3) Given an unknown environment, one possible way to reduce the size of searched area is to send more robots in the space which can provide a better chance to have walls separated among communication pairs, which suffers less signal attenuation and thus have a better ETX.

Before proving Theorem 2, let us simplify equation (5) first. According to [9], the packet reception rate of a link (i,j) can be finely approximated as a sigmoidal function of distance $\lambda_{i,j} = 1 - \frac{1}{1+e^{-\alpha(d_{i,j}-\beta)}}$, where $\alpha, \beta \in \mathcal{R}^+$ are shape and center parameters depending on the communication range and variance of the environmental fading. Figure 1 illustrates how good the approximation is as we compared the sigmoidal function with the packet reception rate function derived directly from either a Q function or an exponential function.

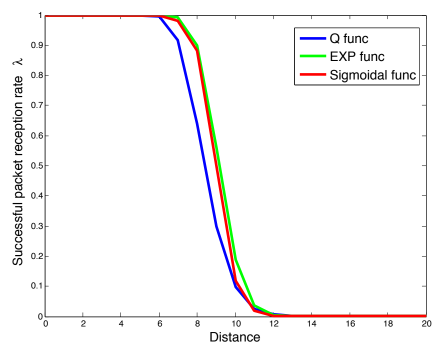


Fig. 1: Comparisons of $\lambda_{i,j}$ derived from sigmoidal, Q and exponential functions

When link (i,j) suffers from wall attenuation, the wall attenuation can be equivalently converted to distance attenuation. According to (1) and (2), the received SNR at robot j is $\frac{10^{\frac{P_0}{10}}}{(\frac{d_{i,j}}{d_0})^\eta 10^{\frac{n_{i,j}W}{10}} N_0 B}$, where $n_{i,j}$ is the number of walls along the link. Consider an equivalent case where there are no walls between robots i and j but the signal attenuation is the same as the obstructed case. Let the distance of robots i and j free from walls be D, then the received SNR at robot j is $\frac{10^{\frac{P_0}{10}}}{(\frac{D}{d_0})^\eta N_0 B}$, which is equal to the received SNR in the obstructed case. Thus, the distance of a wall-free link with same received SNR (or signal attenuation) can be expressed as a function of $n_{i,j}$ and $d_{i,j}$:

$$D(n_{i,j}, d_{i,j}) = 10^{\frac{n_{i,j}W}{10\eta}} d_{i,j} \tag{9}$$

By converting wall attenuation to distance attenuation from (9), the packet reception rate of link (i, j) in the obstructed case can be expressed as

$$\lambda_{i,j} = 1 - \frac{1}{1 + e^{-\alpha(d_{i,j} 10^{\frac{n_{i,j} W}{10\eta}} - \beta)}} \tag{10}$$

And the corresponding ETX of the link becomes

$$\omega_{i,j} = \frac{1}{\lambda_{i,j}} = 1 + e^{\alpha(d_{i,j} 10^{\frac{n_{i,j} W}{10\eta}} - \beta)} \tag{11}$$

Now, let us move on to analyze the scenario of m robots in an obstructed environment. We consider placing all $m-2$ relay robots equally spaced along the straight line connecting source and destination as a benchmark case, and without loss of generality, assume robots can locate at positions that are occupied by walls. If the distance and the number of walls between source and destination are fixed, how distribution of walls affects ETX is shown in the following lemma.

Lemma 1: In the benchmark scenario, given a fixed flow distance and a fixed number of walls causing signal attenuation to the flow, the distribution of walls that maximizes ETX is the one that all walls gather together within one communication link, and the distribution that minimizes ETX is the one that walls evenly distributed among communication links.

Proof: Let $n_{k-1,k}$ denote the number of walls between robots x^{k-1} and x^k, where $2 \le k \le m$. As the distance between each two adjacent robots is fixed as $\frac{d}{m-1}$ in the benchmark case, the total ETX of the flow is

$$\sum_{k=2}^{m} (1 + e^{\alpha(\frac{d}{m-1} 10^{\frac{n_{k-1,k} W}{10\eta}} - \beta)}) \tag{12}$$

which is a convex function of $m-1$ variables $n_{k-1,k}$ $(2 \le k \le m)$.

First, let us consider the minimization problem and for simplicity, we use y_{k-1} to represent $n_{k-1,k}$ $(2 \le k \le m)$. Therefore, finding a walls' distribution that minimizes the ETX is equivalent to solving the following convex optimization problem:

$$\begin{aligned} \min_{\mathbf{y}} \quad & g(\mathbf{y}) = \sum_{i=1}^{m-1} (1 + e^{\alpha(\frac{d}{m-1} 10^{\frac{y_i W}{10\eta}} - \beta)}) \\ \text{s.}t. & \sum_{i=1}^{m-1} y_i = n \\ & y_i \ge 0, \quad \forall 1 \le i \le m-1 \end{aligned} \tag{13}$$

where $\mathbf{y} = (y_1, \ldots, y_{m-1})$ is the allocation of walls among links.

From *KKT Conditions* [19], we have $\forall i \in \{1, \ldots, m-1\}$ $\frac{\partial g(\mathbf{y})}{\partial y_i} = \mu$, where μ is the Lagrange multiplier. Therefore, the optimal solution is to set $y_i = \frac{n}{m-1}, \forall i \in \{1, \ldots, m-1\}$.

Next, let us focus on the maximization problem and we prove the result by induction. Consider the following problem in which it is equivalent to the maximization problem when $k = m-1$.

$$\begin{aligned} \max_{\mathbf{y}} \quad & g_k(\mathbf{y}) = k + e^{-\alpha\beta} \sum_{i=1}^{k} e^{\frac{\alpha d}{m-1} 10^{\frac{y_i W}{10\eta}}} \\ \text{s.}t. & \sum_{i=1}^{k} y_i = n \\ & y_i \ge 0, \quad \forall 1 \le i \le k \end{aligned} \tag{14}$$

When $k = 2$, we have $g_2(\mathbf{y}) = 2 + e^{-\alpha\beta} \sum_{i=1}^{2} e^{\frac{\alpha d}{m-1} 10^{\frac{y_i W}{10\eta}}}$ with $y_1 + y_2 = n$. As $g_2(\mathbf{y})$ is a convex confuction, its maximum achieves at one of its boundary points, either $(n, 0)$ or $(0, n)$.

Assume, $\forall k \le K$, $g_k(\mathbf{y})$ is maximized when one variable takes its value as n and all others are 0. Then when $k = K + 1$, we have $g_{K+1}(\mathbf{y}) = K + 1 + e^{-\alpha\beta} \sum_{i=1}^{K+1} e^{\frac{\alpha d}{m-1} 10^{\frac{y_i W}{10\eta}}}$ where $\sum_{i=1}^{K+1} y_i = n$. Then, we have

$$\begin{aligned} \max_{\mathbf{y}} \quad & g_{K+1}(\mathbf{y}) \\ = & \max_{0 \le y_{K+1} \le n} \{1 + e^{-\alpha\beta} e^{\frac{\alpha d}{m-1} 10^{\frac{y_{K+1} W}{10\eta}}} \\ & + \max_{\sum_{i=1}^{K} y_i = n - y_{K+1}} \{K + e^{-\alpha\beta} \sum_{i=1}^{K} e^{\frac{\alpha d}{m-1} 10^{\frac{y_i W}{10\eta}}}\}\} \\ = & \max_{0 \le y_{K+1} \le n} \{1 + e^{-\alpha\beta} e^{\frac{\alpha d}{m-1} 10^{\frac{y_{K+1} W}{10\eta}}} \\ & + K + e^{-\alpha\beta}((K-1) e^{\frac{\alpha d}{m-1}} + e^{\frac{\alpha d}{m-1} 10^{\frac{(n - y_{K+1}) W}{10\eta}}})\} \end{aligned} \tag{15}$$

where the last equality comes from the fact that $K + e^{-\alpha\beta} \sum_{i=1}^{K} e^{\frac{\alpha d}{m-1} 10^{\frac{y_i W}{10\eta}}}$ under the condition $\sum_{i=1}^{K} y_i = n - y_{K+1}$ takes its maximum when one variable is $n - y_{K+1}$ and all the others are 0. Because of the convexity property, equation (15) achieves its maximum when y_{K+1} equals either 0 or n, which means when $k = K + 1$, $g_{K+1}(\mathbf{y})$ achieves its maximum if one variable equals n, and all the others are 0. ■

When running LEONA, after the first run of $FindPath(\mathbb{G})$ with the communication graph $\mathbb{G}$ constructed under the assumption the environment is wall-free, robots move to be evenly spaced along the straight line connecting the source and destination. And if this is not the optimal configuration, robots' exploration starts from here and expands outwards. If the area robots have searched is large enough such that any communication path even without wall attenuation outside the searched area has a worse ETX than that in the evenly-spaced-along-the-straight-line benchmark case, there is no need to search further and the optimal communication path is guaranteed to be found in the searched area. According to [9], it may be the case that not all relay robots are needed to form the path. However, the benchmark case can give a no better ETX than the optimal case, and thus, the sufficient searched area is still large enough to guarantee to find the optimal configuration. Therefore, based on the benchmark case and Lemma 1, Theorem 2 can be proved by considering the size of sufficient searched area in two edge cases and for any other case, the searched area size falls in between:

Proof: On one hand, where n walls gather together between one communication link that yields worst ETX, the corresponding ETX is $(m-2)(1+e^{\alpha(\frac{d}{m-1}-\beta)})+1+e^{\alpha(\frac{d}{m-1}10^{\frac{nW}{10\eta}}-\beta)}$. Consider a wall-free path of length D whose ETX can be expressed as $(m-1)(1+e^{\alpha(\frac{D}{m-1}-\beta)})$. When $D \geq d10^{\frac{nW}{10\eta}}$, even though it is wall-free, the ETX of the free path is still no better than the obstructed case where walls gather together. Thus, the sufficent explored area is $O(D^2)$. Therefore,the size of the sufficiently large exploration area to find an optimal solution in this case is $\mathcal{A} = O(10^{n\frac{W}{5\eta}}d^2)$, which is also an upper bound for the size of the sufficient searched area .

On the other hand where walls are equally distributed among all communication links which gives best ETX, the corresponding ETX is $(m-1)e^{\alpha(\frac{d}{m-1}10^{\lceil\frac{n}{m-1}\rceil\frac{W}{10\eta}}-\beta)}$ considering the number of walls is an integer between each communication link. Similarly, when a free path with distance satisfying $D \geq d10^{\lceil\frac{n}{m-1}\rceil\frac{W}{10\eta}}$, the corresponding ETX is no less than the benchmark case in the obstructed space. Thus the sufficient explored area is $\mathcal{A} = O(10^{\lceil\frac{n}{m-1}\rceil\frac{W}{5\eta}}d^2)$. Further if $m = O(n)$, the explored area decreases to $\mathcal{A} = O(d^2)$, which means the size of searched area only depends the distance between the source and destination. ■

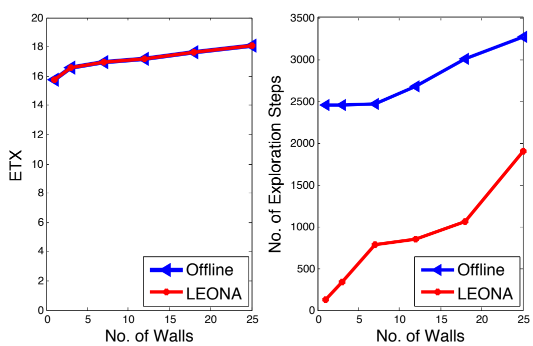
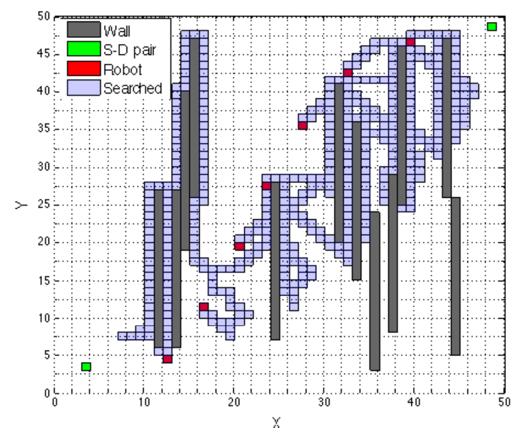


Fig. 2: ETX (left) and moving steps (right)

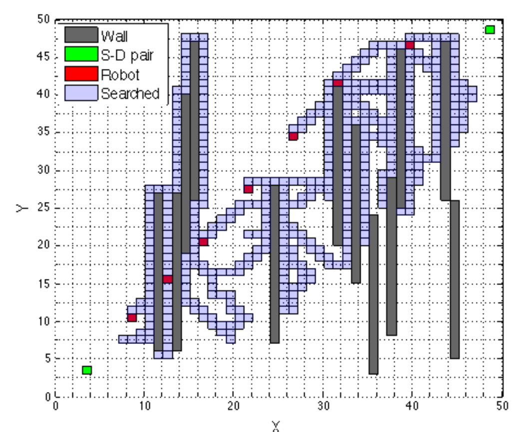
B. Simulation Results

We present numerical simulation results for a network containing 10 robots in a 50×50 environment. The robot 1 and robot 10, working as the source and the destination, are statically located at $(3,3)$ and $(48,48)$ respectively. The rest are mobile robots moving around with the purpose of formulating a communication path between the source and destination with minimized ETX. The initial locations of mobile robots are set to be as equally spaced as possible along the straight line connecting the source and destination. We use $P_0 = -20$, $d_0 = 1$, $\eta = 3.3$, $W = 20$, $N_0 = 10^{-14}$, and $B = 2 \times 10^6$. The packet reception rate of a link (i,j) is set according to eqn (10) with $\alpha = 0.2$ and $\beta = 8$.

We fix the shape and size of walls, and use the number of walls as an indicator of the complexity of an unknown environment. We randomly add walls in the space and the number of walls is varied as shown in Figure 2. We compare LEONA with an *Offline Algorithm*, which is guaranteed to find the optimal configuration. In the *Offline Algorithm*, robots first fully explore the environment to find the prior mapping of the area and build the corresponding communication graph $\mathbb{G}$. Then they apply *Shortest Path Algorithm* to find the optimal configuration. In addition to presenting the ETX of the optimal path, we also present the total number of moving steps robots take before finding the optimal path, which serves as an indicator of searched area size. As can be seen in the Figure 2, LEONA can always find optimal configurations; And as the number of walls (or the complexity of the environment) increases, the number of steps taken during exploration increases under both schemes, but LEONA takes far less.



(a) Configuration with strong wall attenuation ($W = 20$)



(b) Configuration with weak wall attenuation ($W = 3.3$)

Fig. 3: Illustration of robot configurations

In Figure 3, we present the optimal robot configurations obtained in an environment with 12 walls with single wall attenuation effect $W = 20$ in (a) and $W = 3.3$ in (b). In

Figure 3 (a), wall attenuation is high so that the optimal configuration is a path free from walls, along which robots are roughly evenly spaced. In Figure 3 (b) the optimal path still has walls affecting it, since the wall attenuation is small which does not cause robots to move away from the straight line. However, robots are no longer evenly spaced and those communication robot pairs suffering from wall attenuation compensate by having shorter link distance. As can be seen from both cases, robots only explore part of the environment before finding the optimal configuration. One thing needs to be noted is that due to the fact that the utility in the ETX case is a unimodal (increasing-then-decreasing) function of the number of robots, not all available robots are always required in the optimal path configuration; though, generally, more robots are needed as the number of walls increases.

VI. Case Study II: Finding Maximized Transmission-Rate Path

Another important metric to measure link quality is transmission rate. From the classic Shannon-Hartley Formula, the transmission rate of a link (i, j) is a function of the SNR at receiver j corresponding to transmitter i:

$$l_{i,j} = Blog(1 + \gamma_{i,j}) \tag{16}$$

The transmission rate of a communication path formed by multiple links is determined by the transmission rate of its bottleneck link. Thus, to have a maximized transmission rate between the source and destination, the $m-2$ mobile robots need to form a transmission path which has a maximized bottleneck rate. Therefore, the objective function in (4) is instantiated as

$$\max_{(x^1,\ldots,x^m)\in\mathcal{P}} \min(l_{1,2},\ldots,l_{m-1,m}) \tag{17}$$

The optimal configuration of robots in this case can be found by LEONA with the $FindPath(\mathbb{G})$ implemented as *Widest Path Algorithm* [18] with a constraint that the total number of hops along a path is at most $m-1$.

A. Analysis of the Sufficient searched Area

In the case of finding maximized transmission-rate path, we have the same sufficient searched area result:

***Theorem** 3:* When applying LEONA, the size of the sufficient searched area $\mathcal{A}$ that guarantees robots to find the optimal configuration with maximized transmission rate is

$$\mathcal{A} = O(10^{n\frac{W}{5\eta}} d^2) \tag{18}$$

Further, if the distribution of n walls along the straight line connecting the source and destination allows each communication pair to have same number of walls when robots are evenly spaced along the straight line, the sufficient searched area can reduce to $O(10^{\lceil\frac{n}{m-1}\rceil\frac{W}{5\eta}} d^2)$.

Proof: Similarly, we still consider placing robots equally spaced along the straight line connecting source and destination as a benchmark case. Since the distance between any two communication robots is $\frac{d}{m-1}$, the bottleneck link, which determines the transmission rate of the benchmark path, is the communication pair which has most walls between them. The more walls between the bottleneck communication robots' pair, the less the transmission rate of the benchmark path is. Let the number of walls affecting the bottleneck link be denoted as n_b. According to equation (9), the equivalent distance of the bottleneck communication pair free from walls is $D(n_b, \frac{d}{m-1})$. Thus, the size of the sufficiently large exploration area to find an optimal solution is $\mathcal{A} = O(10^{n_b\frac{W}{5\eta}} d^2)$. On one hand, if all the walls gathers in the bottleneck link, n_b takes its maximum as n, which indicates the walls' attenuation is maximized. And the size of the explored area becomes $\mathcal{A} = O(10^{n\frac{W}{5\eta}} d^2)$. On the other hand, if walls are evenly distributed among communication robots' pairs, n_b takes its minimum as $\lceil\frac{n}{m-1}\rceil$, which indicates the walls' attenuation is minimized. And the size of the explored area becomes $\mathcal{A} = O(10^{\lceil\frac{n}{m-1}\rceil\frac{W}{5\eta}} d^2)$. Additionally, if $m = O(n)$, the explored area decreases to $\mathcal{A} = O(d^2)$. ■

B. Simulation Results

We conduct simulations in the same network scenario as Case Study I. The *Offline Algorithm* is the same as Case Study I with only one difference that robots apply *Widest Path Algorithm* to find the optimal configuration. As seen in Figure 4, similar results are found: LEONA can always find the optimal configuration and takes less amount of movements and explorations than the offline scheme. We also present robot configurations in an environment with 12 walls with single wall attenuation $W = 20$ in Figure 5 (a) and $W = 3.3$ in Figure 5 (b). In the strong wall attenuation case, robots form a wall-free optimal path on which they are roughly evenly spaced. However, in the weak wall attenuation case, the optimal path still has walls along it where communication pairs suffering from wall attenuation compensate by having shorter link distance. One difference from Case Study I is that since the maximized transmission rate of a path increases as the number of robots increases, thus, all 9 relay robots are required in the optimal configuration.

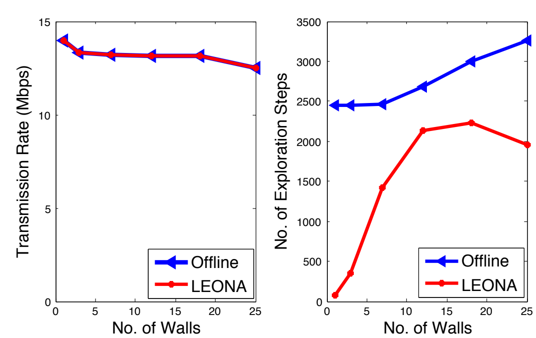
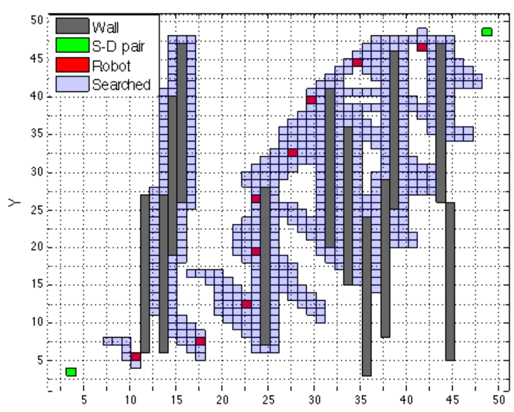


Fig. 4: Transmission rate (left) and moving steps (right)

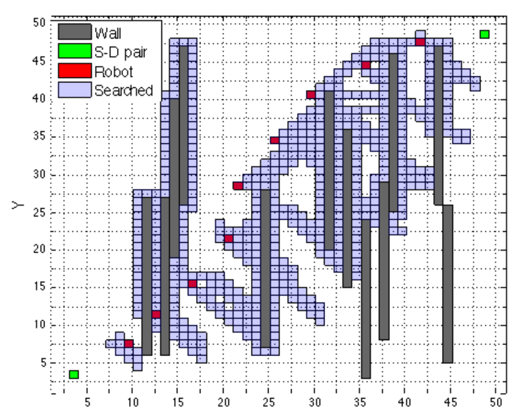
VII. Summary and Future Work

We have shown in this work how the adoption of an iterative online search combined with a graph based approach can allow for the formation of optimal robotic network configurations in unknown environments with obstructions. We have illustrated our general LEONA framework with two specific case studies. It is straightforward to incorporate many other utility functions and constraints into this framework. In this work, we assumed a simple path loss model with wall attenuation in order to make and update predictions — the framework is flexible enough to accommodate other models / approaches to prediction. The only hard requirement is that at each step an optimistic estimate be generated.

For many path and network optimization problems such as the ones considered in this study, it is possible to obtain a global solution in polynomial time. So long as the network of robots moves in such a way as to ensure connectivity is maintained at all times, they can exchange their measurements in an online fashion, and the predicted graph can be updated in a consistent manner by all robots in the network allowing them to each compute the optimal predicted location for themselves in a parallel fashion. It may be possible to adopt and interleave more sophisticated message passing mechanisms with the iterations of the online algorithm to further improve the robustness and efficiency of the system. We leave the design of such enhancements as open problems to be considered in future work.



(a) Configuration with strong wall attenuation ($W = 20$)



(b) Configuration with weak wall attenuation ($W = 3.3$)

Fig. 5: Illustration of robot configurations

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